DEPARTMENT OF EDUCATION, ARTS AND SCIENCE

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THE TEACHING OF MATHEMATICAL SUBJECTS IN SOUTH AFRICA

A. J. VAN ROOY

1965

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NATIONAL BUREAU OF EDUCATIONAL AND SOCIAL RESEARCH

THE TEACHING OF

MATHEMATICS, GENERAL MATHEMATICS AND ARITHMETIC

IN PUBLIC SECONDARY AND HIGH SCHOOLS

FOR WHITES IN SOUTH AFRICA

A. J. VAN ROOY

1965

PREFACE

The teaching of the mathematical subjects is attracting attention to an ever increasing extent. The textbook, the method of teaching, the syllabi and the training of Mathematics teachers are constantly under discussion.

When the South African Mathematical Association approached the National Bureau of Educational and Social Research with the request that the whole matter should be investigated, it was decided to determine in the first place what the existing position was with regard to the teaching of these subjects at high school. This report is the result of the investigation. The last chapter contains certain recommendations in connection with possible future procedure.

I would like to thank all those who had a share in this investigation; the members of the Advisory Committee, members of the staff of the Bureau, many lecturers, principals and teachers.

Two of our most faithful co-workers, Prof. D.J. van Rooy and Mr. A.J. Botha, passed away shortly before the publication of this report. Their contributions to the report are acknowledged with gratitude.

P.M. ROBBERTSE
DIRECTOR

NATIONAL BUREAU OF EDUCATIONAL AND SOCIAL RESEARCH

PRETORIA,

1st February, 1965.

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CHAPTER 1

INTRODUCTION

1.1 THE INVESTIGATION

1.1.1 The reason for the investigation

On the 8th September, 1959, the Director of the National Bureau of Educational and Social Research received a letter dated 4th September, 1959, from Dr. J. van der Mark, Secretary of the South African Mathematics Association. According to the letter, the Executive Committee of the Association was of the opinion that the teaching of Mathematics at high schools should be thoroughly investigated and it strongly recommended that this be done. The Executive Committee of the Mathematics Association was prepared to assist such an investigation by word and deed.

1.1.2 Approval for the investigation to be conducted

On 19th May, 1961, the Secretary of Education, Arts and Science approved the appointment of an Advisory Committee consisting of representatives of the Departments of Education (excluding Bantu Education), the Committee of University Principals, the Federal Council of Teachers' Associations, the Mathematical Association, the Statistical Association and the Joint Council of Professional Engineers. The various bodies were requested to send representatives at their own expense to the meeting of the Committee.

1.1.3 The course of the investigation

On the 25th October, 1961, the Committee met in the Oranje-Nassau Building under the chairmanship of Dr. P.M. Robbertse, Director of the National Bureau of Educational and Social Research.
The Committee consisted of the following persons:

1. Dr. P.M. Robbertse, Director, National Bureau of Educational and Social Research (Chairman).
2. Mr. J.S. de Waal representing the Department of Education, Natal.
3. Mr. J.T. van Wyk representing the Department of Education, S.W.A.
4. Mr. G.L. Edwards representing the Department of Education, Transvaal.
5. Mr. J. Wilkinson representing the Department of Education, O.F.S.
6. Mr. J.J. Dreyer representing the Department of Public Education, Cape of Good Hope.
7. Mr. D.H. Rickett representing the Department of Education, Arts and Science.
8. Prof. A.J. Ockleston representing the Joint Council of Professional Engineers.
9. Dr. D.J. Stoker representing the South African Statistical Association.
10. Prof. J.H. van der Merwe representing the South African Mathematical Association.
11. Dr. J. van der Mark representing the South African Mathematical Association.
12. Prof. D.J. van Rooy representing the Joint Matriculation Board.

13. Prof. P.J. Zietsman representing the Committee of University Principals.

14. Dr. T.F. Gevers representing the Federal Council of Teachers' Associations in South Africa.

Advisory:

15. Dr. C.W. Wright, National Bureau of Educational and Social Research.

16. Mr. W. Verhoef, National Bureau of Educational and Social Research.

17. Mr. J.B. Haasbroek, National Bureau of Educational and Social Research.

18. Dr. A.J. van Rooy, Project Leader.

During 1963 Prof. W.P. Robbertse was nominated in the place of Prof. D.J. van Rooy as representative of the Joint Matriculation Board since Prof. van Rooy's term of office as member of the Board had elapsed. At the request of the Director of the National Bureau of Educational and Social Research, Prof. van Rooy agreed to remain a member of this committee in his personal capacity.

The following acted as co-workers: Prof. A.P. Malan, Prof. H.S. Steyn, Miss S. van Schouwenburg, Miss J.J. Bodenstein, Mr. A.J. Botha, Mr. A.J. Dekker and Mrs. A.E. Thomas.

The Committee decided to determine what the present position was in regard to the teaching of Mathematics in South Africa. For this purpose four questionnaires were compiled and sent to schools, universities, teachers training colleges and technical colleges during June, 1962.

The following four questionnaires were sent to the bodies mentioned:

Questionnaire 375: Principals of secondary and high schools for Whites.

Questionnaire 376: The following departments of the universities: Mathematics and Applied Mathematics, Mathematical Statistics, Physics, Land Surveying, Astronomy and Engineering.

Questionnaire 377: Teachers of Mathematics, General Mathematics and Arithmetic.

Questionnaire 378: Lecturers in the Methods of the teaching of Mathematics.

After the completed questionnaires had been returned and checked, the data were summarized.

1.2 THE AIMS OF THE INVESTIGATION

The teaching of Mathematics is dependent upon the variety of factors which function as a whole. The aim of this survey was to analyse the various factors and the part played by them in the teaching of the subject.

The factors for analysis were classified as follows:

1.2.1 An analysis of the educational value of Mathematics and the aims in the teaching of Mathematics

1.2.2 An analysis of the Mathematics syllabi according to the various Departments of Education in respect of the following aspects

(a) Whether the syllabi are adapted to the latest developments in the technical field and whether the syllabi are overloaded with obsolete subject matter;

(b) whether uniformity exists among the syllabi of the various provinces/departments of education;

(c) whether it is possible to amend the syllabi in such a manner that the gap between the high school and university can be reduced; and

(d) whether the syllabi of the respective departments of education are of such a nature that the aims and envisaged educational value of the subject can be achieved by means of them.

1.2.3 A survey of the various Mathematics textbooks used in high schools

1.2.4 A survey of the academic training of the staff teaching Mathematics at high schools in order to determine the following:

(a) The academic courses in Mathematics and related subjects such as Mathematical Statistics, Statistics, Applied Mathematics, in which the teachers concerned have passed.

(b) Whether the teacher is interested in broadening his knowledge of the subject by means of further study.

(c) The technical literature read by the teachers concerned.

1.2.5 A survey of the professional and academic training received by prospective Mathematics teachers at the various training institutions

(a) Does the Method of Mathematics, as offered by lecturers at universities and teachers colleges, serve the purpose of producing thoroughly trained Mathematics teachers?

(b) Are the following aspects dealt with completely:-

(i) The contents of the syllabus.

(ii) The various teaching methods with reference to tuition in elementary Mathematics, as well as the use of modern aids.

(iii) The history, educational value and objectives of teaching Mathematics.

1.2.6 An analysis of the teaching methods applied by teachers in the teaching of Mathematics

An analysis of the following aspects is essential:-

(a) The role of "drilling" in the teaching process in order to meet the requirements of the examination, and the extent to which it results in only the knowledge required for the examination being imparted to the pupils.

(b) The use and value of aids in the teaching of the subject.

(c) The importance of homework, revision and tests in the teaching of Mathematics.

(d) The methods used in the teaching of Mathematics and the possible influence of such methods.

(e) The extent to which integration of the various fields of Mathematics is desirable.

(f) The extent to which self-activity of pupils should be encouraged.

1.2.7 An investigation of possible facilities to promote the teaching of Mathematics

Initially the following may be considered as possible facilities

(a) Compulsory courses in Mathematics for members of the staff not qualified in that subject.

(b) Compulsory refresher courses for Mathematics teachers.

(c) The publication and distribution of literature.

(d) Periodicals for the discussion of the methods of teaching as well as the latest developments in the field of Mathematics.

(e) The publication of articles on the methodology and development of the subject in the existing journals of the teachers' associations.

1.3 PRELIMINARY REPORTS

1.3.1 The object of the preliminary reports

After the statistical processing of data was completed, a series of confidential preliminary reports was sent to members of the committee for their comments. The contribution made in this manner by members of the commitee, to a very large extent determined the spirit and direction of the final report.

1.3.2 The reports

The following reports have appeared -

(1) Office report 1963/18: The teaching of Mathematics: A summary of the replies to a questionnaire sent to university lecturers.

(2) Office report 1963/19: A survey of the teachers of Mathematics, General Mathematics and Arithmetic in the high and secondary schools of the Provincial Departments of Education of the Republic of South Africa.

(3) Office report 1963/24: A summary of the comments received on Office report 1963/18: The replies of university lecturers to a questionnaire.

(4) Office report 1963/30: A survey of the teachers of Mathematics, General Mathematics and Arithmetic in the vocational high schools.

(5) Office report 1963/32: Comments on Office report 1963/19: Suggestions in regard to what can be done in connection with the promotion of the teaching of Mathematics at the high schools.

(6) Office report 1963/45: Teachers who have completed at least two degree courses in Mathematics and who did not teach a mathematical subject in 1962.

(7) Office report 1963/46: The training of teachers in the mathematical subjects at high schools.

(8) Office report 1963/47: Comments on Office report 1963/19.

(9) Office report 1964/3: The views of teachers of Mathematics according to questionnaire NB.377.

(10) Office report 1964/6: A comparison of matriculation and matriculation exemption mathematics syllabuses.

1.3.3 Further reports

A concise synoptic report was submitted to the members of the Committee at the end of 1963, and after fresh comments had been received, a report was compiled for submission to the Committee meeting of 9th October, 1964.

1.4 THE QUESTIONNAIRES SENT TO THE HIGH SCHOOLS AND SECONDARY SCHOOLS

1.4.1 Questionnaires N.B.375 and N.B.377

A copy of Questionnaire N.B.375 was sent to every public high school and secondary school for Whites in the Republic of South Africa and in South West Africa. This questionnaire had to be completed by the principal of the school. A number of copies of Questionnaire N.B.377 were also included for completion by the teachers of Mathematics, General Mathematics and Arithmetic. If the school principal himself taught one or more of these subjects, he was required to complete Questionnaire N.B.377 in addition to Questionnaire N.B.375.

Owing to a shortage of copies, questionnaires were sent to only a limited number of private schools. Since the completed questionnaires which were returned could not be regarded as representative of the private schools, these schools were not taken into account. It is recommended that the co-operation of the private schools should be obtained with eventual further and follow-up investigation in order to compensate for the gap unfortunately left in this report.

It also became clear that Questionnaire N.B.375 was not suitable for the technical colleges since no provision had been made for part-time and extramural lecturers and students. The Technical Colleges are therefore not included in the data furnished in this report. In order to obtain a complete picture a supplementary investigation will therefore be necessary.

1.4.2 The schools in the Transvaal

The schools which returned completed questionnaires and the schools which failed to return theirs, were the following:

Afrikaans medium high schools which returned completed questionnaires:

1.	Afrikaans Meisies, Pretoria	2.	Afrikaans Hoër, Germiston
3.	Alberton	4.	Balfour
5.	Ben Viljoen	6.	Bergsig
7.	Brits	8.	Carletonville
9.	Carolina	10.	Christiana
11.	Delmas	12.	Die Burger, Newville
13.	Die Goudveld, Langlaagte	14.	D.F. Malan, Crosby
15.	Dr. E.G. Jansen, Boksburg	16.	Drieriviere, Vereeniging
17.	Dr. Malan, Meyerton	18.	Edenvale
19.	Elandspoort, Pretoria West	20.	Eric Louw, Messina
21.	Fakkel, Johannesburg	22.	Florida, Afrikaans
23.	Gerrit Maritz, Pretoria North	24.	Gimnasium, Potchefstroom
25.	Goedehoop, Germiston	26.	Goudrif, Germiston
27.	Hans Strijdom, Naboomspruit	28.	Heidelbergse Hoër Volkskool
29.	Helpmekaar Meisies, Johannesburg	30.	Helpmekaar Seuns, Johannesburg
31.	Hercules, Pretoria	32.	Hendrik Verwoerd, Pretoria
33.	Hoogenhout, Bethal	34.	Hugenote Meisies, Springs
35.	Hugenote Seuns, Springs	36.	Jan de Klerk, Krugersdorp
37.	J.G. Strijdom, Johannesburg	38.	Kemptonpark
39.	Kensington	40.	Klerksdorp
41.	Langenhoven, Pretoria	42.	Linden
43.	Louis Trichardt	44.	Lydenburg
45.	Lyttelton	46.	Monument, Krugersdorp
47.	Noordoosrand	48.	Nylstroom
49.	Oos-Moot, Pretoria	50.	Orkney
51.	Pietersburg	52.	Piet Potgieter, Potgietersrus
53.	Potchefstroomse Hoër Volkskool	54.	Pretoria-Noord
55.	Pretoria-Wes	56.	Riebeeck, Randfontein
57.	Rodeon, Swartruggens	58.	Roodepoort
59.	Schoonspruit, Klerksdorp	60.	Schweizer-Reneke

61. Staatspresident C.R. Swart, Pretoria
62. Stilfontein
63. Stoffberg Meisies, Brakpan
64. Stoffberg Seuns, Brakpan
65. Sundra
66. Thabazimbi
67. Transvalia, Vanderbijlpark
68. Tuine, Pretoria.
69. Vanderbijlpark
70. Ventersdorp
71. Vereeniging
72. Voortrekker, Boksburg
73. Vorentoe, Johannesburg
74. Vryburger, Primrose
75. Westonaria
76. Wonderboom, Pretoria

Afrikaans medium high schools which dit not return completed questionnaires

1. Afrikaans Seuns, Pretoria
2. Ben Vorster, Tzaneen
3. Brandwag, Benoni
4. F.H. Odendaal, Pretoria
5. Voortrekkerhoogte, Pretoria

Parallel medium high schools which returned completed questionnaires

1. Barberton
2. Belfast
3. Erasmus, Bronkhorstspruit
4. General Hertzog, Witbank
5. Lichtenburg
6. Nelspruit
7. Nigel
8. Piet Retief
9. Rob Ferreira, White River
10. Rustenburg
11. Standerton
12. Volksrus
13. Waterval-Boven
14. Zeerust

Parallel medium high schools which did not return completed questionnaires

1. Ermelo
2. Middelburg
3. Warmbad
4. Wolmaransstad

English medium high schools which returned completed questionnaires

1. Athlone Boys', Johannesburg
2. Athlone Girls', Johannesburg
3. Benoni
4. Boksburg
5. Brakpan
6. Capricorn, Pietersburg
7. Carleton Jones, Carletonville
8. Florida Park, Florida
9. Forest, Johannesburg
10. General Smuts, Vereeniging
11. Germiston Boys'
12. Germiston Girls'
13. Greenside, Johannesburg
14. Highlands North, Johannesburg
15. Hillview, Pretoria
16. Hyde Park, Johannesburg
17. Jeppe Boys', Johannesburg
18. Jeppe Girls', Johannesburg
19. Johannesburg Girls'
20. King Edward VII, Johannesburg
21. Krugersdorp
22. Malvern, Johannesburg
23. Mayfair
24. Milner, Klerksdorp
25. Northview, Johannesburg
26. Parktown Boys', Johannesburg
27. Parktown Girls'
28. Potchefstroom Boys'
29. Potchefstroom Girls'
30. Pretoria Boys'
31. Pretoria Girls'
32. Queens, Johannesburg
33. Roosevelt, Johannesburg
34. Sir John Adamson, Johannesburg
35. Springs Boys'
36. Springs Girls'
37. The Hill, Johannesburg
38. The Vaal, Vanderbijlpark
39. Waverley Girls', Johannesburg

English medium high schools which did not return completed questionnaires

1. Clapham, Pretoria
2. Dawnview, Primrose
3. Edenvale
4. Lyttelton Manor
5. Randfontein

1.4.3 Estimate for Transvaal schools

In order to obtain a picture of conditions in all the public schools, the data obtained in respect of the schools which returned completed questionnaires, were increased proportionately to make provision for all the schools. The number of pupils in the various types of schools was used as a basis for comparison. The factors used for multiplication are set out in Table 1.1.

TABLE 1.1

THE NUMBER OF PUPILS IN TRANSVAAL HIGH SCHOOLS (1962)

Type of high school	Number of pupils in schools which			Factor
	replied to questionnaires	did not reply to questionnaires	Total	
Afrikaans medium	55,083	3,832	58,915	1.070
Parallel medium	9,652	2,698	12,350	1.280
English medium	27,577	2,178	29,755	1.079
TOTAL	92,312	8,708	101,020	1.094

The completed questionnaires received represent 91.4% of the pupils in public high schools in the Transvaal. The estimate of the total should therefore differ little from the actual numbers. The "factors" were obtained by dividing the total number of pupils by the number of pupils in the schools which submitted completed questionnaires.

1.4.4 The schools in the Cape Province

A list of the public secondary and high schools is given below. It is also indicated whether completed questionnaires were returned by the schools concerned.

Afrikaans medium high schools which returned completed questionnaires

1.	Aberdeen	2.	Albertinia
3.	Aliwal North	4.	Andrew Rabie, Port Elizabeth
5.	Aurora	6.	Barkly East
7.	Barrydale	8.	Bloemhof Meisies, Stellenbosch
9.	Bonnievale	10.	Bredasdorp
11.	Burgersdorp	12.	Calvinia
13.	Carel du Toit, Steytlerville	14.	Carnarvon
15.	Charlie Hofmeyr, Ceres	16.	Cilliė, Port Elizabeth
17.	Citrusdal	18.	Clanwilliam
19.	David Ross, Lady Grey	20.	Delportshoop
21.	De Rust	22.	Despatch
23.	De Villiers Graaff, Villiersdorp	24.	De Vos Malan, King William's Town
25.	D.F. Malan, Bellville	26.	D. F. Malherbe, Port Elizabeth
27.	Diamantveld, Kimberley	28.	Die Brandwag, Uitenhage
29.	Dirkie Uys, Moorreesburg	30.	D. M. Pretorius, Noupoort
31.	Dordrecht	32.	Douglas
33.	Fort Beaufort	34.	Gimnasium Seuns, Paarl
35.	Grabouw	36.	Grens, East London
37.	Griekwastad	38.	Groblershoop
39.	Hangklip, Queenstown	40.	Hartswater
41.	Heidelberg	42.	Henvallei
43.	Hofmeyr	44.	Hope Town
45.	Jansenville	46.	Jan van Riebeeck, Cape Town
47.	Kalahari, Kuruman	48.	Kareedouw
49.	Keimoes	50.	Kenhardt
51.	Kirkwood	52.	Knysna
53.	Kraaifontein	54.	Kuilsrivier
55.	Ladismith	56.	Laingsburg

57.	Langenhoven, Riversdale	58.	La Rochelle Meisies, Paarl
59.	Lutzville	60.	Mafeking
61.	McLachlan, Joubertina	62.	Middelburg
63.	Montagu	64.	Murraysburg
65.	Namakwaland, Springbok	66.	Napier
67.	Nassau, Mowbray	68.	Nico Malan, Humansdorp
69.	Nuwerus	70.	Olifantshoek
71.	Otto du Plessis, Port Elizabeth	72.	Outeniqua, George
73.	Overberg, Caledon	74.	Paarl Meisies
75.	Paul Kruger, Steynsburg	76.	Piketberg
77.	Porterville	78.	Postmasburg
79.	Prieska	80.	Punt, Mossel Bay
81.	Reivilo	82.	Richmond
83.	Riviersonderend	84.	Robertson
85.	Rockland Meisies, Cradock	86.	Strydenburg
87.	Sutherland	88.	Swartland, Malmesbury
89.	Swellendam	90.	Tarkastad
91.	Templeton, Bedford	92.	Theron, Britstown
93.	Touwsrivier	94.	Transkei Afrikaans, Umtata
95.	Tulbagh	96.	Uniondale
97.	Upington	98.	Vaalharts
99.	Vanrhynsdorp	100.	Victoria West
101.	Volkskool, Graaff-Reinet	102.	Voortrekker, Wynberg
103.	Vredendal	104.	Vryburg
105.	Williston	106.	Willowmore
107.	Wolseley	108.	Worcester Meisies

English medium high schools which returned completed questionnaires

1.	Alexander Road, Port Elizabeth	2.	Bergvliet
3.	Butterworth	4.	Cambridge, East London
5.	Camps Bay, Cape Town	6.	Cape Town
7.	Clarendon Girls', East London	8.	Dale College Boys', King William's Town
9.	Ellerslie Girls', Cape Town	10.	Fish Hoek
11.	George Randell, Orange Grove	12.	Good Hope Seminary, Cape Town
13.	Graeme College, Grahamstown	14.	Green and Sea Point Boys', Cape Town
15.	Grey Boys', Port Elizabeth	16.	Kaffrarian Girls', King William's Town
17.	Kimberley Boys'	18.	Kimberley Girls'
19.	Muir College Boys', Uitenhage	20.	Muizenberg
21.	Observatory Boys', Cape Town	22.	Observatory Girls', Cape Town
23.	Pinelands	24.	Plumstead
25.	Queen's Park, Woodstock	26.	Rhenish Girls', Stellenbosch
27.	Riebeek College Girls', Uitenhage	28.	Rondebosch Boys', Cape Town
29.	Rustenburg Girls', Rondebosch	30.	Selborne, East London
31.	Umtata	32.	Union, Graaff-Reinet
33.	Victoria Park, Port Elizabeth	34.	Westerford, Cape Town
35.	Wynberg Girls'		

Parallel medium high schools which returned completed questionnaires

1.	Adelaide	2.	Alexandria
3.	Bellville	4.	Cathcart
5.	Daniëlskuil	6.	Durbanville

7.	Elliot	8.	Gill College Boys', Somerset East
9.	Hermanus	10.	Hottentots-Holland, Somerset West
11.	Indwe	12.	J.G. Meiring, Goodwood
13.	J.J. du Preez, Parow	14.	King Edward, Matatiele
15.	Lansdowne, Cape Town	16.	Maclear
17.	Molteno	18.	Oudtshoorn Girls'
19.	Paarl Boys'	20.	Paul Roos Gynmasium Boys', Stellenbosch
21.	Pearson	22.	Stutterheim
23.	Vredenburg	24.	Worcester Boys'
25.	Ysterplaat	26.	Zwaanswyk, Retreat

High schools which did not submit completed questionnaires

1.	Adamantia, Kimberley	2.	Bellevue Girls', Somerset East
3.	Calitzdorp	4.	Central, Beaufort West
5.	Colesberg	6.	Collegiate Girls', Port Elizabeth
7.	Cradock Boys'	8.	Franschhoek
9.	Fraserburg	10.	Garies
11.	Goodwood	12.	Goudini, Rawsonville
13.	Graafwater	14.	Groote Schuur Afrikaans-medium, Rondebosch
15.	Hopefield	16.	Huguenot, Wellington
17.	Jan Malan, Koringberg	18.	Karos-Connan, Gordonia
19.	Kokstad	20.	Lawson Brown, Port Elizabeth
21.	Maitland, Cape Town	22.	Martin Oosthuizen, Kakamas
23.	Milnerton, Cape Town	24.	Oudtshoorn Boys'
25.	P.J. Olivier, Grahamstown	26.	Queen's College Boys', Queenstown
27.	Queenstown Girls'	28.	S.A. College Boys', Newlands
29.	Sans Souci Girls', Newlands	30.	Sterkstroom
31.	Tygerberg, Parow	32.	Ugie
33.	Vaal River, Barkly West	34.	Victoria Girls', Grahamstown
35.	Warrenton	36.	Wittedrift, Knysna
37.	Wynberg Boys'		

Afrikaans medium secondary schools which returned completed questionnaires

1.	Brandvlei	2.	Bray
3.	Cookhouse	4.	Darling
5.	Greyton	6.	Grootbrakrivier
7.	Herbertsdale	8.	Kamieskroon
9.	Kanoneiland	10.	Klawer
11.	Klipplaat	12.	Lambertsbaai
13.	Loeriesfontein	14.	Loxton
15.	McGregor	16.	Merweville
17.	Nababeep	18.	Niekerkshoop
19.	Nieuwoudtville	20.	Patentie
21.	Philipstown	22.	Pofadder
23.	Redelinghuys	24.	Riebeek-Kasteel
25.	Riebeek East	26.	Stanford
27.	Stella	28.	Venterstad

English medium secondary schools which returned completed questionnaires

1.	Clifton	2.	Queen Alexandra, Port Alfred
3.	W. Myburgh, Frankfort		

Parallel medium secondary schools which returned completed questionnaires

1.	Berlin	2.	Hanover

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3.	Katberg, Balfour	4.	Komgha
5.	Peddie	6.	Riebeek West
7.	Velddrift	8.	West Bank, East London

Secondary schools which did not submit completed questionnaires

1.	Alice	2.	Cedarville
3.	Engcobo	4.	Gansbaai
5.	Hankey	6.	Jamestown
7.	Pearston	8.	Ritchie
9.	Simonstown	10.	Sondagsrivier
11.	Vanwyksdorp	12.	Vanwyksvlei
13.	Vosburg		

1.4.5 Estimates for schools in the Cape Province

It is clear that by no means all of the schools in the Cape Province replied to the questionnaires. It is not possible to estimate how many pupils and teachers there are in Afrikaans medium, English medium and parallel medium schools since the official yearbook of the Department of Public Education of the Cape of Good Hope mentions the medium of instruction only by way of exception. The estimates had therefore to be made on a more global scale. Table 1.2 shows the factors used for multiplication.

TABLE 1.2

THE NUMBER OF PUPILS IN HIGH AND SECONDARY SCHOOLS IN THE CAPE PROVINCE (1961/62)

Group of schools	Number of pupils in schools which replied to questionnaires	Number of pupils in schools which did not reply to questionnaires	Total	Factor
High schools				
Afrikaans medium	26,458			
English medium	13,087			
Parallel medium	9,283			
	48,828	9,979	58,807	1.204
Secondary schools				
Afrikaans medium	1,420			
English medium	181			
Parallel medium	484			
	2,085	532	2,617	1.255

The high schools from which completed questionnaires were received, represent 83.0% of the pupils, and the secondary schools 79.9%.

1.4.6 The schools in Natal

The high schools and secondary schools in Natal can be reported upon as follows:

Afrikaans medium high schools from which completed questionnaires were received

1.	Dirkie Uys, Durban	2.	Port Natal, Durban
3.	Stamford Hill, Durban	4.	Voortrekker, Pietermaritzburg

No questionnaire was returned from Saamwerk, Durban.

English medium high schools from which completed questionnaires were received

1.	Durban	2.	Durban Girls'
3.	Grosvenor Boys', Durban	4.	Mansfield, Durban
5.	Northlands Boys', Durban North	6.	Northlands Girls', Durban
7.	Windermere, Durban	8.	Grosvenor Girls', Durban
9.	Alexandra Boys', Pietermaritzburg	10.	Harward Boys', Pietermaritzburg
11.	Maritzburg College, Pietermaritzburg	12.	Pietermaritzburg Girls'
13.	Russell, Pietermaritzburg	14.	Eshowe
15.	Ixopo	16.	Kingsway, Amanzimtoti
17.	Queensburgh, Malvern	18.	Wartburg-Kirchdorf, Wartburg

English medium high schools which submitted no completed questionnaires

1.	Glenwood, Durban	2.	Mitchell, Durban
3.	New Forest, Durban	4.	Westville

Parallel medium high schools from which completed questionnaires were received

1.	Empangeni	2.	Estcourt
3.	Glencoe	4.	Greytown
5.	Ladysmith	6.	Newcastle
7.	Pinetown	8.	Port Shepstone
9.	Vryheid		

No questionnaire was returned by Dundee High School.

Secondary schools from which completed questionnaires were received

English medium	1.	Kloof
	2.	Margate
Parallel medium	3.	Stanger
	4.	Utrecht

1.4.7 Estimates for Natal

Table 1.3 shows the factors by which the data obtained from the questionnaires completed by Natal schools had to be multiplied in order to make an estimate for all the schools.

TABLE 1.3

THE NUMBER OF PUPILS IN HIGH AND SECONDARY SCHOOLS IN NATAL (1962)

Group of schools	Number of pupils in schools which replied to questionnaires	did not reply to questionnaires	Total	Factor
High schools				
Afrikaans medium	1,818	457	2,275	1.251
English medium	8,536	2,751	11,287	1.322
Parallel medium	3,791	428	4,219	1.113
Secondary schools				
English medium	118	0	118	1.000
Parallel medium	214	0	214	1.000
TOTAL	14,471	3,636	18,113	1.251

The schools which submitted completed questionnaires represent 79.9% of the pupils.

1.4.8 The schools in the Orange Free State

Afrikaans medium high schools from which completed questionnaires were received

1. Afrikaans, Kroonstad
2. Bothaville
3. Christelike en Nasionale Oranje Meisies, Bloemfontein
4. Gimnasium, Welkom
5. Goudveld, Welkom
6. Harrismith
7. Hennenman
8. Hertzogville
9. J.B.M. Hertzog, Bloemfontein
10. Parys
11. President Swart, Brandfort
12. Sasolburg
13. Sentrale, Bloemfontein
14. Willem Pretorius, Heilbron
15. Winburg

Parallel medium high schools from which completed questionnaires were received

1. Grey College, Bloemfontein
2. Hentie Cilliers, Virginia
3. Kroonstad
4. Voortrekker, Bethlehem
5. Wessel Maree, Odendaalsrus

High schools from which no completed questionnaires were received

1. Eunice, Bloemfontein
2. Ficksburg
3. Jim Fouché, Bloemfontein
4. Ladybrand
5. Model, Bloemfontein
6. Vrede
7. Welkom English Medium

Schools with secondary and primary departments from which completed questionnaires were received

Afrikaans medium

1. Ben Havemann
2. Boshof

3.	Chris van Niekerk, Vredefort	4.	Clocolan
5.	Dealesville	6.	Edenburg
7.	Emily Hobhouse	8.	Excelsior
9.	Fouriesburg	10.	Hendrik Potgieter, Reddersburg
11.	Hoopstad	12.	J.A. Malherbe
13.	Jagersfontein	14.	Lindley
15.	Marquart	16.	Petrusburg
17.	President Steyn, Bloemfontein	18.	Retief, Kestell
19.	Rouxville	20.	Shannon
21.	Springfontein	22.	Steynrus
23.	Theunissen	24.	Trompsburg
25.	Tweeling	26.	Villiers
27.	Wepener	28.	Wesselsbron
29.	Zastron	30.	Bainsvlei Junior
31.	Dr. Viljoen Junior, Bloemfontein	32.	Memel Junior
33.	Oranjeville Junior	34.	Paul Roux Junior
35.	Rosendal Junior		

Parallel medium

1.	Bultfontein	2.	Dewetsdorp
3.	Edenville	4.	Frankfort
5.	General Hertzog, Smithfield	6.	Koffiefontein
7.	Reitz	8.	Sarel Cilliers
9.	Werda, Fauresmith	10.	Gerrit Maritz Junior, Thaba 'Nchu

Schools with secondary and primary departments from which no questionnaires were received

1.	Cornelia	2.	Dirkie Uys, Warden
3.	Hertzogville	4.	Koot Niemann
5.	M.T. Steyn, Philippolis	6.	Pellissier, Bethulie
7.	Van Kerken, Bloemfontein	8.	Ventersburg

1.4.9 Estimates for schools in the Orange Free State

It was not always possible to determine from the departmental data whether a school was an Afrikaans medium or a parallel medium school. For that reason it was necessary to make use of grouping.

Table 1.4 shows the factors used for multiplying in order to make an estimate for all the schools, including those which did not submit completed questionnaires.

TABLE 1.4

THE NUMBER OF HIGH SCHOOL PUPILS IN THE SCHOOLS OF THE ORANGE FREE STATE (1962)

Group of schools	Number of pupils in schools which replied to questionnaires	did not reply to questionnaires	Total	Factor
High schools				
Afrikaans medium	6,118	-		
Parallel medium	3,163	-		
Total	9,281	-		
English and Afrikaans medium	-	2,720	12,001	1.293
Secondary departments and combined high and primary schools				
Afrikaans medium	3,926			
Parallel medium	1,304			
Total	5,230			
English and Afrikaans medium		970	6,200	1.185
GRAND TOTAL	14,511	3,690	18,201	1.254

The schools which did return completed questionnaires represent 79.7% of the pupils.

1.4.10 The schools in South West Africa

Completed questionnaires were received from the following schools:

Afrikaans medium high schools

1. Gobabis
2. Keetmanshoop
3. Otjiwarongo
4. Outjo

Afrikaans medium secondary schools

1. Tsumeb
2. Usakos
3. Walvis Bay

No completed questionnaires were received from the following schools:

High schools

1. Mariental
2. Windhoek
3. Windhoek English medium
4. Jan Möhr

1.4.11 Estimates for schools in South West Africa

Table 1.5 shows the factor to be used for multiplying in order to make an estimate for all the public schools in South West Africa.

TABLE 1.5

THE NUMBER OF HIGH SCHOOL PUPILS IN THE PUBLIC SCHOOLS OF SOUTH WEST AFRICA (1962)

Group of schools	Number of pupils in the schools which replied to the questionnaires	did not reply to questionnaires	Total	Factor
Afrikaans medium				
High schools	1,517			
Secondary schools	607			
Total	2,124			
High schools (Afrikaans and English medium)		1,565	3,689	1.737
GRAND TOTAL	2,124	1,565	3,689	1.737

Since the schools which replied to the questionnaires are representative of only 57.6% of the pupils, no estimates of the total number of pupils taking Mathematics or of the total number of Mathematics teachers were made.

1.5 QUESTIONNAIRES SENT TO THE VOCATIONAL HIGH SCHOOLS

1.5.1 Vocational high schools from which completed questionnaires were received

Housecraft high schools

1. Christiana
2. Ferdinand Postma, Potchefstroom
3. George
4. Knysna
5. President Steyn, Bethlehem
6. Zastron

Commercial and technical high schools

1. Brakpan
2. Carletonville
3. Klerksdorp
4. Randfontein, Jan Viljoen
5. Witbank

Commercial high schools

1. Afrikaans, Parktown, Johannesburg
2. Bethlehem
3. Discovery
4. Krugersdorp
5. Oudtshoorn
6. Potchefstroom
7. Rustenburg
8. Springs

Technical high schools

1. Bloemfontein
2. Drostdy, Worcester
3. Ficksburg
4. Langlaagte
5. Middelburg
6. Oudtshoorn
7. Piet Retief, Adelaide
8. Potchefstroom

9. Tom Naudé, Pietersburg
10. Uitenhage
11. Wolmaransstad

Industrial schools

1. Daeraad, Wolmaransstad
2. Dewetsdorp
3. Emmasdale, Heidelberg
4. Excelsior, King William's Town
5. George
6. George Hofmeyr, Standerton
7. J.J. Serfontein, Queenstown
8. Paarl
9. Rustenburg
10. Tempe, Bloemfontein
11. Vaal River, Standerton
12. Constantia, Retreat

Agricultural high schools

1. Clanwilliam
2. Marlow, Cradock
3. Oakdale, Riversdale
4. Mooi River
5. Jacobsdal (Farm management is a school subject)

Special schools

1. Elizabeth Conradie, Diskobolos, via Kimberley
2. Alexanderfontein, Diskobolos, via Kimberley

1.5.2 Vocational high schools from which no completed questionnaires were received

Housecraft high schools

1. Adelaide
2. Brits
3. Riebeek West

Commercial and Technical high schools

1. Alberton
2. Boksburg
3. Vanderbijlpark
4. Vereeniging

Commercial high schools

1. Belgravia, Johannesburg
2. Benoni
3. De Villiers Street, Johannesburg
4. Ermelo
5. Paarl
6. Tygerberg

Technical high schools

1. De Wet Nel, Kroonstad
2. John Orr, Johannesburg
3. Maitland
4. Rietfontein, Pretoria
5. N. Diederichs, Krugersdorp

Industrial schools

1. Oudtshoorn
2. Eendrag, Ladybrand
3. Die Vlakte, Standerton
4. J.W. Luckhoff, Heidelberg
5. Diskobolos, Kimberley

Agricultural high schools

1. Amsterdam
2. Bekker, Magaliesburg
3. Brits
4. Hoëveld, Morgenzon
5. Koos de la Rey, Sannieshof
6. Kuschke, Eerstegoud, Pietersburg
7. Merensky, Tzaneen
8. Vaalharts, Vryburg
9. Tweespruit Boys'
10. Anna Scheepers, Tweespruit

Through an oversight the Transvaal schools did not receive any questionnaires.

1.5.3 Estimates for vocational schools

The factors by which numbers of pupils must be multiplied in order to make provision for all the schools in the different groups are shown in Table 1.6.

TABLE 1.6

THE NUMBER OF PUPILS IN THE VOCATIONAL HIGH SCHOOLS (1962)

Schools for Vocational Education	Number of pupils in schools which replied to questionnaires	did not reply to questionnaires	Total	Factor
1. Housecraft High Schools	779	469	1,248	1.602
2. Commercial and Technical High Schools	1,972	1,844	3,816	1.935
3. Commercial High Schools	3,073	2,306	5,379	1.750
4. Technical High Schools	4,670	2,375	7,045	1.509
5. Industrial Schools	1,916	471	2,387	1.246
6. Agricultural High Schools	810	2,796	3,606	4.452
7. Special Schools	345	0	345	1.000
TOTAL	13,565	10,261	23,826	1.756

The schools from which completed questionnaires were received represent 56.9% of the total number of pupils. The estimates included in the report in respect of these schools must therefore be accepted with a certain amount of reservation.

1.6 THE SCHOOLS FROM WHICH QUESTIONNAIRES COMPLETED BY THE TEACHERS WERE RECEIVED. (QUESTIONNAIRE N.B. 377)

1.6.1 Schools according to the sex of the pupils

Table 1.7 reflects the number of teachers in boys' schools, girls' schools and mixed schools from which completed questionnaires were received.

TABLE 1.7

QUESTIONNAIRES RECEIVED FROM TEACHERS IN SCHOOLS CLASSIFIED ACCORDING TO THE SEX OF THE PUPILS

Type of school	Teachers: Number	Percentage
Boys' schools	267	15.5
Girls' schools	154	8.9
Schools for boys and girls	1,285	74.7
Not indicated on questionnaires	15	0.9
TOTAL	1,721	100.0

The circumstances in the Republic probably contribute to the fact that the mixed schools far exceed the others in numbers, but it also clearly indicates that the demand for co-education at schools is much greater than the requirement for separate educational facilities for boys and girls.

1.6.2 The medium of instruction

The medium of instruction in the schools from which completed questionnaires were received is shown in Table 1.8.

TABLE 1.8

QUESTIONNAIRES RECEIVED FROM TEACHERS CLASSIFIED ACCORDING TO THE MEDIUM OF INSTRUCTION IN THE SCHOOLS

Medium	Teachers	
	Number	Percentage
Afrikaans	770	44.7
English	428	24.9
Parallel	185	10.7
Dual, mainly Afrikaans	238	13.8
Dual, equally Afrikaans and English	66	3.8
Dual, mainly English	8	0.5
Other	1	0.1
Not indicated	25	1.5
TOTAL	1,721	100.0

Afrikaans medium and mainly Afrikaans medium schools constitute 58.5% of the total, while only 25.4% of the schools are either English medium or mainly English medium.

This ratio does not agree with the proportions of the two population groups.

The small rural schools are predominantly Afrikaans, while the English-speaking pupils for the most part receive their education in the large urban schools. This fact explains why the percentage of schools does not agree with the percentage of pupils. The Afrikaans Mathematics classes exist in many schools which are smaller than the average.

In addition, it should be borne in mind that many English-speaking pupils receive their education in private schools.

1.6.3 The type of school

Table 1.9 shows the number of completed questionnaires received from ordinary high schools and vocational high schools.

TABLE 1.9

THE QUESTIONNAIRES RECEIVED FROM TEACHERS, CLASSIFIED ACCORDING TO THE VARIOUS TYPES OF SCHOOLS

Type of school	Questionnaires received	
	Number	Percentage
High schools	1,269	73.7
Schools with primary and secondary departments	251	14.6
Agricultural high schools	6	0.3
Technical colleges	28	1.6
Technical high schools	65	3.8
Commercial high schools	16	0.9
Commercial and technical high schools	32	1.9
Industrial schools	22	1.3
Housecraft high schools	8	0.5
Not indicated	24	1.4
TOTAL	1,721	100.0

1.6.4 The Departments of Education

Table 1.10 reflects the number of completed questionnaires received from teachers of the various departments of education. A few questionnaires were received from teachers who are not in the service of the departments of education. In Table 1.10 such questionnaires are also taken into account.

TABLE 1.10

QUESTIONNAIRES RECEIVED FROM TEACHERS CLASSIFIED ACCORDING TO EMPLOYERS

Employer	Questionnaires received	
	Number	Percentage
Department of Education,		
Cape of Good Hope	528	30.7
Natal	160	9.3
Orange Free State	166	9.7
Transvaal	648	37.6
South West Africa	19	1.1
Department of Education, Arts and Science	181	10.5
Provincially aided schools	4	0.2
Schools aided by the Department of Education, Arts and Science	8	0.5
Private schools	7	0.4
TOTAL	1,721	100.0

Since the private schools which submitted completed questionnaires are not regarded as representative of them as a group, they are not classified separately in this report.

1.6.5 Teachers qualified and unqualified in Mathematics

For the purposes of this report teachers who have passed in at least two degree courses in one or more of the following subjects are regarded as qualified:

Mathematics (academic or engineering course)
Applied Mathematics (academic or engineering course)
Mathematical Statistics
Statistical Mathematics
Statistical Methods
Economic Statistics

1.7 UNIVERSITY LECTURERS HAVING A DIRECT INTEREST IN HIGH SCHOOL MATHEMATICS

1.7.1 Questionnaire N.B.376

This questionnaire was sent to the following departments of universities:

Mathematics and Applied Mathematics,
Mathematical Statistics,
Physics,
Land Surveying, Astronomy,
The various Departments of Engineering.

The questionnaires had to be completed by the heads of departments after consulting their staff.

Fifty completed questionnaires and several memoranda were received.

1.7.2 Bodies from which completed questionnaires were received

The university departments which submitted their contribution in this manner were the following:

University of Cape Town

Mathematics, Applied Mathematics, Mechanical Engineering, Chemical Engineering, Civil Engineering, Electrotechnical Engineering, Physics.

University of Natal

Mathematics and Applied Mathematics, Land Surveying, Electrotechnical Engineering, Chemical Engineering, Engineering, Physics.

University of the Orange Free State

Mathematics, Applied Mathematics, Physics.

Potchefstroom University for Christian Higher Education

Mathematics and Applied Mathematics, Physics.

University of Pretoria

Mathematics, Applied Mathematics, Mathematical Statistics, Land Surveying, Electrotechnical Engineering, Engineering.

Rhodes University

Mathematics and Applied Mathematics, Physics.

University of Stellenbosch

Mathematics, Mathematical Statistics, Electrotechnical Engineering, Physics.

University of South Africa

Mathematics and Applied Mathematics, Mathematical Statistics.

University of the Witwatersrand

Mathematics, Applied Mathematics, Mathematical Statistics, Mining Engineering, Chemical Engineering, Mechanical Engineering, Civil Engineering, Electrotechnical Engineering, Physics, Nuclear Physics.

1.8 LECTURERS WHO TRAIN TEACHERS

Questionnaire N.B.378 was sent out during June, 1962, to lecturers in the Method of the teaching of Mathematics.

The number of completed questionnaires received is shown in Table 1.11.

TABLE 1.11

THE COMPLETED QUESTIONNAIRES SUBMITTED BY LECTURERS WHO TRAIN TEACHERS

Institution	No. of questionnaires
University of Cape Town	2
University of Natal	3
University of the Orange Free State	1
Potchefstroom University for C.H.E.	1
University of Pretoria	1
Rhodes University	1
University of Stellenbosch	1
University of South Africa	1
TOTAL	11
Durban Teachers' Training College	1
Golden City Teachers' College, Johannesburg	1
Heidelberg Teachers' College	3
Johannesburg Teachers' College	4
Natal Training College, Pietermaritzburg	1
Pretoria Teachers' College	3
Wellington Training College	1
TOTAL	14
Pretoria Technical College	1

Although the Wellington Training College does not train any high school teachers, a partially completed questionnaire was received from it. Several training colleges and technical colleges have reported that they do not train high school teachers.

1.9 THE RESULTS

The results of the investigation are discussed in the chapters which follow.

1.10 USE OF TERMS

Unless otherwise appears from the context, the terms syllabus and curriculum have the following meanings:

syllabus	-	detailed section of a course of study
curriculum	-	whole course of study in broad outline

CHAPTER 2

THE AIMS AND VALUE OF THE TEACHING OF MATHEMATICS

2.1 INTRODUCTION

The function of the mathematical methods used in sciences such as Mechanics, Physics and Astronomy is well known. It is equally well known that Mathematics is indispensable for the practical work of the engineer and the technician. (55, p. 257)

In recent years Mathematics has become equally important in the humanities. A knowledge of mathematical methods is essential for sociological research, while in the psychological field Psychometry is making new demands upon the mathematician. The opinion is being expressed to an increasing extent that these aspects should receive more attention.

"The traditional college curriculum in mathematics is largely an ossification of the program designed to meet the needs of engineers before World War I. Since that time mathematics has developed as never before in its history.

"The inadequacy of the traditional mathematics curriculum is most obvious in the relation to the needs of the social scientist." (61, p. ix)

"It might seem that the increasing needs of the social scientist would make it desirable to plan different introductory courses for social and natural scientists. However, it turns out that the topics considered desirable by the social scientist are exactly those that are fundamental for all users of mathematics, for the educated citizen, and for the prospective mathematician." (61, p. x)

The following statements are often heard:

Mathematics is the subject par excellence to mould the pupils' mental capacities and to develop the ability to think logically; it is absolutely essential for every civilized person in the modern world; and it is a virtually indispensable aid in almost every line of study or practical career.

In addition, Mathematics offers a unique mental challenge and intellectual satisfaction from the first school year to the highest rung of a person's career of study.

Mathematics demands the ability to think neatly, accurately and systematically, and the teaching of this subject offers teachers an excellent opportunity to inculcate these essential characteristics in the pupil. Few other subjects lend themselves to the same extent to the cultivation of the right habits of thinking and the acquisition of the ability to make valid deductions.

Mathematics offers a useful means of ensuring the development of our capacities for leadership, but then it must be <u>Mathematics</u> and not merely a technique of solving problems according to drilled-in formulas.

For persons who make use of Mathematics, it is very important that they should be able to formulate a given problem in mathematical language.

Simple geometrical insight and the ability to handle formulas is an essential requirement for every supervisor and skilled labourer.

(Translation) But many people have a vague idea of what really is characteristic of that which concerns a mathematician by virtue of his profession. (55, p. 257)

Many people are under the impression that the manuals and textbooks for Mathematics already contain sufficient rules and formulas for the solution of all the mathematical problems which crop up in the technique of Mathematics. Even well educated persons sometimes ask: (Translation) But can one really create anything new in Mathematics? (55, p. 257) This is the reasons why the mathematician is sometimes represented as a tiresome person who knows a large number of formulas and hypotheses and whose task it is to impart this hackneyed knowledge to others. (55, p. 257) In this investigation a closer view is taken of high school Mathematics and attention given to the teachers of Mathematics pupils and the way in which this subject is taught.

2.2 THE NEED FOR MATHEMATICIANS

A very special need exists for mathematicians who are able to give guidance in carrying out comprehensive calculations.

Formerly the work could be undertaken with the assistance of teams of calculators provided with table calculating machines. (Translation) But in modern science and technology one encounters problems which would require several months or even years of work of scores of human computers with these limited aids to complete the calculation. This situation has led to the phenomenal development of modern calculating machines. (55, p. 258)

Many collaborators are necessary for the conversion of a mathematical problem into such a form that it can be solved by means of a computer. The mathematical theory of the methods of calculation is at present of great importance and the need of specialists who are acquainted with these methods is increasing with the number of computers. (55, p. 259) There is, in particular, a need of programmers, that is, persons who know how to make the data manageble for computers. The new problems also constantly demand a further development of the more theoretical Mathematics, with a resultant need of independent investigators in this field.

In this report it will be shown inter alia how great is the need of qualified Mathematics teachers. It will become clear that the teaching of Mathematics should receive very serious attention in order to ensure that no mathematical talent is lost. The view is held that the safety and continued existence of the Republic of South Africa are largely dependent upon the success with which the present critical shortage of scientific and technical personnel can be solved. One of the most essential conditions is the effective teaching of Mathematics in the high schools.

2.3 AN APTITUDE FOR MATHEMATICS

Is it possible to teach mathematics to all high school pupils?

(Translation) The need for special aptitude for the study and understanding of Mathematics is often overestimated. (55, p. 260) An aptitude for Mathematics becomes really important only at the stage when this subject is chosen as a major subject for further study. In this case it must first be established whether the necessary aptitude exists. What actually constitutes mathematical talent? (Translation) The ability to carry out algebraic calculations in the sense of handling complicated expressions, the discovery of fruitful methods and of solving equations which do not belong to a known type, etc., is approximately all the talent which is demanded of a mathematician in serious scientific work. (55, p. 261)

It will also be no surprise to the reader if it is asserted that the geometrical view; or geometrical intuition, plays a very large role in almost every field of Mathematics, even the most abstract.

The art of correctly setting up a logical argument is also one of the principal aspects of mathematical talent. (55, p. 262) The ability to understand the principle of complete induction thoroughly and to apply it is a useful criterion for logical maturity which is indispensable to the mathematician.

The various aspects of mathematical talent occur in different combinations. A very one-sided talent is of course dangerous. It goes without saying that talent alone is not sufficient but that it should be accompanied by a love of

Mathematics and systematic and daily work.

2.4 THE OBJECTIVES AND VALUE OF THE TEACHING OF MATHEMATICS FOR DIFFERENT PUPILS

It is obvious that all pupils are not equally talented and that they should be separated into different groups for educational purposes. A classification often encountered is the following:

A. Pupils who are capable of following a university course for degree purposes at a later stage, and those who take the matriculation course or school-leaving course which leads to matriculation exemption.

B. Pupils who are capable of obtaining the school-leaving certificate without (matriculation or) matriculation exemption.

C. Pupils who leave school as soon as they are old enough.

Group A should actually be subdivided into two different subgroups, namely pupils who intend to take Mathematics at the university and those who intend to take other subjects. In this investigation however no such fine distinction was made, although we can accept that the persons who replied to the questionnaires laid the emphasis on those pupils who intended to take Mathematics at the university.

University lecturers in subjects for which training in Mathematics at a high school is essential were asked for their opinion in regard to the aim and value of the teaching of Mathematics to pupils in the three different groups (Questionnaire N.B. 376).

The same questions were put to teachers of Mathematics in Questionnaire N.B. 377 which was sent to them.

2.4.1 The university lecturers' opinions

For certain scientific subjects preliminary tuition in Mathematics at a high school is set as a requirement for any student who wishes to study the subject at a university. In view of this fact the heads of the following departments at universities were asked for their opinions in connection with the purpose and value of the teaching of Mathematics at high schools: Mathematics, Applied Mathematics, Mathematical Statistics, Physics, Land Surveying and Engineering. Fifty lecturers submitted completed questionnaires. Table 2.1 indicates the number of lecturers who selected particular aims for the teaching of Mathematics. The five most important aims for each group of pupils had to be indicated.

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TABLE 2.1

THE AIMS AND EDUCATIONAL VALUE OF THE TEACHING OF MATHEMATICS ACCORDING TO UNIVERSITY LECTURERS

Aim	University lecturers who indicated the aims for groups of pupils					
	A		B		C	
	No.	%	No.	%	No.	%
1. Formation of mathematical concepts	50	100	21	42	5	10
2. To promote the ability to think clearly and to make logical deductions	46	92	39	78	24	48
3. To inspire a love for and interest in Mathematics	38	76	12	24	7	14
4. To provide a basic training for a future career	28	56	29	58	16	32
5. To experience the intellectual enjoyment of discovering the laws of number and space ..	27	54	13	26	7	14
6. To train pupils to calculate correctly	18	36	28	56	35	70
7. Extension and completion of the primary school work	15	30	24	48	27	54
8. To increase the number of persons with training in Mathematics	14	28	6	12	3	6
9. Discipline	11	22	16	32	16	32
10. To promote the building up of personality and character	8	16	11	22	10	20
11. To teach factual information of value in everyday life	5	10	21	42	30	60
12. To enable the pupil to make his way in the world	2	4	19	38	36	72
13. To pass an examination	0	0	2	4	4	8
TOTAL	262		241		220	

By rights each of the totals should have been 250. Some lecturers, however, indicated more than five aims and others fewer than five aims per group of pupils.

2.4.2 The teachers' views

Teachers were given the opportunity to give their replies to the same questions in Questionnaire 377. In Table 2.2 an analysis is given of the replies of 743 teachers who had themselves completed at least two degree courses in a mathematical subject.

TABLE 2.2

THE AIMS AND EDUCATIONAL VALUE OF THE TEACHING OF MATHEMATICS ACCORDING TO TEACHERS

Aim	Teachers who indicated the aims					
	Pupil groups					
	A		B		C	
	No.	%	No.	%	No.	%
1. Formation of mathematical concepts	525	71	280	38	157	21
2. To promote the ability to think clearly and to make logical deductions	639	86	501	67	300	40
3. To inspire a love for and interest in Mathematics	531	71	173	23	82	11
4. To provide a basic training for a future career	418	56	406	54	279	38
5. To experience the intellectual enjoyment of discovering the laws of number and space	461	62	84	11	21	3
6. To train pupils to calculate correctly	254	34	543	73	534	72
7. Extension and completion of the primary school work	44	6	178	24	534	72
8. To increase the number of persons with training in Mathematics	207	28	67	9	29	4
9. Discipline	150	20	234	31	301	40
10. To promote the building up of personality and character	177	24	197	27	172	23
11. To teach factual information of value in everyday life	62	8	334	45	421	57
12. To enable the pupil to make his way in the world	68	9	117	16	430	58
13. To pass an examination	87	12	251	34	305	41
TOTAL	3623		3365		3565	

If the teachers had indicated five aims for all three groups, these totals should have been 3715 in each case. Some teachers, however, confined themselves to only one or two groups.

2.4.3. A comparison of the order of importance of the aims according to university lecturers and according to teachers

In Table 2.3 the aims are compared in declining order of importance according to the opinions of the teachers and those of university lecturers.

The pupils who are competent enough to proceed to a university are borne in mind in the table.

TABLE 2.3

THE AIMS AND VALUE OF THE TEACHING OF MATHEMATICS FOR PUPILS WHO ARE COMPETENT TO PROCEED TO A UNIVERSITY

Aim	According to the teachers		According to university lecturers	
	Order of importance	%	Order of importance	%
To promote the ability to think clearly and to make logical deductions	1	86	2	92
To inspire a love for and interest in Mathematics	2	71	3	76
Formation of mathematical concepts	3	71	1	100
To experience the intellectual enjoyment of discovering the laws of number and space	4	62	5	54
To provide a basic training for a future career	5	56	4	56
To train pupils to calculate correctly	6	34	6	36
To increase the number of persons with training in Mathematics	7	28	8	28
To promote the building up of personality and character	8	24	10	16
Discipline	9	20	9	22
To pass an examination	10	12	13	0
To enable the pupil to make his way in the world	11	9	12	4
To teach factual information of value in everyday life	12	8	11	10
Extension and completion of primary school work	13	6	7	30

Table 2.4 reflects the aims in descending order of importance in respect of those pupils who may pass a school-leaving examination but who will not meet the requirements for admission to a university.

TABLE 2.4

THE AIMS AND VALUE OF THE TEACHING OF MATHEMATICS FOR PUPILS WHO WILL PASS A SCHOOL-LEAVING EXAMINATION WITHOUT MEETING THE REQUIREMENTS FOR ADMISSION TO A UNIVERSITY

Aim	According to the teachers		According to university lecturers	
	Order of importance	%	Order of importance	%
To train pupils to calculate correctly	1	73	3	56
To promote the ability to think clearly and to make logical deductions	2	67	1	76
To provide a basic training for a future career	3	54	2	58
To teach factual information of value in everyday life	4	45	5	42
Formation of mathematical concepts	5	38	5	42
To pass an examination	6	34	13	4
Discipline	7	31	8	32
To promote the building up of personality and character	8	27	11	22
Extension and completion of primary school work	9	24	4	48
To inspire a love for and interest in Mathematics	10	23	10	24
To enable the pupil to make his way in the world	11	16	7	38
To experience the intellectual enjoyment of discovering the laws of number and space ..	12	11	9	26
To increase the number of persons with training in Mathematics	13	9	12	12

In Table 2.5 a similar comparison is drawn in respect of pupils who will leave school as soon as they are old enough to satisfy the requirements of compulsory school attendance.

TABLE 2.5

THE AIMS AND VALUE OF THE TEACHING OF MATHEMATICS FOR PUPILS WHO LEAVE SCHOOL AS SOON AS THEY ARE OLD ENOUGH

Aim	According to the teachers		According to university lecturers	
	Order of importance	%	Order of importance	%
Extension and completion of primary school work	1	72	4	54
To train pupils to calculate correctly	1	72	2	70
To enable the pupil to make his way in the world	3	58	1	72
To teach factual information of value in everyday life	4	57	3	60
To pass an examination	5	41	12	8
Discipline	6	40	6	32
To promote the ability to think clearly and to make logical deductions	6	40	5	48
To provide a basic training for a future career	8	38	6	32
To promote the building up of personality and character	9	23	8	20
Formation of mathematical concepts	10	21	11	10
To inspire a love for and interest in Mathematics	11	11	9	14
To increase the number of persons with a training in Mathematics	12	4	13	6
To experience the intellectual enjoyment of discovering the laws of number and space ..	13	3	9	14

2.5 CONCLUSION

There is remarkable agreement between the aims of the teachers and the views of university lecturers. For those pupils who are able to proceed to a university later on, the first six aims are the same, the only difference being in the order of importance. This difference is significant especially when the first three aims are analysed more closely. The university lecturers see the matter logically and approach it from the direction of their subject. The formation of the "mathematical concept" is placed at the head of the list. Then follows the more general "promotion of logical thinking", the order eventually ending in "a love for and interest in the subject".

The teacher has to do primarily with the pupil and he knows that if the pupil cannot become accustomed to the way of thinking of the mathematician, it will be very difficult to cultivate a love for and interest in the subject. When these requirements have been met, then the essential conditions for mathematical concept formation have at least been satisfied.

A striking agreement in the views of university lecturers and those of teachers is also revealed in the aims in respect of those pupils who do complete their full high school career but who do not proceed to a university. Once again the first three aims are the same and the order of importance similarly shows the expected difference in approach. Five of the first six aims are the same.

In regard to those pupils who leave school as soon as they are entitled to do so, six of the first seven aims are the same for the two groups of tutors. It is interesting to note that it is really only in the case of this group of pupils that the utilitarian motive, namely, "to enable the pupil to make his way in the world", is given particular prominence. In respect of those pupils who do complete Std. 10 but who do not proceed to a university, the general formative value of the teaching of Mathematics is regarded as being of primary importance, while in the case of those pupils who go to a university later on the specific mathematical concept formation is emphasized.

If the aims are confined to the first five for each group, one common aim emerges: to promote the ability to think clearly and to make logical deductions. The value of the teaching of Mathematics in training the mind is highly rated and is important for all pupils.

If the first six aims are considered, the following are also found to be common:

The cultivation of the ability to calculate accurately.

The provision of the basic training for a future career.

The difference between the third and second groups is the fact that in respect of the pupils who leave school as soon as they are old enough, note must be taken of the immediate advantage of the teaching of Mathematics. Their tuition must therefore be very closely linked with every-day life.

The pupils in the group which have completed their high school career but who do not proceed to a university, require a more general formative approach.

The ability to understand mathematical concepts must be developed particularly in the case of the pupils who go to a university.

Mathematics is therefore studied with the following aims in view:

Its formative value; the training of professional persons in Mathematics and related fields; the use of this subject as an instrument in the case of persons who are not mathematicians to enable them to obtain certain results which in most cases occur in the form of numbers.

CHAPTER 3

THE MATHEMATICS PUPIL

3.1 THE NECESSITY FOR PUPILS TO TAKE MATHEMATICS

3.1.1 The increasing application of Mathematics in science

Very great developments in Mathematics have taken place in both the theoretical and the practical field during the twentieth century. "While mathematics, in the form of pure mathematics, has reached dizzying heights of abstraction, it has kept its feet on the ground by multiplying and extending its applications.

"Now more than ever, it is true that mathematics is the handmaiden of the sciences. Before this century the science of physics had already made abundant use of mathematics in mechanics, optics, the theory of heat, and electromagnetic theory. Analysis used to be the chief mathematical tool of the physicist. Now, with the development of relativity theory and quantum mechanics, he has had to learn Riemannian geometry and modern algebra. The increasing role that chance processes play in physical theory compels him to learn probability theory as well.

"Mathematics had spilled over from physics into the other physical sciences, chemistry and geology. It has invaded the life sciences, biology and psychology, and has expanded into the social sciences too. There is no area of science today that can avoid using mathematical methods." (1, p. 450)

(Translation) At present there is an increasing shortage of mathematicians who work in scientific or technical institutes and are concerned with related subjects (physics, geophysics, etc.) and with various subdivisions of modern technology. The work of the mathematicians in these institutions is not limited to the performance of calculations or the solution of mathematical problems which are set by the physicists or technologists. Many research workers are men learned in, besides mathematics, one or another practical science for which considerable schooling in mathematics is necessary. The mathematics department is one of the most important centres for the training of specialists in all the fields of science or technology where the equipment of modern mathematics is necessary." (55, p. 266)

The construction and handling of modern computers is an important form of specialized engineering for which specialists are trained in the appropriate divisions of technical high schools.

The mathematical theory of methods of calculation is of great importance today, and the need for specialists who are proficient in these methods increases with the number of computers. The problem of programming arises, i.e. the transformation of a calculating procedure in the form which makes it possible for the solution to be obtained automatically by means of a computer of a certain type." (55, p. 258-9)

3.1.2 The Orange River Scheme

Any large development scheme in South Africa, such as, for example, the Orange River Scheme will also demand the attention of mathematicians in order to be carried out in the most efficient manner. A good example of such a development scheme is the famous Netherlands Delta Plan.

"On February 1st 1953 the South Western part of the Netherlands, and, to a lesser extent, parts of England and Belgium, were struck by a flood disaster, which exceeded by far any one hitherto observed.

"In order to find out the best methods for preventing, inasfar as possible, a similar disaster in future, the government immediately appointed a committee, consisting of the most prominent hydraulic engineers, called the 'Delta-committee', because its realm is the delta formed by the rivers Rhine, Meuse and Scheldt.

"The reason why all this is mentioned in this report is the fact that it gave rise to a number of mathematical and physical problems.

"The problems fall into three groups:

(1) statistical extrapolation problems concerning the frequencies of excessive high floods;

(2) economic decision problems, concerning the optimal height to which dikes must be heightened, taking account of the cost and of damage caused by breaks;

(3) mathematical physical problems concerning the question, which types of depressions moving over the North Sea are the most dangerous, and which heightening of sealevel they may cause.

"They also form an example of the sometimes insufficiently stressed fact that modern society has a great need, not only of large scale computing, but also of 'large scale mathematics'." (103, p. 94)

Considerable agreement exists between the South African Orange River Scheme and the Delta Plan of the Hollanders. The principal difference is that in South Africa this gigantic scheme is being tackled in order to keep the water of the Orange River away from the sea and to utilize it for the development of the surrounding region. In the Netherlands the object of the Delta Plan is to keep the waters of certain rivers and the sea away from the land and so safegaurd the region concerned. Whereas, owing to particular circumstances, the South African plan contains more positive elements, it may be taken for granted that a large number of mathematically trained persons will also be required for the proper planning of this scheme in all its facets and for launching it economically. South Africa is rich in raw materials and these will be sufficient for the execution of the Orange River Scheme. Will there, however, be sufficient skilled engineers, technicians and specialists to mention only one professional group in which mathematically skilled persons are required?

3.1.3 The business world

Until recently a person who had obtained a degree with Mathematics as major subject, had almost no option but to enter the teaching profession. Although a very great need of Mathematics teachers exists today, and although anyone whose interests lie in that direction should be fully encouraged to become a teacher, a teaching career is today by no means the only one which need be considered. "In our society where the poorest citizen can be stockholder in the largest corporation, the mathematical techniques used in finance are of interest to everyone, as are the mathematical techniques for sound strategy, scientific planning and for decision-making under uncertainty.

"With added examples and exercises provided by the instructor, the present texts on finite mathematics could be used to offer modern mathematics to those business administration, engineering and physical science students who want to prepare for management roles in their chosen field." (34, p. 149)

The services of a mathematician are in demand today in almost every field.

3.1.4 Careers for women mathematicians

In the United States of America an increasing number of women are following careers as mathematicians.

"In the 1956-58 National Register of Scientific and Technical Personnel, 1277 women were listed in mathematical occupations, which was nearly 11% of the total mathematical listings. In this group, 53% were employed by colleges and universities, 29% by private industry, 13% by governmental agencies, and 5% in other occupations. The Ph.D. degree was held by 19%, the master's degree by

30%, and about 1% held less than a bachelor's degree.

"According to a study by the Educational Testing Service, less than 50% of the upper one third of high ability women high school graduates continue their education, whereas over 60% of the men of the same ability level do so. This is one indication of the loss of potential talent for science and mathematics among women." (30, p. 225)

The woman principal of a housecraft high school wrote as follows with reference to a questionnaire sent to her:

"Most of the questions in this questionnaire are not applicable to this school as Arithmetic is only taught in Std. 6 as a preparatory subject for Bookkeeping in Stds. 7 and 8.

"My personal opinion is that, under the circumstances, Arithmetic should be cut out because the time could be better used in beginning Bookkeeping in Std. 6 in this school, as the time factor is a problem. The intelligence of the pupils, as well as the great part of the year which is devoted to practical examinations makes it extremely difficult to cover the Std. 6 and 7 Bookkeeping curriculum in the space of two years."

In an article (43, p. 198-9) on this subject E.R. Hedrick wrote as follows on the question whether a girl should learn Arithmetic and Algebra:

"For every woman who does anything, whether it be a manlike task outside the home, or those tasks of housewifery that are often so misprized, deals with quantities and with relations between quantities every day of every week. The question as to whether twice as much gas burned under a kettle will make potatoes cook twice as fast is of the same order of difficulty as the question as to whether compound interest on money for twice as long is twice as much. The question as to the comparative contents of two cans of tomatoes of the same shape is actually the same question as the question as to the relative volumes of similar cylinders. If one is twice as high as the other, will it hold just twice as much? Will twice as much ice make the icebox twice as cold? What is meant by twice as cold? The reading of a gasmeter is a functional question of no mean order. Will twice as much sugar make a thing twice as sweet? Will doubling the amount of soap wash the clothes twice as clean? Will doubling the amount of yeast make the loaves twice as large, or raise the bread twice as fast? What is the effect on cake of increasing the amount of butter? Are oranges twice as thick though twice as heavy? If the skins are twice as thick on the large oranges, did we miscalculate the relative values of the oranges? Does it pay to buy big potatoes as compared with small ones? Will a mail order package twice as heavy cost twice as much postage? If sent by express, would the expressage be doubled? When does it become advisable to ship by freight? Will a kitchen window twice as large let in twice as much light? Will twice as many guests double the expense of a party: hence is it just as well to have the same number of guests at two different times as to have all at one time? Will twice the amount of food at each of half as many meals sustain life as well? Does one feed a man who works more than one who does not? How much more? If he works twice as long, should he have twice as much food? And there are those finer questions of balancing rations for children, for invalids, for old people; questions of calories and proteins and carbohydrates for the real expert; questions of bacterial growth in diseases, and in sour milk, and - sometimes - in beverages; questions of food for a mother and food for a child, content of butter fat and protein in milk and in skimmed milk, acids, sugars, starches. Quantities - quantities - relations between quantities.

"Perhaps many a woman hesitates to comprehend or even to study such relations. 'There are too many quantities.' 'I can never figure things out.' 'I have no head for figures.' 'That is too complicated for me.' But all these things are well within the range of experience of most women. Why are they too hard? Why cannot women understand and master such things? Is it perhaps that they have not formed habits of quantitative thinking? Is it perhaps that some all-too-wise person has told them that algebra is of no use to girls? Is it perhaps that the teacher of algebra was too busy with the shorthand of algebraic symbols to spend time over mere questioning about relationships between quantities?

"Some girls need no experience in dealing with quantities, and no habit of accurate thinking about quantitative relations. Some girls never have to do with quantities or with relations between quantities. Who are these girls? They are those who toil not, neither do they spin." (43, p. 198-9)

The teaching of Arithmetic should receive very serious attention. A pupil who is well grounded in Arithmetic has a solid foundation on which to build his future development in Mathematics. It creates the necessary self-confidence in the pupil and makes it possible for him to continue even with the senior syllabus.

3.2 THE NUMBER OF PUPILS WHO TAKE MATHEMATICS IN SOUTH AFRICAN PUBLIC HIGH SCHOOLS

3.2.1 Estimate

Table 3.1 gives an estimate of the number of pupils who attended public high schools in 1962 and the number which took Mathematics in Std. 10. Pupils at technical colleges and at private schools have not been taken into account.

3.2.2 Conclusion

A striking fact which emerges is that without exception a higher percentage of English-speaking pupils take Mathematics in Std. 10 than is the case with Afrikaans-speaking pupils. In view of the present shortage of manpower in the very fields in which a mathematical schooling is essential, e.g. in the case of medical practitioners, technicians and engineers, this phenomenon is very important.

The question arises whether there is not, among Afrikaans-speaking persons, a great manpower potential which still has to be developed.

It is clear that this considerable difference between the percentage of Afrikaans and the percentage of English-speaking pupils who take Mathematics at school, demands further research. Is the phenomenon due to a difference in aptitude or to a difference in interest?

This is a matter which should be investigated. The principal of an Afrikaans medium school complains that many boys with little talent for Mathematics are virtually compelled by their parents to take this subject in spite of advice to the contrary by the school. Is the English-speaking child really better endowed mathematically than the Afrikaans-speaking pupil?

The differences occur not only between Afrikaans and English-speaking pupils however, but also from province to province in respect of the percentage of pupils who take Mathematics in Std. 10. These differences are interesting but, on the other hand, also very disquieting. Where is the explanation for this phenomenon to be sought? It is a matter which demands urgent attention, since it may be expected that the intelligence and mathematical talent of the pupils in the four provinces will not differ very much.[1)]

3.2.3 Aptitude

The reason why relatively more English-speaking than Afrikaans-speaking pupils take Mathematics at high school may, as has already been said, be due to a difference in aptitude.

(Translation) The necessity for special aptitude for the study and understanding of Mathematics is often overestimated. Mathematics sometimes makes an impression of being very difficult as a result of a bad, much too formal way of interpretation." (55, p. 260) This observation may be valid for both Afrikaans and English schools and therefore offers no explanation for the phenomenon.

[1)] Comments made by members of the committee.

TABLE 3.1

THE NUMBER OF PUPILS IN THE VARIOUS HIGH SCHOOLS AND THE NUMBER OF PUPILS TAKING MATHEMATICS IN STD. 10 (ESTIMATE FOR 1962)

High Schools	Number			Pupils taking Mathematics in Std. 10	
	Schools	Pupils	Pupils in Std. 10	Number	Percentage
Transvaal					
Afrikaans medium	81	58,915	5,384	2,880	53.5
English medium	44	29,755	2,989	2,444	81.8
Parallel medium	18	12,350	1,622	755	46.5
	143	101,020	9,995	6,079	60.8
Cape Province					
Afrikaans medium	109	26,458	3,209	1,423	44.3
English medium	35	13,087	1,579	1,032	65.4
Parallel medium	26	9,283	875	625	71.4
Schools which replied to questionnaires	170	48,828	5,663	3,080	54.4
Estimate for all the high schools in the Cape Province	207	58,807	6,818	3,708	
Natal					
Afrikaans medium	5	2,275	266	175	65.7
English medium	22	11,287	1,062	921	86.7
Parallel medium	10	4,219	418	331	74.7
	37	17,781	1,746	1,427	81.7
Orange Free State					
Afrikaans medium	15	6,118	687	326	47.5
Parallel medium	5	3,163	329	212	64.4
Schools which replied to questionnaires	20	9,281	1,016	538	53.0
Estimate for all the schools	27	12,001	1,314	696	
Secondary departments of schools with primary and secondary departments					
Afrikaans medium	35	3,926	437	235	53.8
Parallel medium	10	1,304	112	66	58.9
	45	5,230	549	301	54.8
Estimate for all the schools	53	6,200	651	357	
Vocational schools					
Housecraft schools	9	1,248	77	0	0
Commercial and technical	9	3,816	232	168	67.4
Commercial	14	5,379	1,369	189	13.8
Technical	17	7,045	398	398	100.0
Industrial	17	2,387	76	29	37.7
Agricultural	15	3,606	481	160	33.3
Special	2	345	11	8	72.7
	83	23,826	2,644	952	

It is clear that this relative retrogression in the number of pupils taking Mathematics is most pronounced in the Afrikaans medium schools. The school principal mentioned above complains that many pupils (especially boys) have little talent for Mathematics but nevertheless are desirous of taking the subject, even against the advice of the school but according to the wishes of their parents. The argument usually advanced is that the employer desires it or that it is a requirement for apprenticeship. It would therefore appear that parents sometimes have a better sense of values than the school. On the other hand it must be admitted that the school has a tremendous task.

It is essential that valid tests should be made available to determine the mathematical aptitude of pupils. Poor examination results are not always accepted as an indication of the pupil's talent - frequently the blame is laid on the teacher.

The fault may also lie in the promotion of unfit candidates. For that reason it is very important that use should also be made of suitable standardized scholastic tests as early as possible in order to distinguish between future university students and the others.

3.2.4 Other problems experienced with pupils

Lack of motivation

The principal of a technical high school states that the pupils frequently complain that Mathematics is uninteresting, valueless, difficult and abstract. He considers that in many cases the teacher must bear the blame for this state of affairs.

The pupils cannot understand why they should "do" factors, simultaneous equations, etc.! They regard Mathematics merely as another subject which fills the timetable without having any practical value - this astonishing view coming from pupils at a technical high school!

(Translation) I wish to mention that if it is at all possible for a breach to be made in the fear of Mathematics with which the pupils come from primary school, much progress will already have been made.

Inefficient teaching of Arithmetic

The principal of an English medium school asserts that he is quite satisfied with the ability of his Mathematics teachers but that the results which they obtain are poor because they work with a group of pupils who have not grasped the fundamental ideas in connection with number prior to or in Std. 6. They cannot master Algebra because when they reach the high school, they still do not realise that 12 is equivalent to 10+2. It is not unusual in Std. 6 to have girls in particular who must make three attempts before they can say how many times 7 goes into 63. For that reason the school principal considers that this survey should really have begun in the primary school.

It is difficult for a pupil to develop a love for and interest in a subject if he is not adept in the processes required. Often the teacher assumes that the pupil had indeed mastered the basic operations, while this is still by no means the case. That explains why so many pupils tackle Mathematics under the handicap of an inadequate ability to do Arithmetic and even with a feeling of aversion to the subject.

On the other hand, an enthusiastic high school teacher will immediately endeavour to overcome these difficulties by means of remedial exercises and more efficient instruction.

And yet even in the highest classes the arithmetical incompetence of pupils is found to be a stumbling block.

To illustrate this fact a report of the subdivision Test Services of the

National Bureau of Educational and Social Research is included herewith.

Problems in connection with the mastering of basic concepts and operations

During the first quarter of 1963 a number of preliminary forms of performance tests in Algebra were applied to a representative random sample of pupils in Stds. 7, 8 and 9 in schools throughout the Republic and South West Africa. The application of this test did not form part of the present investigation but was conducted by the subdivision for Test Services of the National Bureau of Educational and Social Research for their own purposes.

The object was to obtain from each item data (e.g. difficulty values) on the strength of which it could be decided which items should be included in the final tests and which should be omitted. The analysis of the items revealed so many interesting and valuable data, however, that it was decided to draw up a complete report on the findings. This report would be completed towards the end of 1964.

The aim of this report is to give an analysis or diagnosis of problems and/or shortcomings in the mastering of basic concepts and operations in Algebra.

To illustrate the foregoing, a few data are presented below in connection with the problems and/or shortcomings referred to above. By way of explanation it should be mentioned that all the items are of the multiple-choice type where the pupil is required to indicate the correct answer among five "possibilities". The Arithmetic items were done by all pupils in the random sample, while the Algebra items were answered only by pupils taking Mathematics (or General Mathematics) as subject. The random sample was made up as follows:

	Arithmetic			Algebra		
	Afrikaans	English	Total	Afrikaans	English	Total
Std. 7	744	630	1374	444	446	890
Std. 8	743	624	1367	427	475	902
Std. 9	726	594	1320	309	415	724

For each item the percentage of the pupils which indicated each of the five possible answers as correct was calculated. A, B, C, D, E below are the "possible" answers in the tables which follow. The correct answer and the percentage of pupils who gave the correct answer are underlined. N is the total number of pupils tested.

Particular attention is given in the discussion to the percentage of pupils who indicated the four wrong answers as correct, since this is an indication of the extent of the abovementioned shortcomings and/or problems.

(a) Arithmetic:

(i) Complete: $\frac{3}{9} = \frac{9}{?}$

Std.	Not done	A <u>27</u>	B 15	C 3	D 81	E 18	N
7	4.6	<u>51.6</u>	1.9	29.6	7.7	4.6	1374
8	3.9	<u>59.0</u>	0.9	22.1	9.5	4.6	1367
9	2.0	<u>63.4</u>	1.7	14.3	14.7	3.9	1320

Note: Since pupils are introduced to fractions as early as Std. 4 it is a revelation that almost 30% of the Std. 7 pupils think that $\frac{3}{9} = \frac{9}{3}$

(ii) In order to obtain $\frac{1}{5}$ of a number, the number must be ....

Std.	Not done	A Multiply by $\frac{5}{1}$	B Divide by .20	C Multiply by .02	D Divide by $\frac{1}{5}$	E Multiply by .20	N
7	3.7	25.7	14.1	2.6	41.0	12.9	1374
8	4.2	21.5	12.9	2.9	47.0	11.5	1367
9	2.4	11.2	10.5	4.3	52.9	18.7	1320

Note:

(i) It is often not realised that the word "of" in this expression means "multiply". The fact that $\frac{1}{5}$ = .20 is a further problem.

(ii) Why is it that half of the Std. 9 pupils imagine that "$\frac{1}{5}$ of" is the same as "divide by $\frac{1}{5}$"?

(iii) .1 x .1 x .1 =

Std.	Not done	A 0.3	B .003	C .1	D .001	E .100	N
7	2.3	20.2	12.0	30.2	32.4	2.9	1374
8	2.2	12.4	7.5	27.5	47.3	3.1	1367
9	0.9	12.1	5.9	24.2	55.2	1.7	1320

Note: Here two difficulties occur, namely the fact that lxlxl=l and the position of the decimal point. Approximately one third of Std. 7 pupils think that lxlxl=3. About 18% of the Std. 8 pupils agree with this! Almost a quarter of the Std. 9 pupils admittedly know that the answer is "l" but place the decimal point incorrectly. Is this perhaps due to the method of teaching by which a rule had been drilled into the pupils without any real insight?

(iv) Eighty sheep are divided between two farmers so that P receives four times as many as Q. How many sheep does P obtain?

Std.	Not done	A 60	B 320	C 64	D 20	E 16	N
7	5.2	32.2	6.8	27.5	22.0	6.3	1374
8	5.4	28.2	5.6	29.5	18.3	13.0	1367
9	2.6	23.1	2.4	45.1	15.9	10.9	1320

Note: It is clear that the majority of the pupils in Stds. 7, 8 and 9 do not test the answers obtained. Is this due to a shortcoming in disposition, interest or aptitude, or is it the result of poor or incorrect teaching?

(v) 100% of P = ... x P.

Std.	Not done	A 100	B $\frac{1}{100}$	C 10	D $\frac{1}{10}$	E 1	N
7	13.1	39.7	29.1	7.2	3.2	7.7	1374
8	13.9	40.6	27.8	5.8	1.4	10.5	1367
9	6.3	40.3	28.0	3.3	1.2	20.9	1320

Note: The percentage of pupils who do not venture to give an answer ("not done") is striking. For 40% of the pupils in all three standards there is no difference between "100%" and "100" (see A); and more than a quarter of the pupils think that 100% means $\frac{1}{100}$. It is therefore clear that a poor understanding of % occurs in all standards.

(b) Algebra:

(i) The index in $12x^3$ is .....

Std.	Not done	A 1	B 2	C 12	D 3	E x	N
7	7.3	0	2.0	26.4	49.5	14.8	890
8	6.9	1.6	0	19.2	43.7	28.6	902
9	3.6	0	1.4	17.1	50.9	27.0	724

Note: If the concept "index" is present (or absent!) to this extent, what meaning can a pupil attach to any rules in which this word occurs? If this is the case with the word "index", what must the position be in regard to "coefficient", "square", "product", "sum","quotient", etc. (Also see (b)(iii) and (iv).)

(ii) If $b = 3$ and $c = -2$, calculate the value of $b - 2c$ =

Std.	Not done	A 7	B -1	C -12	D -7	E None of these	N
7	1.5	9.7	50.1	5.5	4.8	28.4	890
8	0	23.3	54.6	4.6	2.3	15.2	902
9	0.9	53.4	30.2	0.9	6.1	8.5	724

Note: The fact that 36% (B+D) of the pupils in Std. 9 who have already had two years' tuition in Algebra, experience difficulty with signs in doing such a very simple sum, is truly disquieting. One wonders what answer was obtained by the 8.5% of the Std. 9 pupils (see E).

(iii) Add $-5p$ to $-8p$

Std.	Not done	A 3p	B 13p	C -13p	D $-13p^2$	E -3p	N
7	0.8	5.9	34.8	10.0	30.8	17.7	890
8	6.6	3.5	24.1	23.0	31.4	11.4	902
9	5.3	0.9	21.4	43.1	18.8	10.5	724

Note: Rules relating to other operations play a large role here, this still being the case even in Std. 9. It is clear that the majority of Mathematics pupils do not have an adequate understanding of directed numbers.

(iv) $\frac{m^6}{m^3}$ = ...

Std.	Not done	A 3	B m^2	C m^3	D $m^6 - m^3$	E 2	N
7	4.3	4.1	24.5	32.5	16.2	18.4	890
8	2.1	4.1	37.7	34.8	4.1	17.2	902
9	0.9	6.1	38.4	42.3	3.0	9.3	724

Note: The erroneous idea or lack of any idea of the meaning of the word "index" (see (b)(i)) is confirmed here. Rules which have been so carefully drilled into pupils, are incorrectly applied here. In Std. 9 the misconception assumes a form different from that in Std. 7 (see B, D and E).

(v) $-5p = 3$

$p = \dots\dots\dots$

Std.	Not done	A $\frac{3}{5}$	B $-\frac{5}{3}$	C $-\frac{3}{5}$	D -15	E 8	N
7	15.5	15.6	11.7	25.0	15.5	16.7	890
8	0.9	19.9	10.9	32.0	13.0	23.3	902
9	0	30.9	10.0	42.0	5.2	11.9	724

Note: A points to a mistake in the sign, drilling can still correct this kind of error, but B, D and E are indications of an erroneous conception or lack of conception. A quarter of the Std. 9 pupils have not yet mastered this basic technique. This is an alarming state of affairs, especially when it is borne in mind that the equation is the central concept or starting point for so many teachers, textbooks and inspectors!

(vi) Ben is 5 years older than Jan, and Piet is twice as old as Ben. Their ages total 43 years. How old is each?
Which of the following equations would you use to solve this problem?

Std.	Not done	A $x-5+x+2(x-5)=43$	B $x+(x+5)+2x=43$	C $x+5x+10x=43$	D $(x-5)+x+2x=43$	E $x+5x+(5x+2)=43$	N
7	5.9	20.2	20.8	27.5	12.4	13.2	890
8	7.1	15.3	22.8	27.1	17.4	10.3	902
9	5.9	20.9	23.9	26.2	6.7	16.4	724

Note: More than 40% of the Std. 9 pupils (C+E) cannot clearly distinguish the difference between "....... years older" and "....... times as old". Not even 10% of the Std. 9 pupils are able to convert these relatively simple data in verbal form into the form of an equation. Why then should all the trouble be taken to attempt to teach all the methods of solving equations of different kinds?

(vii) $\frac{2}{k} + \frac{k}{3} = \frac{?}{3k}$

Std.	Not done	A $6+k$	B $6+k^2$	C $2+k$	D $2k$	E $3+k^2$	N
7	2.3	10.6	6.4	21.8	50.0	8.9	890
8	5.3	7.1	12.4	14.2	57.5	3.5	902
9	2.8	6.7	36.7	7.3	43.3	3.2	724

Note: Of the pupils in Std. 9 50% (C+D) have a poor idea of fractions. How did they ever pass Std. 8 Mathematics? For the 10% who answered A and E there is still hope because their difficulty is not so great.

Poorly trained teachers

Teachers without the necessary training are incapable of giving proper instruction in the fundamentals of the subject to junior classes and they cannot inspire the pupils with a love for the subject. The result is that there is a continual decrease in the number of pupils taking Mathematics, with disastrous results for the future.

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(Translation) Only he who passionately lives for Mathematics and for whom it is a living science is in a position to teach Mathematics in the right way. Without doubt, many pupils of the secondary school know from experience how fascinating and therefore easy Mathematics becomes under such masters. (55, p. 257-8)

3.3 THE OPTIONAL SUBJECTS

3.3.1 Introduction

If Mathematics had been a compulsory subject for all pupils up to Std. 10, the position would not have been encountered that in some schools so few pupils take Mathematics. The possibility exists that there would then have been fewer pupils in Std. 10.

It is therefore important to see what subjects are prescribed as subjects alternative to Mathematics by the various departments of education and whether there is any difference in policy between the Afrikaans and English schools.

TABLE 3.2

SCHOOLS IN WHICH ARITHMETIC IS AN OPTIONAL SUBJECT

Optional subjects in Departments of Education	School principal					
	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
Cape of Good Hope						
Optional subjects prescribed	11	18.0	18	11.8	29	13.5
Optional subjects not prescribed	8	13.1	32	20.9	40	18.7
No replies received	42	68.9	103	67.3	145	67.8
TOTAL	61	100.0	153	100.0	214	100.0
South West Africa						
Optional subjects prescribed	0	-	2	33.3	2	25.0
Optional subjects not prescribed	1	50.0	1	16.7	2	25.0
No replies received	1	50.0	3	50.0	4	50.0
TOTAL	2	100.0	6	100.0	8	100.0
Education, Arts and Science						
Optional subjects prescribed	1	5.6	8	22.2	9	16.7
Optional subjects not prescribed	4	22.2	10	27.8	14	25.9
No replies received	13	72.2	18	50.0	31	57.4
TOTAL	18	100.0	36	100.0	54	100.0

3.3.2 An optional subject as opposed to Arithmetic

The number of schools at which a subject is prescribed as an option in the place of Arithmetic, are indicated in Table 3.2 which also shows whether or not the school principal himself is a qualified Mathematics teacher. It will be interesting to see whether this fact has any influence on the policy of the schools.

Note

The calculation of the percentages (especially to the first decimal place) in all cases was probably not mathematically justified. For the sake of uniformity however this was done throughout so that it would be easier to draw comparisons (with the necessary reservations).

Conclusion

No replies were received from a large number of schools. In all probability General Mathematics is taught instead of Arithmetic in the majority of those schools.

Of the schools which did submit replies, 38 allow an optional subject in the place of Arithmetic. Of these 38 schools, 29 are in the Cape Province and 9 in the Department of Education, Arts and Science. In the case of the other departments of education it is the rule that all children must take Arithmetic, either as a separate subject or as part of General Mathematics.

3.3.3 Optional subjects as opposed to Mathematics and General Mathematics

Departments of Education

Table 3.3 reflects the number of schools at which optional subjects are offered in the place of Mathematics and General Mathematics. It also indicates whether or not the school principal himself is a qualified Mathematics teacher.

TABLE 3.3

SCHOOLS AT WHICH MATHEMATICS AND/OR GENERAL MATHEMATICS IS AN OPTIONAL SUBJECT

Departments of Education	School principal					
	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
Cape of Good Hope						
Optional subject offered	47	77.0	103	67.3	150	70.1
Optional subject not offered	4	6.6	11	7.2	15	7.0
No reply received	10	16.4	39	25.5	49	22.9
TOTAL	61	100.0	153	100.0	214	100.0
Natal						
Optional subject offered	5	55.6	15	55.6	20	55.6
Optional subject not offered	3	33.3	7	25.9	10	27.8
No replies received	1	11.1	5	18.5	6	16.6
TOTAL	9	100.0	27	100.0	36	100.0
Orange Free State						
Optional subject offered	20	95.2	40	90.9	60	92.3
Optional subject not offered	0	-	1	2.3	1	1.5
No replies received	1	4.8	3	6.8	4	6.2
TOTAL	21	100.0	44	100.0	65	100.0
Transvaal						
Optional subject offered	23	71.9	79	80.6	102	78.5
Optional subject not offered	5	15.6	7	7.2	12	9.2
No replies received	4	12.5	12	12.2	16	12.3
TOTAL	32	100.0	98	100.0	130	100.0
South West Africa						
Optional subject offered	2	100.0	4	66.7	6	75.0
Optional subject not offered	0	-	0	-	0	-
No replies received	0	-	2	33.3	2	25.0
TOTAL	2	100.0	6	100.0	8	100.0
Education, Arts and Science						
Optional subject offered	4	22.2	11	30.6	15	27.8
Optional subject not offered	10	55.6	12	33.3	22	40.7
No replies received	4	22.2	13	36.1	17	31.5
TOTAL	18	100.0	36	100.0	54	100.0

3.3.4 Optional subjects and the percentage of pupils taking Mathematics

Table 3.4 reflects the kinds of schools in the order of the percentage of pupils taking Mathematics in Std. 10.

TABLE 3.4

THE PERCENTAGE OF PUPILS TAKING MATHEMATICS

	Medium	Department	Percentage
1.	Parallel	Education, Arts and Science: Technical high schools	100.0
2.	English	Natal	86.7
3.	English	Transvaal	81.8
4.	Parallel	Natal	74.7
5.	Parallel	Education, Arts and Science: Special schools	72.2
6.	Parallel	Cape of Good Hope	71.4
7.	Parallel	Education, Arts and Science: Commercial and technical schools	67.4
8.	Afrikaans	Natal	65.7
9.	English	Cape of Good Hope	65.4
10.	Parallel	Orange Free State	64.4
11.	Parallel	O. F. S. secondary departments	58.9
12.	Afrikaans	Orange Free State secondary departments	53.8
13.	Afrikaans	Transvaal	53.5
14.	Afrikaans	Orange Free State	47.5
15.	Parallel	Transvaal	46.5
16.	Afrikaans	Cape of Good Hope	44.3
17.	Parallel	Education, Arts and Science: Industrial schools	37.7
18.	Parallel, Afrikaans, English	Various departments: Agricultural schools	33.3
19.	Parallel	Education, Arts and Science: Commercial schools	13.8
20.	Parallel	Education, Arts and Science: Housecraft schools	-

Conclusion

It makes little difference to the policy in respect of optional subjects in the place of Mathematics or General Mathematics whether or not the principal of the school is himself a qualified Mathematics teacher. Of the provincial departments of education Natal has the fewest optional subjects. It is noteworthy that a high percentage of the Std. 10 pupils take Mathematics, namely 86.7% of the pupils in English schools, 65.7% of the pupils in Afrikaans schools and 74.7% of the pupils in parallel medium schools - the highest of the provinces in the three different medium groups.

Of the four provincial departments of education that of the Orange Free State apparently allows the most optional subjects. In that province 64.4% of the pupils in parallel medium high schools take Mathematics in Std. 10, while the figure in respect of Afrikaans medium high schools is 47.5% of the pupils. Unfortunately the data for English medium high schools in the Orange Free State are not available. The foregoing comparison already shows how a liberal policy in respect of optional subjects results in pupils not taking Mathematics, whereas in reality many of them could have taken it. There is no reason to assume that an Afrikaans pupil in Natal will have a greater aptitude for Mathematics than one in the Orange Free State.

In the Cape Province the number of schools at which optional subjects are offered is ten times as large as those where it is not offered (70.1% as against 7%). Less than half of the pupils in Std. 10 at Afrikaans high schools (44.3%) take Mathematics.

3.4 THE SUBJECTS WHICH PUPILS CAN TAKE INSTEAD OF ARITHMETIC

3.4.1 Optional subjects for boys

Table 3.5 shows the number of Afrikaans medium high schools at which certain subjects are offered as optional subjects for boys. Only those schools from which completed questionnaires were received, are taken into account in the table.

TABLE 3.5

NUMBER OF AFRIKAANS MEDIUM SCHOOLS AT WHICH BOYS MAY TAKE THE FOLLOWING OPTIONAL SUBJECTS AS ALTERNATIVES TO ARITHMETIC

Department	Cape Province			South West Africa		Education, Arts and Science			Total		
Standards	6	7	8	6	7	6	7	8	6	7	8
Social Studies	0	2	0	0	0	0	0	0	0	2	0
Latin	0	1	1	0	0	0	0	0	0	1	1
Mathematics	2	4	2	1	1	1	1	5	4	6	7
Woodwork	0	3	2	0	0	1	0	0	1	3	2
German	0	11	11	0	0	0	0	0	0	11	11
Typing	0	1	1	0	0	0	0	0	0	1	1
TOTAL	2	22	17	1	1	2	1	5	5	24	22

Table 3.6 indicates the number of English medium high schools at which boys may take certain subjects instead of Arithmetic.

The bottom row of the tables reflects the number of schools from which completed questionnaires were received, while the rows immediately above show the number of schools offering optional subjects.

TABLE 3.6

THE NUMBER OF ENGLISH MEDIUM HIGH SCHOOLS AT WHICH BOYS MAY TAKE CERTAIN SUBJECTS INSTEAD OF ARITHMETIC

Department	Cape Province		
Standards	6	7	8
Latin	0	2	2
German	1	2	2
TOTAL	1	4	4
Number of schools		38	

In Table 3.7 an indication is given of the number of parallel medium schools at which boys may take certain subjects instead of Arithmetic.

TABLE 3.7

THE NUMBER OF PARALLEL MEDIUM SCHOOLS AT WHICH BOYS MAY TAKE CERTAIN SUBJECTS INSTEAD OF ARITHMETIC

Department	Cape Province			Transvaal		Total		
Standard	6	7	8	7	8	6	7	8
Latin	2	1	1	0	0	2	1	1
Mathematics	0	1	1	0	0	0	1	1
Woodwork	1	3	2	0	0	1	3	2
Typing	0	0	0	1	1	0	1	1
TOTAL	3	5	4	1	1	3	6	5
Number of schools	34			14		48		

Conclusion

According to Table 3.1 a very low percentage of the Afrikaans pupils in the Cape Province take Mathematics in Std. 10. According to Table 3.7 it is in these very schools that a liberal choice of subjects as opposed to Arithmetic is allowed. In the English schools the optional subjects are Latin, Mathematics and German which for many pupils will be more difficult than Arithmetic. In the Afrikaans schools Social Studies, Woodwork and Typing are added as optional subjects for boys. This must definitely have an influence on the number of pupils taking Mathematics in Std. 10, as is in fact apparent from the table.

3.4.2 Optional subjects for girls

Table 3.8 shows the optional subjects which girls at the Afrikaans medium high schools are allowed to take instead of Arithmetic.

TABLE 3.8

THE NUMBER OF AFRIKAANS MEDIUM HIGH SCHOOLS AT WHICH GIRLS MAY TAKE CERTAIN SUBJECTS INSTEAD OF ARITHMETIC

Department	Cape Province			Education, Arts and Science			Total		
Standards	6	7	8	6	7	8	6	7	8
Social Studies	0	1	0	0	0	6	0	1	6
Latin	0	2	1	0	0	0	0	2	1
Mathematics	2	0	2	1	0	1	3	0	3
Needlework	0	0	2	0	0	0	0	0	2
German	1	1	11	0	0	0	1	1	11
Typing	0	2	2	0	0	0	0	2	2
TOTAL	3	6	18	1	0	7	4	6	25
Number of schools	138			24			162		

In Table 3.9 the number of English medium high schools at which girls may take certain subjects instead of Arithmetic are indicated.

TABLE 3.9

THE NUMBER OF ENGLISH MEDIUM HIGH SCHOOLS AT WHICH GIRLS MAY TAKE CERTAIN SUBJECTS INSTEAD OF ARITHMETIC

Department	Cape Province		Natal	Total	
Standards	6	8	8	6	8
Social Studies	0	0	2	0	2
Latin	0	1	0	0	1
Mathematics	0	1	0	0	1
Needlework	0	1	0	0	1
German	1	2	0	1	2
TOTAL	1	5	2	1	7
Number of schools	38		20	58	

Table 3.10 shows the number of parallel medium schools at which certain subjects may be taken instead of Arithmetic.

TABLE 3.10

THE NUMBER OF PARALLEL MEDIUM HIGH SCHOOLS AT WHICH GIRLS CAN TAKE CERTAIN SUBJECTS INSTEAD OF ARITHMETIC

Department	Cape Province			Education, Arts and Science		Total		
Standard	6	7	8	6	8	6	7	8
Social Studies	0	0	0	0	2	0	0	2
Latin	2	0	1	0	0	2	0	1
Mathematics	0	0	1	1	0	1	0	1
German	0	1	1	0	0	0	1	1
TOTAL	2	1	3	1	2	3	1	5
Number of schools	34			20		54		

Conclusion

In the largest choice of subjects for girls occurs in Afrikaans schools in the Cape Province. The reason is apparently the staff problems in the relatively large number of schools in that province if the number of pupils is taken into account. Arithmetic is a compulsory subject up to Std. 8 in the other provinces.

3.5 THE SUBJECTS WHICH PUPILS MAY TAKE INSTEAD OF MATHEMATICS AND GENERAL MATHEMATICS

3.5.1 Optional subjects for boys

Tables 3.11 and 3.12 show the subjects which boys may take instead of Mathematics or General Mathematics at Afrikaans medium high schools.

TABLE 3. 11

THE NUMBER OF SCHOOLS AT WHICH CERTAIN SUBJECTS MAY BE TAKEN BY BOYS INSTEAD OF MATHEMATICS OR GENERAL MATHEMATICS IN STDS. 6, 7 AND 8 (AFRIKAANS MEDIUM)

Department	Cape Province			Orange Free State			Transvaal			South West Africa		Education, Arts and Science			Total		
Standard	6	7	8	6	7	8	6	7	8	7	8	6	7	8	6	7	8
Typing	0	17	20	0	8	12	0	1	1	0	1	0	0	0	0	26	34
Geography	0	0	0	0	0	0	0	5	4	0	0	0	0	0	0	5	4
Social Studies	3	16	20	0	0	2	0	0	0	2	1	0	0	0	3	18	23
History	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
Woodwork	2	28	32	0	1	2	0	0	0	0	0	0	0	0	2	29	34
Woodwork or Typing	0	4	3	0	1	0	0	0	0	0	0	0	0	0	0	5	3
Bookkeeping	1	12	9	0	0	0	1	5	0	4	2	0	0	1	2	21	12
Commerce	0	10	0	1	14	16	0	4	4	0	0	0	0	0	1	28	20
Commerce or Typing	0	3	0	0	3	3	0	1	1	0	0	0	0	0	0	7	4
Shorthand	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2	0
Bookkeeping or Shorthand	0	11	9	0	0	0	0	0	0	0	0	0	0	0	0	11	9
Agriculture	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0
Free choice	0	0	0	0	0	0	1	5	6	0	0	0	0	0	1	5	6
TOTAL	6	103	93	1	27	35	2	21	16	8	4	1	1	2	10	160	150
Number of schools		138			50			76		7			24			295	

TABLE 3.12

THE NUMBER OF SCHOOLS AT WHICH CERTAIN SUBJECTS MAY BE TAKEN BY BOYS INSTEAD OF MATHEMATICS IN STDS. 9 AND 10 (AFRIKAANS MEDIUM)

Department	Cape Province		Natal	Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Standards	9	10	10	9	10	9	10	9	10	9	10	9	10
Typing	13	14	0	4	2	0	0	0	0	0	0	17	16
Geography	10	10	0	4	0	5	4	1	1	0	0	20	15
Social Studies	0	0	0	0	4	0	0	0	0	0	0	0	4
Biology	6	5	0	0	0	3	3	0	0	0	0	9	8
Physiology	9	7	0	0	0	0	0	0	0	0	0	9	7
History	9	10	0	3	5	4	3	2	2	0	0	18	20
Woodwork	14	16	1	1	1	9	9	1	1	0	0	25	28
Geography or Typing	0	0	0	0	3	1	1	0	0	0	0	1	4
Physiology or Typing	1	1	0	0	0	0	0	0	0	0	0	1	1
Woodwork or Typing	0	0	0	1	3	1	0	0	0	0	0	2	3
Geography or Woodwork	1	1	0	0	0	1	1	0	0	0	0	2	2
Bookkeeping and Commercial Arithmetic	4	2	0	0	0	2	1	0	0	2	0	8	3
Commerce	0	0	0	12	11	7	7	0	0	0	0	19	18
Commerce or Typing	0	0	0	0	2	2	1	0	0	0	0	2	3
Shorthand	7	6	0	0	0	0	0	0	0	0	2	7	8
Bookeeping and Commercial Arithmetic or Shorthand	0	0	0	0	0	1	1	0	0	1	1	2	2
Agriculture	7	7	0	0	0	0	0	0	0	0	0	7	7
Agriculture or Typing	2	2	0	0	0	0	0	0	0	0	0	2	2
Agriculture or Geography	1	0	0	0	0	0	0	0	0	0	0	1	0
Agriculture or History	1	2	0	0	0	0	0	0	0	0	0	1	2
Bookkeeping and Commercial Arithmetic or Art	0	0	0	1	1	0	0	0	0	0	0	1	1
Free choice	1	2	0	0	0	25	26	0	0	0	0	26	28
TOTAL	86	85	1	26	32	61	57	4	4	3	3	180	182
Number of schools	109		4	50		76		7		24		270	

Tables 3.13 and 3.14 reflect the subjects which may be taken instead of Mathematics by boys at English medium schools.

TABLE 3.13

SUBJECTS WHICH MAY BE TAKEN BY BOYS INSTEAD OF MATHEMATICS OR GENERAL MATHEMATICS IN STDS. 6, 7 AND 8 AT ENGLISH MEDIUM SCHOOLS

Department	Cape Province			Natal		Transvaal		Total		
Standards	6	7	8	7	8	7	8	6	7	8
Social Studies	0	1	1	0	0	0	0	0	1	1
Woodwork	0	1	2	2	2	0	0	0	3	4
Typing or Woodwork	0	0	3	0	0	0	0	0	0	3
Shorthand	0	0	0	1	0	0	0	0	0	1
Bookkeeping and Commercial Arithmetic	1	3	3	0	1	0	0	1	3	4
Free choice	0	0	1	0	0	1	1	0	1	2
TOTAL	1	5	10	2	4	1	1	1	8	15
Number of schools		38		20		39			97	

TABLE 3.14

SUBJECTS WHICH MAY BE TAKEN BY BOYS INSTEAD OF MATHEMATICS IN STDS.9 AND 10 AT ENGLISH MEDIUM SCHOOLS

Department	Cape Province		Natal		Transvaal		Total	
Standards	9	10	9	10	9	10	9	10
Geography	1	1	0	0	0	0	1	1
History	6	6	0	0	0	0	6	6
Woodwork	1	1	2	1	0	0	3	2
Bookkeeping and Commercial Arithmetic	2	2	2	2	2	2	6	6
Shorthand	0	0	0	0	0	2	0	2
Free choice	0	0	0	0	2	2	2	2
TOTAL	10	10	4	3	4	6	18	19
Number of schools	35		18		39		92	

Tables 3.15 and 3.16 show the number of parallel medium schools at which boys may take certain subjects instead of Mathematics.

TABLE 3.15

SUBJECTS WHICH BOYS IN STDS. 6, 7 AND 8 AT PARALLEL MEDIUM SCHOOLS MAY TAKE INSTEAD OF GENERAL MATHEMATICS OR MATHEMATICS

Department	Cape Province			Natal		Orange Free State		Transvaal		Education, Arts and Science	Total		
Standards	6	7	8	7	8	7	8	7	8	8	6	7	8
Typing	0	4	5	0	0	3	3	2	2	0	0	9	10
Geography	0	0	0	1	0	0	0	0	1	0	0	1	1
Social Studies	0	1	2	0	0	0	0	0	0	0	0	1	2
Woodwork	1	6	4	3	3	5	4	1	1	0	1	15	12
Bookkeeping	0	8	7	1	1	0	0	0	0	1	0	9	9
Commerce	0	0	0	0	0	2	4	0	0	0	0	2	4
Shorthand	0	0	0	0	0	0	0	0	3	0	0	0	3
Agriculture	1	2	2	0	0	0	0	0	0	0	1	2	2
Free choice	0	2	0	0	0	1	1	0	6	0	0	3	7
TOTAL	2	23	20	5	4	11	12	3	13	1	2	42	50
Number of schools	34			11		15		14		20	94		

TABLE 3.16

THE NUMBER OF PARALLEL MEDIUM SCHOOLS AT WHICH BOYS MAY TAKE CERTAIN SUBJECTS INSTEAD OF MATHEMATICS IN STDS. 9 AND 10

Department	Cape Province		Natal		Orange Free State		Transvaal		Education, Arts and Science	Total	
Standards	9	10	9	10	9	10	9	10	9	9	10
Typing	3	3	0	0	2	2	0	0	0	5	5
Geography	2	2	0	0	2	2	1	1	0	5	4
Biology	4	4	0	0	0	0	0	0	0	4	4
Physiology	5	5	0	0	0	0	0	0	0	5	5
History	1	1	0	0	0	0	1	1	0	2	2
Woodwork	1	1	2	2	2	2	0	0	0	5	5
Geography or Typing	0	0	0	0	4	5	0	0	0	4	5
Woodwork or Typing	0	0	0	0	1	0	0	0	0	1	0
Commerce	0	0	0	0	1	1	4	4	0	5	5
Bookeeping and Commercial Arithmetic	0	0	1	1	0	0	0	0	0	1	1
Shorthand	1	1	0	0	0	0	1	1	1	3	2
Agriculture	2	2	0	0	0	0	0	0	0	2	2
Agriculture or History	1	1	0	0	0	0	0	0	0	1	1
Free choice	1	1	0	0	1	1	1	1	0	3	3
TOTAL	21	21	3	3	13	13	8	7	1	46	44
Number of schools	26		9		15		14		20	84	

3.5.2 Optional subjects for girls

In Tables 3. 17 and 3. 18 an indication is given of the subjects which girls may take at Afrikaans medium schools instead of Mathematics or General Mathematics.

TABLE 3.17

THE NUMBER OF AFRIKAANS MEDIUM SCHOOLS AT WHICH CERTAIN SUBJECTS MAY BE TAKEN BY GIRLS INSTEAD OF MATHEMATICS OR GENERAL MATHEMATICS IN STDS. 6, 7 AND 8

Department	Cape Province			Natal	Orange Free State		Transvaal			South West Africa		Education, Arts and Science	Total		
Standards	6	7	8	8	6	8	6	7	8	7	8	8	6	7	8
Typing	3	39	41	0	3	27	2	0	31	0	0	0	8	39	99
Social Studies	0	14	14	0	0	0	0	0	0	0	0	0	0	14	14
Domestic Science	0	15	13	2	0	0	0	0	0	0	0	0	0	15	15
Geography or Typing	0	0	1	0	0	1	0	0	0	0	0	0	0	0	2
Domestic Science or Typing	0	2	0	0	0	0	0	0	0	0	0	0	0	2	0
Domestic Science or Short-hand	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
Commerce	0	0	0	0	0	11	0	0	4	0	0	0	0	0	15
Commerce or Typing	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
Bookkeeping	1	2	10	3	0	0	0	2	2	1	0	0	1	5	15
Shorthand	0	11	0	0	0	0	0	2	2	0	0	5	0	13	7
Agriculture or Typing	0	2	2	0	0	0	0	0	0	0	0	0	0	2	2
Free choice	0	0	0	0	0	1	0	7	7	1	1	0	0	8	9
TOTAL	4	85	81	6	3	40	2	11	47	2	1	5	9	98	180
Number of schools	138			4	50		76			7		24	299		

TABLE 3.18

THE NUMBER OF AFRIKAANS MEDIUM SCHOOLS AT WHICH CERTAIN SUBJECTS MAY BE TAKEN BY GIRLS INSTEAD OF MATHEMATICS IN STDS. 9 AND 10

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Standards	9	10	9	10	9	10	9	10	9	10	9	10	9	10
Typing	17	18	0	0	18	17	6	7	0	0	0	0	41	42
Geography	7	6	0	0	0	0	0	0	1	1	0	0	8	7
Biology	6	5	0	0	0	0	4	3	0	0	0	0	10	8
Physiology	7	3	0	0	0	0	0	0	0	0	0	0	7	3
History	10	11	0	0	1	2	0	0	2	2	0	0	13	15
Domestic Science	10	9	2	2	0	0	10	10	0	0	1	0	23	21
Geography or Typing	3	3	0	0	6	6	4	3	0	1	0	0	13	13
Physiology or Typing	3	3	0	0	0	0	0	0	0	0	0	0	3	3
Domestic Science or Typing	3	3	0	0	1	0	2	3	1	1	0	0	7	7
Domestic Science or Shorthand	0	0	0	0	0	0	0	1	0	0	0	0	0	1
Bookkeeping and Commercial Arithmetic	4	2	0	0	0	0	2	0	0	0	0	0	6	2
Commerce	0	0	0	0	1	0	1	1	0	0	0	0	2	1
Commerce or Typing	0	0	0	0	6	6	4	4	0	0	0	0	10	10
Shorthand	6	7	0	0	0	0	1	0	0	0	0	1	7	8
Bookkeeping and Commercial Arithmetic or Shorthand	0	0	0	0	0	0	0	0	0	0	0	6	0	6
Agriculture or Typing	1	1	0	0	0	0	0	0	0	0	0	0	1	1
Agriculture or Geography	1	0	0	0	0	0	0	0	0	0	0	0	1	0
Agriculture or History	0	1	0	0	0	0	0	0	0	0	0	0	0	1
Commerce or Typing or Geography	0	0	2	0	0	0	0	0	0	0	0	0	2	0
Bookkeeping and Commercial Arithmetic or Art	0	0	0	0	2	0	0	0	0	0	0	0	2	0
Free choice	1	1	1	0	1	0	26	27	0	0	0	0	29	28
TOTAL	79	73	5	2	36	31	60	59	4	5	1	7	185	177
Number of schools	109		4		50		76		7		24		270	

Tables 3.19 and 3.20 show the subjects which may be taken by girls in English medium high schools instead of Mathematics or General Mathematics.

TABLE 3.19

THE NUMBER OF ENGLISH MEDIUM SCHOOLS AT WHICH CERTAIN SUBJECTS MAY BE TAKEN BY GIRLS INSTEAD OF GENERAL MATHEMATICS OR MATHEMATICS IN STDS. 6, 7 AND 8

Department	Cape Province			Orange Free State		Transvaal		Total		
Standards	6	7	8	7	8	7	8	6	7	8
Typing	0	0	0	1	1	9	10	0	10	11
Geography	0	0	0	0	0	1	1	0	1	1
Social Studies	0	1	1	0	0	0	0	0	1	1
Domestic Science	0	2	4	4	3	0	0	0	6	7
Domestic Science or Typing	0	1	0	0	0	0	0	0	1	0
Domestic Science or Shorthand	0	0	0	0	2	0	0	0	0	2
Commerce	0	0	0	2	2	0	0	0	2	2
Bookkeeping and Commercial Arithmetic	1	13	11	2	2	1	1	1	16	14
Shorthand	0	0	0	0	0	1	1	0	1	1
Free choice	0	0	0	0	0	1	0	0	1	0
TOTAL	1	17	16	9	10	13	13	1	39	39
Number of schools	38			20		39		97		

TABLE 3.20

THE NUMBER OF ENGLISH MEDIUM SCHOOLS AT WHICH GIRLS MAY TAKE CERTAIN SUBJECTS INSTEAD OF MATHEMATICS IN STDS. 9 AND 10

Department	Cape Province		Orange Free State		Transvaal		Total	
Standards	9	10	9	10	9	10	9	10
Typing	2	2	0	1	6	6	8	9
Geography	1	1	0	0	0	0	1	1
History	4	4	0	0	0	0	4	4
Domestic Science	3	2	0	4	3	3	6	9
Physiology or Typing	1	1	0	0	0	0	1	1
Domestic Science or Typing	0	1	0	0	0	0	0	1
Domestic Science or Shorthand	0	1	0	0	0	0	0	1
Commerce	0	0	0	0	1	1	1	1
Bookkeeping and Commercial Arithmetic	2	1	3	1	2	2	7	4
Shorthand	3	3	0	2	1	1	4	6
Bookkeeping and Commercial Arithmetic or Shorthand	1	1	0	0	0	0	1	1
Free choice	0	0	0	0	4	4	4	4
TOTAL	17	17	3	8	17	17	37	42
Number of schools	35		18		39		92	

Tables 3.21 and 3.22 reflect the number of parallel medium schools at which girls may take other subjects instead of Mathematics and General Mathematics.

TABLE 3.21

THE NUMBER OF PARALLEL MEDIUM SCHOOLS AT WHICH GIRLS MAY TAKE CERTAIN SUBJECTS INSTEAD OF MATHEMATICS IN STDS. 7 AND 8

Department	Cape Province		Natal		Orange Free State		Transvaal		Education, Arts and Science	Total	
Standards	7	8	7	8	7	8	7	8	8	7	8
Typing	9	10	0	0	8	8	7	6	0	24	24
Social Studies	0	1	0	0	0	0	0	0	0	0	1
Domestic Science	0	0	2	2	0	0	0	0	0	2	2
Geography or Typing	0	1	0	0	0	0	0	0	0	0	1
Domestic Science or Typing	1	0	0	0	0	0	0	0	0	1	0
Domestic Science or Shorthand	0	0	0	0	0	5	0	0	0	0	5
Commerce	1	2	0	0	0	0	0	0	0	1	2
Commerce or Typing	0	0	0	0	1	0	0	0	0	1	0
Bookkeeping and Commercial Arithmetic	4	4	1	1	0	0	0	0	0	5	5
Shorthand	0	0	0	0	0	0	0	0	5	0	5
Free choice	0	1	0	0	1	1	0	0	0	1	2
TOTAL	15	19	3	3	10	14	7	6	5	35	47
Number of schools	34		11		15		14		20	94	

TABLE 3.22

THE NUMBER OF PARALLEL MEDIUM SCHOOLS AT WHICH GIRLS MAY TAKE CERTAIN SUBJECTS INSTEAD OF MATHEMATICS IN STDS. 9 AND 10

Department	Cape Province		Natal		Orange Free State		Transvaal		Education, Arts and Science		Total	
Standards	9	10	9	10	9	10	9	10	9	10	9	10
Typing	3	3	0	0	5	4	2	0	0	0	10	7
Geography	0	1	0	0	0	0	0	0	0	0	0	1
Social Studies	1	0	0	0	0	0	0	0	0	0	1	0
Biology	4	4	0	0	0	0	0	0	0	0	4	4
Physiology	1	1	0	0	0	0	0	0	0	0	1	1
History	1	1	0	0	0	0	0	0	0	0	1	1
Domestic Science	0	0	2	2	0	0	0	0	0	0	2	2
Geography or Typing	0	0	0	0	4	5	1	0	0	0	5	5
Physiology or Typing	2	2	0	0	0	0	0	0	0	0	2	2
Domestic Science or Typing	1	1	0	0	0	0	0	0	0	0	1	1
Domestic Science or Shorthand	0	0	0	3	0	0	0	0	0	0	0	3
Commerce	0	0	0	0	1	0	0	1	0	0	1	1
Commerce or Typing	0	0	0	0	0	0	3	3	0	0	3	3
Bookkeeping and Commercial Arithmetic	0	0	1	1	0	0	0	0	0	0	1	1
Shorthand	2	2	0	0	0	0	0	0	1	1	3	3
Bookkeeping and Commercial Arithmetic or Shorthand	0	0	0	0	0	0	1	1	1	1	2	2
Free choice	1	0	0	0	1	0	1	0	0	0	3	0
TOTAL	16	15	3	6	11	9	8	5	2	2	40	37
Number of schools	26		9		15		14		20		84	

3.5.3 **The influence of optional subjects in Stds. 9 and 10 on the percentage of pupils who take Mathematics in Std. 10**

Table 3.23 gives an indication of the percentage of pupils taking Mathematics in the four provincial departments of education, the "optional subject percentage" also being shown. This percentage is obtained by multiplying the number of schools offering optional subjects by 100 and then dividing by the number of schools in that group.

e.g. The optional subject percentage of Std. 9 girls in parallel medium schools in Cape Town $= \frac{16 \times 100}{26}$

$= 61.5$

TABLE 3.23

THE PERCENTAGE OF PUPILS TAKING MATHEMATICS IN STDS. 9 AND 10 COMPARED WITH THE AVAILABILITY OF OPTIONAL SUBJECTS

Department	Medium	Percentage of pupils taking Mathematics	Optional subject percentage			
			Std. 9		Std. 10	
			Boys	Girls	Boys	Girls
Natal	English	86.7	22.2	16.7	16.7	44.4
Transvaal	English	81.8	10.3	43.6	15.4	43.6
Natal	Parallel	74.7	33.3	33.3	33.3	66.7
Cape of Good Hope	Parallel	71.4	80.8	61.5	80.8	57.7
Natal	Afrikaans	65.7	0	125.0	25.0	50.0
Cape of Good Hope	English	65.4	28.6	48.6	28.6	48.6
Orange Free State	Parallel	64.4	86.7	73.3	86.7	60.0
Transvaal	Afrikaans	53.5	80.3	78.9	75.0	77.6
Orange Free State	Afrikaans	47.5	52.0	72.0	64.0	62.0
Transvaal	Parallel	46.5	57.1	57.2	50.0	35.7
Cape of Good Hope	Afrikaans	44.3	78.9	72.5	78.0	67.0

3.5.4 **Discussion**

A relation exists between the variety of optional subjects offered at schools and the percentage of pupils taking Mathematics in Std. 10. The number of them taking Mathematics in Std. 10 depends largely upon the pupils themselves. If the pupils all insist upon taking Mathematics, the optional subjects as such will disappear automatically.

If the maximum number of pupils have to take Mathematics up to Matric, a start must be made with the pupils themselves by thorough motivation and effective teaching.

The question may be asked whether it is really essential that girls should take such a "difficult subject" as Mathematics. This is a problem which has also been contended with in other countries.

"The American public was surprised by James B. Conant's assertion in his study of the American high school that too many bright girls avoided the stiff courses. It was startled when it learned that less than half of the top twenty-five per cent of all high school graduates completed college; and of the students in the top ten per cent in high school who did not go to college, two-thirds were women." (4, p. 141)

An important reason why so many girls have to take Mathematics is the possible effect which the subject has on the next generation of students.

"Moreover, those girls who disparage serious academic learning as an aid to the homemaker, should note Professor Nevitt Sanford's study about the good students. They most frequently had mothers with intellectual interests and aspirations whereas the poorest students had mothers who urged college attendance for its social prestige." (4, p. 141)

"Adults should help young people dispel the misconception that it is unwomanly for a girl to have an intelligent grasp of nuclear physics or English constitutional law yet it is 'ladylike for her to fly around the world serving cocktails'." (4, p. 143)

"Fortunately, at most universities there is a nucleus of academically talented girls who are eager to acquire the technique of learning and to develop the art of thinking. They quickly discover that studying is a prerequisite to learning, and that even a potentially brilliant mind functions inadequately when not disciplined by thinking. But their number is vastly smaller than it should be ...." (4, p. 144)

3.6 THE ATTITUDE OF THE PUPILS TOWARDS MATHEMATICS

3.6.1 Introduction

(Translation) The role that Mathematics plays in our lives is becoming more important and extended. Everywhere there is a growing demand for people who can think mathematically, who can apply mathematical methods and who can do original mathematical work. In order to continue to provide for this demand we must constantly ensure that the teaching of Mathematics comes into its rightful place, and we may neglect nothing in order to awaken the interest of youth in mathematics and keep it alive. (72, p. 152)

It is not only in the Netherlands that this attitude is adopted in connection with the Mathematics pupil.

(Translation) In 1952 the admission to specialised Mathematics courses at the universities of the U.S.S.R. was greatly extended in comparison with previous years. It is of great importance, in view of this extension, that the youth are brought up to a love of Mathematics. Therefore it is necessary that everywhere the opportunity to develop their taste and to measure their abilities and talents be offered. (55, p. 260)

And in South Africa in particular it is essential that the pupil should be led to be in favour of taking Mathematics.

"The curriculum in mathematics is known to unfold itself over four or five years; it develops in such a way that failure to grasp essentials will result in four or five unhappy years of little progress; and the target is 'getting through matric', or its equivalent. Having made this grade another psychological barrier awaits the pupil: with no experience of independent work in mathematics he is launched into a university and again made to feel inadequate. Often he is further disheartened by being told that only when he reaches second-year mathematics at the university will he began to learn 'real' mathematics. I submit that these two psychological barriers, from junior to high school and from high school to university, are most significant, extremely damaging, and quite unnecessary.

"Another important psychological influence is that of the 'climate of opinion' in the minds of teachers, pupils, parents and the general public: mathematics is a difficult subject which only the best pupils can manage; mathematics is of no consequence if the pupil is not going on to a university; mathematics is a subject started by the Greeks and it never changes; mathematics is certainly not a subject you would read books about or do research in; mathematics is taught, and not discovered by pupils.

These stereotyped statements are very important to the pupil and provide a most unhealthy environment for successful teaching. It is my belief that the climate of opinion is an accurate indication of what is in fact going on, or more precisely, what was actually going on a few years ago. There is a time-lag as a rule. We do well do take this climate of opinion seriously in any programme of evaluation. (62, p. 31)

3.6.2 A Mathematics periodical for pupils and students

The replies given by teachers to the question whether a need exists for a South African periodical for pupils and students are reflected in the data given in Table 3.24.

TABLE 3.24

THE NEED OF A MATHEMATICS PERIODICAL FOR PUPILS AND STUDENTS

Teachers	Qualified in Mathematics		Unqualified in Mathematics	
	Number	%	Number	%
Those who consider that a need exists	531	75	696	71
Those who consider that no need exists	99	14	101	10
Those who did not reply	78	11	186	19
TOTAL	708	100	983	100

3.6.3 Conclusion

Approximately three quarters (75% and 71%) of the teachers are of the opinion that a need exists for a South African periodical for pupils and students. Possibly the new periodical Spectrum will largely meet this need.

3.6.4 The need of short papers for pupils and students

Teachers were also asked whether a need existed for a series of short papers written by prominent mathematicians in order to draw attention to Mathematics suitable for high school pupils. This of course also connotes Mathematics which does not at present occur in high school syllabi. The object of these proposed treatises is to stimulate the interest of talented pupils and to present Mathematics as a satisfactory and meaningful human activity. The replies given by teachers to this question are shown in Table 3.25.

TABLE 3.25

THE NEED OF SHORT PAPERS TO STIMULATE THE INTEREST OF TALENTED PUPILS

Teachers	Qualified in Mathematics		Unqualified in Mathematics	
	Number	%	Number	%
Those who consider that a need exists	632	89	764	78
Those who consider that no need exists	30	5	33	3
Those who did not reply	46	6	186	19
TOTAL	708	100	983	100

Good examples of such treatises or monographs are found in the series "Contemporary School Mathematics" published by Edward Arnold Ltd., (41 Maddox Street, London W. 1).

The first series is intended for pupils 11 to 15 years old and consists of the following booklets:

G. Matthews	:	Matrices 1
C.A.R. Bailey	:	Sets and Logic 1
F.B. Lavis	:	Computers 1
J.A.C. Reynolds	:	Shape, size and place

A similar series for older pupils is also published by the above publishers. It is worthwhile taking cognizance of these short works. There is a need of similar works in both official languages in South Africa.

3.6.5 Conclusion

Exceptionally keen interest is displayed in the proposed publications, and it would appear that they would meet a very great need.

In order to create the right climate for a considerable increase in the number of pupils taking Mathematics at least to Std. 10, and to encourage talented pupils to strive towards better achievements than merely good Matriculation results, a Mathematics periodical and Mathematics articles written specially for the pupils 'and students' market can make a great contribution in this direction.

3.6.6 The Mathematics Olympiad

In the Netherlands leading mathematicians were inspired with a wonderful idea to improve the attitude of pupils towards Mathematics.

(Translation) For years the Mathematical Association and the Education Commission spoke about organizing an annual Mathematics competition for secondary pupils. This was not an original idea and even the name "Mathematics Olympiad" which was chosen for the competition was not new. Similar competitions had been held since the beginning of this century in Hungary. The Olympiad held in 1960 was the sixtieth of the Hungarian series. The fact that a disproportionately large number of Hungarians may be noted among the prominent mathematicians of the world can be regarded as partly a result of the Mathematics competitions. (73, p. 152)

This matter was vigorously tackled in the Netherlands and carried into effect. Numerous schools and their cleverest pupils were induced to participate in these competitions, and much was done on the part of the authorities to promote the undertaking.

(Translation) The organization of the Mathematics Olympiad was made possible in 1963, as also in 1962, by an extra subsidy to the Netherlands Education Committee on Mathematics by the Ministry of Education, Arts and Science.(111, p. 161)

(Translation) The prizegiving took place on Friday, 15th November, in the building of the Ministry of Education, Arts and Science, Nieuwe Uitleg 1, The Hague by the Director-General of the Ministry, Mr. J.G.M. Broekman. (111, p. 163)

This healthy competitive spirit was also fostered in other countries.

Similar undertakings were tackled even in Russia, the ultimate purpose of such enterprises being constantly borne in mind.

(Translation) The Mathematics olympiads, where difficult questions are set and where "winners" obtain prizes and honourable mention, succeed in their purpose where the clubs work well. The olympiads should be the crown of the work which has been done during the course of the years, and should not be an event on its own. (55, p. 263)

This movement has also gained ground in the U.S.A.

"The Bergen County Interscholastic Mathematics League, an organization representing most of the high schools in the country, was formed for the purpose of furthering the study and enjoyment of mathematics among the students in these schools.

"The main activity of the league is to sponsor meets at which students vie among each other in solving mathematics problems. Our basic purpose is to develop alertness and speed in solving a variety of problems. This is an excellent way to produce mathematical prowess in our students and to promote interest in this subject. We anticipate that the meets will encourage students in other activities that give training and practice in this art. The meets might, in a way, be compared to track meets. Five "events" are held at each meet; each "event" consists of solving a mathematics problem in a given time (usually about five or six minutes). Running scores for the teams are posted, and individual scores are kept. The names of the students who solve four or five of the problems correctly are published after each meet. Team scores are also publicized. These meets differ from athletic meets in that the league does not put one school team directly against another school team. If this were done, we would have to group schools by size. This difficulty has been obviated by the way in which teams are made up." (83, p. 223)

3.7 THE PUPIL IN THE CLASSROOM

3.7.1 The number of pupils in Arithmetic classes

The teachers who completed Questionnaire N.B.377 were asked to state the average number of pupils in their classes. The details are reflected in Table 3.26

TABLE 3.26

THE AVERAGE SIZE OF THE ARITHMETIC CLASSES

Department	Cape Province			Natal			Orange Free State			Transvaal			South West Africa	Total		
Standards	6	7	8	6	7	8	6	7	8	6	7	8	6	6	7	8
Number of pupils in class:																
0 to 9	0	3	4	0	0	0	0	1	5	1	0	0	1	2	4	9
10 to 19	4	16	11	2	2	4	2	17	11	2	4	4	0	10	39	30
20 to 29	17	26	10	10	30	28	8	19	25	12	36	59	2	49	111	122
30 to 39	11	24	7	12	16	11	15	13	3	11	117	81	3	52	170	102
40 to 49	1	2	1	0	0	0	1	0	0	0	3	2	0	2	5	3
TOTAL	33	71	33	24	48	43	26	50	44	26	160	146	6	115	329	266
Average number of pupils in class	26	25	22	29	27	25	29	24	20	27	31	30	28			

The teachers were also asked to indicate the number of pupils in their largest classes. Table 3.27 shows how many of these largest classes contained 40 or more pupils.

TABLE 3.27

NUMBER OF LARGEST CLASSES WITH FORTY OR MORE PUPILS

Department	Cape Province			Natal			Orange Free State			Transvaal			South West Africa			Total		
Standards	6	7	8	6	7	8	6	7	8	6	7	8	6	7	8	6	7	8
Number of pupils in largest class																		
40 - 49	2	1	2	0	1	1	4	1	0	1	18	11	0	0	0	7	21	14
50 and more	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1
TOTAL	2	1	3	0	1	1	5	1	0	1	18	11	0	0	0	8	21	15

3.7.2 Conclusion

Forty-four of the 710 teachers who furnished particulars had classes containing forty or more pupils, while 15 classes had fewer than ten pupils. The overwhelming majority of the largest classes had between twenty and forty pupils.

3.7.3 The number of pupils in the classes for General Mathematics

Table 3.28 reflects the number of teachers with classes of the average given size.

TABLE 3.28

THE AVERAGE SIZE OF THE CLASSES FOR GENERAL MATHEMATICS

Department	Cape Province			Natal			Orange Free State			Transvaal			South West Africa			Total		
Standards	6	7	8	6	7	8	6	7	8	6	7	8	6	7	8	6	7	8
Number of pupils in class:																		
0 to 9	5	19	31	0	1	2	0	0	0	0	0	0	0	0	0	5	20	33
10 to 19	17	42	56	0	0	0	3	0	1	4	3	2	0	3	5	24	48	64
20 to 29	71	40	56	0	0	0	3	2	0	33	20	11	2	1	3	109	63	70
30 to 39	85	47	31	1	0	0	1	0	0	161	29	25	5	1	0	253	77	56
40 to 49	9	7	1	0	0	0	0	0	0	4	1	0	0	0	0	13	8	1
50 and more	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
TOTAL	188	155	175	1	1	2	7	2	1	202	53	38	7	5	8	405	216	224
Average number of pupils in class	28	22	20	34	8	9	21	23	16	33	29	29	31	22	18			

Since the figures given here merely show the average size of class, they do not give a picture of the largest classes which teachers have. In Table 3.29 an indication is given of the number of teachers who have the largest classes containing forty or more pupils.

TABLE 3.29

THE LARGEST CLASSES IN GENERAL MATHEMATICS

Department	Cape Province			Transvaal			Total		
Standards	6	7	8	6	7	8	6	7	8
Number of pupils in class:									
40 to 49	19	10	4	16	3	2	35	13	6
50 and more	2	1	0	0	0	0	2	1	0
TOTAL	21	11	4	16	3	2	37	14	6

3.7.4 Conclusion

In the Cape Province a considerable number of small classes are encountered. There are fifty-five teachers whose classes contain fewer than ten pupils. On the other hand there are 33 teachers with classes containing from forty to forty-nine pupils and three teachers who have even more than fifty pupils in a class. The Cape Province is of course a very extensive area and this fact explains why some classes are so small. This is, however, no reason why some classes should be so large.

In the Transvaal there are nine teachers with classes containing fewer than twenty pupils and twenty-one teachers with classes in which there are forty or more pupils.

The majority of the classes have from twenty to thirty-nine pupils.

3.7.5 The number of pupils in the classes for Mathematics

The number of teachers with classes of the average given size are shown in Tables 3.30 and 3.31.

TABLE 3.30

THE AVERAGE SIZE OF THE MATHEMATICS CLASSES IN STDS. 6 TOT 8

Department	Cape Province			Natal			Orange Free State			Transvaal			South West Africa		Total		
Standards	6	7	8	6	7	8	6	7	8	6	7	8	6	7	6	7	8
Number of pupils in class:																	
0 to 9	0	6	10	0	0	1	0	9	16	0	2	3	0	0	0	17	30
10 to 19	11	24	12	0	11	6	9	17	18	0	16	22	0	0	20	68	58
20 to 29	13	17	17	9	22	35	22	13	14	3	52	73	0	1	47	105	139
30 to 39	24	15	8	12	11	6	14	6	4	16	75	71	1	0	67	107	89
40 to 49	6	2	1	0	0	0	1	0	1	1	5	2	0	0	8	7	4
TOTAL	54	64	48	21	44	48	46	45	53	20	150	171	1	1	142	304	320
Average number of pupils in class	29	21	21	30	24	23	26	18	16	32	29	27	30	25			

TABLE 3.31

THE AVERAGE SIZE OF THE MATHEMATICS CLASSES IN STDS. 9 AND 10

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Total	
Standards	9	10	9	10	9	10	9	10	9	10	9	10
Number of pupils in class:												
0 to 9	49	53	1	3	15	21	5	6	0	1	70	84
10 to 19	48	56	10	12	25	14	34	66	5	1	122	149
20 to 29	48	36	25	19	9	13	105	68	0	1	187	137
30 to 39	18	7	4	1	3	1	25	9	0	0	50	18
40 to 49	3	0	0	0	0	0	0	0	0	0	3	0
TOTAL	166	152	40	35	52	49	169	149	5	3	432	388
Average number of pupils in class	18	15	21	20	14	13	24	20	13	16		

Table 3.32 shows the number of teachers whose largest classes have forty or more pupils.

TABLE 3.32

THE LARGEST CLASSES IN MATHEMATICS

Department	Cape Province				Natal	Orange Free State			Transvaal				Total			
Standards	6	7	8	9	6	6	7	8	6	7	8	9	6	7	8	9
Number of pupils in class:																
40 to 49	6	5	0	4	5	3	1	2	2	14	8	2	16	20	10	6
50 and more	1	0	0	0	0	1	0	0	0	0	0	0	2	0	0	0
TOTAL	7	5	0	4	5	4	1	2	2	14	8	2	18	20	10	6

3.7.6 Conclusion

There is no Std. 10 Mathematics class with forty or more pupils. A total of 54 teachers however had 40 or more pupils in the class in the lower standards.

Of the 820 teachers who had Std. 9 and Std. 10 classes, 425 (51.8%) had an average of fewer than twenty pupils in their Mathematics classes.

Of the 766 teachers who furnished data in respect of the size of the Std. 6, Std. 7 and Std. 8 classes, 193 (25.2%) had classes with fewer than 20 pupils, while 48 (6.3%) had classes containing 40 or more pupils.

It is obviously realised that classes with 40 or more pupils cannot result in good work being produced. Consequently not a single Std. 10 class had that number of pupils.

Classes with fewer than 10 pupils are encountered more often in the case of Std. 9 and Std. 10 than in the lower classes. It is noteworthy that in

Std. 6 no teacher had fewer than 10 pupils in his Mathematics class. In the standards that follow, the number of teachers with fewer than 10 pupils in a class rapidly increases, namely 17, 30, 70, 84. These figures show how the numbers of pupils gradually dwindle. This strong trend, particularly in the Cape Province, where the pattern is 6, 10, 49, 53, is possibly due to the fact that in the rural areas some parents send their children to schools in the larger centres after the latter have passed Std. 8.

3.8 THE EFFECTIVENESS OF THE MATHEMATICS TUITION WHICH PUPILS RECEIVE AT HIGH SCHOOLS

3.8.1 The opinion of university lecturers

Lecturers in subjects in respect of which it is a requirement that the students should at least have passed the matriculation standard (or obtained exemption) were asked to indicate what causes possibly contribute to the failure of first-year students. They had to distinguish between factors which

(1) frequently occur amongst matriculants,
(2) may sometimes be a cause, and
(3) do not occur to any significant extent amongst matriculants.

Table 3.33 reflects the response of fifty university lecturers.

TABLE 3.33

CAUSES WHICH POSSIBLY CONTRIBUTE TO THE FAILURE OF FIRST-YEAR STUDENTS

Factors in the case of first-year students	Number of lecturers who consider that the factors occur frequently	occur sometimes	seldom occur	No reply received
Inadequate conception of the mathematical approach	34	6	3	7
Inadequate mathematical conceptual ability	32	8	1	9
Factual knowledge without insight	32	8	3	7
Inability to study independently	31	12	2	5
Inadequate deductive reasoning	22	17	1	10
Inadequate inductive reasoning	21	12	6	11
Inadequate knowledge of elementary algebraic operations	20	21	4	5
Inadequate mathematical ability	20	20	3	7
Negligence in calculations	20	19	6	5
Inability to obtain necessary data or literature on a subject	20	6	16	8
Insufficient allocation of time to Mathematics at high school	13	7	18	12
Resentment towards subject in general	8	19	15	8

3.8.2 Condusion

The foregoing factors can be divided into four groups, namely

First group: Factors 1 to 3: The most important feature in the case of first-year students who are in danger of not making the grade, is a lack of mathematical insight. This reveals itself in the form of a lack of the necessary mathematical method of approach and comprehensive ability, while incorrect teaching methods result in students having the factual knowledge without the necessary insight. The following examples of certain factors are specially mentioned:

Students are able to calculate the gradient of a straight line without associating it with the slope of a line.

They are unable to solve an equation by any other method than the standard case taught to them. They can manipulate logarithms without understanding the meaning of indices.

They are able to read off numerical values from a graph without linking them with the problem concerned.

The seriousness of the matter lies in the fact that at least 64% of the lecturers assert that these factors frequently occur and contribute to the failure of first-year students.

Second group of factors: Factors 4 to 6: Here we have to do with shortcomings which are not so specifically of a mathematical nature. The inability of students to study independently is accompanied by an underdeveloped reasoning capacity. From 42% to 62% of the lecturers assert that these factors frequently occur while a further quarter (24% to 34%) observe that they sometimes occur.

A professor of Mathematics asserts that members of his department have long since come to the conclusion that the main problem of first-year students is their general inability to reason logically. They find that this inability is caused by a lack of deductive coaching at school. Too many isolated techniques are taught and too many things are learned by heart.

Points 4, 5 and 6 of the table show how representative this verdict is.

The principal of an engineering department of a university states that he and his colleagues share the unanimous feeling that the students whom they receive for training as engineers have seldom been shown how to reason mathematically and logically and that the work of such students is a perfect example of how something which has been told to them or taught to them is merely repeated instead of being presented in a series of independent arguments or steps aimed at reaching the solution to a problem.

Nowit can be argued that the remarks of university lecturers are not so very important since only a small (selected) group of persons take Mathematics at a university. For those who do not proceed to a university, it is of paramount importance that they should have the ability to study independently and it is this very ability which is lacking in so many pupils who leave school. A further possible argument is that not all pupils wish to continue their studies, whether privately or at an institution for higher education. But even in that event it is nevertheless important that they should at least know how to think and yet 42% of the university lecturers assert that unsatisfactory deductive and inductive reasoning is frequently encountered. Only 2% of the lecturers assert that unsatisfactory deductive reasoning seldom occurs.

The picture of the Mathematics pupil as a thinker is by no means satisfactory.

Third group of factors: Factors 7 to 9: Forty per cent of the lecturers stated that they frequently had to contend with a lack of mathematical proficiency on the part of the students. A further forty per cent at least sometimes had an uphill struggle as a result of this the same shortcoming. The lack of mathematical proficiency may be ascribed primarily to a defective knowledge of elementary algebraic processes and inaccuracy in calculations. Examples of shortcomings which frequently occur are the following:

Students have no idea of proportion and rectilinearity, e.g. $y = kx$ and $y = 1+kx$.

They do not understand the presentation of functional relations by means of graphs.

They are unable to resolve vectors into components, since they are not proficient enough in the application of elementary trigonometry.

They do not know that sin (a+b) is not equal to sin a+ sin b.

They do not know what addendo and dividendo mean.

They do not understand exponentials.

In regard to the shortcomings revealed by pupils, the following can also be added to the list of things "not known" and "not understood": the inability to handle brackets.

What strikes one particularly is that the keynote of all these complaints in regard to the lack of mathematical skill is the lack of mathematical comprehension.

Fourth group of factors: Factors 10 to 12: Although teachers and university lecturers agree on many matters, as has for example been so clearly revealed in the chapter on the aims and value of the teaching of Mathematics, considerable difference of opinion exists among them in regard to the time which should be devoted to the teaching of Mathematics in the high school. Only 26% of the lecturers consider that it often occurs that pupils have suffered harm because of too little time having been given to the teaching of Mathematics in high schools. Thirty-six per cent of the lecturers assert that this occurs to an insignificant extent. There are admittedly complaints that the pupils find it difficult to do independent literature study. Attention is given to the cause of this state of affairs in the chapter on textbooks.

The attitude adopted by first-year students towards the subject also causes no real concern. When pupils have the necessary insight, interest will also be displayed because insight and interest go hand in hand.

3.8.3 Summary

Although the various factors were submitted to the lecturers in a completely random order in the Questionnaire (NB. 376, page 5), it was possible on the strength of their responses to group these factors together in a logical manner, thereby enabling a convincing picture of the shortcomings in the present teaching of Mathematics to be obtained. In the first place there is the lack of mathematical insight and understanding, in the second place there is a lack of logical reasoning and the ability to undertake independent study and in the third place there is the lack of mathematical skill, while things such as the time allotted to the teaching of Mathematics, the use of the library and the attitude of the pupils towards the subject may be important matters for the teachers but they do not unduly disturb the peace of mind of university lecturers.

CHAPTER 4

THE MATHEMATICAL SUBJECT MATTER

4.1 PUPILS AND SUBJECT MATTER

4.1.1 The new approach in the teaching of Mathematics

Recently a Mathematics booklet intended for pupils ranging from the age of eleven to thirteen years was shown to a certain prominent person. He is the chairman of many committees, including inter alia a school committee.

After he had glanced through the booklet, he gave it back with the words: "This is the most alarming reading matter which I have ever seen."

What had caused such consternation? The reading matter was after all intended as subject matter for the teaching of children from eleven to thirteen years old:

The foreword of the booklet read as follows:-

"Multiply £29.13.6½d. by 37"

"Simplify $\dfrac{3x^2 + 5}{x^2 - 5x + 6} - \dfrac{7x^2 + 4x + 1}{x^2 - 9x + 14}$"

"Prove that the base-angles of an isosceles triangle are equal."

"These types of question no longer represent Mathematics. In fact, much of the 'mathematics' traditionally taught in schools is moribund and due for reform. This does not, however, mean that everything familiar should be scrapped indiscriminately."

"Children who 'don't like mathematics' can have a change of heart when they see its relevance to the present day." (8, p. 5)

It is obvious that something exceptional has happened in the mathematical field. One generation after the other had learned mainly the same subject material at school, but a change has now set in. The most spectacular part of this is that new symbols such as $\cup$, $\cap$ and $\in$ are used. New concepts, or rather old concepts in a new guise, are now offered.

South Africa is not the only country where this possible reform in the teaching of Mathematics is causing concern.

> "Parents of 11-plus pupils in many different parts of the country are going to be shocked when they are asked to help with homework problems on such topics as sets, co-ordinates and non-decimal numbers. Teachers, too, are preparing themselves to instruct young pupils along these new lines, and last week some 150 of them attended the Schools Mathematics Project Conference at Southampton University.
>
> "Many of them had met together a year ago but last year's apprehension and fears for the success of the scheme had largely been replaced by a feeling of confidence based on the results achieved." (26, p. 297)

It would therefore appear that the inhabitants of the British Isles initially also felt rather uneasy about the matter, but that a measure of adjustment has in the meantime already taken place.

In the United States of America serious attention is also being given to the new movement in the teaching of Mathematics.

> "It is interesting to note that one of the most important factors in initiating thorough going projects in the United States originated in a university's

department of engineering. At the University of Illinois the engineering department issued a report about the inadequacy of the mathematical training of their engineering students entering it. In due course a committee was appointed to investigate the mathematical curriculum in high schools; it consisted of four members appointed by the deans of education and engineering, and by the head of the mathematics department. This committee was known as the University of Illinois Committee on School Mathematics (UICSM) and began its work in December 1951." (62, p. 32)

The leading figure in this committee is Dr. Max Beberman.

With the support of the National Science Foundation and under the guidance of E.G. Begle the School Mathematics Study Group has made modernised subject matter available in the form of experimental textbooks.

4.1.2 The pupils

An interesting feature in connection with this new subject matter is the fact that pupils frequently cope more easily with it than do their teachers.

"One of the sourer comments made about the new mathematics from time to time is that it may be all very well for the children, who haven't had time to get set in bad ways, but it is far too advanced for the teachers. This is something which, as time goes on, is unlikely to bo confined to mathematics."

But what would be the reason why pupils find this Mathematics easier than is the case with adults?

"... the new mathematics is based upon some concepts that adults find difficult and abstract because they are unfamiliar, and it is really too much to expect a primary or secondary modern school teacher who is nothing remotely like a mathematics graduate to master it after school hours."

Why is this new material easier for children than for adults?

"... the whole point about fundamental mathematical ideas is that they only appear difficult because they are unfamiliar; they are logically simpler as well as being logically more satisfactory; and it is a sound general rule that what is logically simpler can be made simpler to teach." (7, p. 580)

For the above-mentioned reason members of the Conference on Mathematics for the business world came to the following conclusion:

"One interesting observation about the teaching of some of the new symbols (i.e., additive, sets, quantifier, etc.) is that though the teachers find them frightening, the students comprehend them easily." (87, p. 9)

The idea may of course take root that a teacher should teach his pupils the subject of "sets" so that he can approach Mathematics as a subject in a completely modern manner.

"Dr. Moredock warned teachers to be wary of the teaching of 'sets' separately. 'Sets' is a part of the language of mathematics and cannot be taught as a separate entity." (87, p. 11)

Basic concepts such as "sets" should therefore not be treated as separate subjects but should permeate all the material taught.

4.1.3 The tuition

The question may now be asked whether these new concepts will not make mathematics so abstract that only the very talented pupils will be able to understand it.

"Dr. Marks concluded his talk with the observation that though mathematics has become more abstract, we have improved methods of teaching. It is now respectable to study mathematics and to enjoy learning. Students are discovering a new field of enormous fascination." (87, p. 48)

The efforts towards effecting a reform are moreover being supported by public opinion to an increasing extent.

"The need for more mathematics is the tenor of the times. Mathematics once more is the queen of the sciences. It is no longer one of the social studies - no longer merely a means of communication. There is a new respect for what mathematics can do, and a new respect for the teacher who can teach it." (87, p. 49)

These efforts to effect a metamorphosis in the teaching of Mathematics at high schools are founded on four basic suppositions:

"1. That mathematics itself is interesting and that given a proper approach and content the student taking general mathematics will recapture much of his former interest in arithmetic. This interest will serve as motivation for additional work in mathematics and a more favourable attitude toward learning additional mathematics later.

"2. That general mathematics students have been seriously underrated and, given the proper materials, they will achieve considerably above the expected level.

"3. That the world in which today's students will live will require a far greater degree of mathematical understanding and competence than the present. What is deemed 'practical' today may be totally useless tomorrow. For mathematics to be of real value to the citizen of tomorrow, it is necessary that he understand the principles, see how they relate to physical situations, and know how to apply them in a variety of situations.

"4. That these students will remember more for a longer period of time, if the few underlying principles are understood and used frequently in varying settings." (113, p. v, vi)

"It is not so much a matter of the difficulty of the subject as it is of the student being ready to develop an appetite for the subject. In order to be ready he must have some preparation. The problem is to discover what sort of preparation will serve the purpose." (96, p. 6)

"The best results, for teacher and student alike, will come from a measure of care and balance in preservation of what is good in the old ways, along with adoption of new ideas and new ways. In this connection several things deserve to be mentioned.

(1) Boredom through familiarity is much more likely to affect the teacher than the student.

(2) Exercise to attain mastery of a technique is not boring to a student so long as he is aware that he is making progress toward a desirable end. He needs to be interested in this goal, whether it be the arrival of a suitable stage of understanding for moving on to the next objective, or the acquisition of enough skill to apply what he has learned to specific problems which interest him.

(3) The justification of any particular subject or method in mathematical instruction should rest on one or more of the following:

(a) its merit, on intellectual grounds, and its value as a stimulant of mathematical interest and talent,

(b) its necessity as preparation for things which follow,

(c) its relevance to significant applications outside of mathematics."

All this should be kept within reasonable limits.

"Concepts, theories and techniques should not be multiplied beyond necessity." (96, p. 7)

4.2 TERMINOLOGY

The foregoing outline of the subject matter of Mathematics gives the impression that a new language is spoken today. It has become clear to the study groups that mathematical purity of language also has a very good educational value. The linguistically purer the manner in which mathematical terms and concepts are expressed, the more readily understandable will they be to the pupil.

"The Illinois group perhaps can be characterized by its insistence upon precise language both by the teacher and the pupil. In fact, when the words 'numeral' and 'pronumeral' are mentioned to teachers, they usually think of Beberman. Although many of us feel that the Beberman Comittee has gone too far in this preciseness of language, yet in the long run it will probably make a teacher a little more careful as to how he expresses himself. An example which I would like to offer as an example of the desirability of preciseness of statement is the reading of the mathematical statement $2^4=16$. Many teachers, both high school and college, would read this 'two to the fourth power is sixteen'. How much better it would be for the students if the teacher read this 'two raised to the fourth power is sixteen' or better still 'two exponent four is sixteen'.

To the teachers this is a matter of paramount importance.

"Elementary and secondary teachers are having to learn a new vocabulary - the vocabulary of algebra and geometry. During the 1930's when education was geared to life adjustment, mathematical terms were oversimplified to make them palatable for everyone and the vocabulary of mathematics was all but forgotten." (96, p. 30)

Today much stricter attention is given to linguistic accuracy and criticism such as the following is often heard.

"The phrase 'Changing fractions to decimals' is not only a sloppy use of the term 'decimal' - it is also misleading." (87, p. 40)

4.3 THE STAGE AT WHICH THE SUBJECT MATTER SHOULD BE OFFERED

In Great Britain this problem is approached as follows:

"Much work has been carried out on readiness for learning in connection with early reading. In this country no formal investigation has been made into conditions of 'readiness' for the learning of any section of the mathematical work. It has been generally assumed that previous attainment is a suitable criterion for 'readiness' to move on to the next step. In the early school years, intuition on the part of the teacher, has also helped. It seems likely, however, that 'readiness to learn a new process or topic in Mathematics' depends on these factors:

(i) Suitable acquaintance with past work. This past work may be in the nature of relevant perceptual framework as well as the more obvious 'preceding formal work'.

(ii) Environmental factors controlling the provision of suitable perceptual experiences at the stage about to be reached.

(iii) Factors personal to the individual, such as degree of intelligence, concentration, span of attention, willingness to go on, etc.

(iv) Individual-environment relationships controlling such elements as interest, attitude, teacher-learner communication, etc.

(v) Maturational levels largely affecting the ability for making concepts, and therefore the quality of thinking, at different stages. This is particularly important in the early years, though it may also have a relevance throughout.

(vi) Family pattern. The developmental family attitude towards the subject. This is a strong element in the case of many girls, and taking account of it is important at all stages." (57, p. 213)

In straightforward terms, the principle we have to offer is that hard thinking by teachers themselves about apparently modern mathematical concepts and methods can be productive of suitable topics and approaches to the subject at a much lower level, which will prove stimulating and worthwhile to teacher and pupil alike." (21, p 307)

The problem of handling this reform in the teaching of Mathematics is also receiving urgent attention in the Netherlands, and people in that country are not hesitating to take careful cognizance of what even the Americans have to say in connection with the subject.

(Translation) In The American Mathematical Monthly, of October, 1954 there is a detailed report on 'Mathematics for Social Scientists'. The list of items included in the report begins as follows:

1. Set algebra, relations, functions, one-to-one correspondence, equivalence relations, partitions, order relations.

2. Axiomatic development of number system including the concept of limit of a sequence. Careful definitions, but proof limited to heuristic discussions.

In a lecture on the subject Prof. Beth expressed himself as follows:

(Translation) It is doubtless fresh in your memory that the tide turned in the United States suddenly when, on the 4th October, 1957, Russia successfully launched Sputnik I. (11, p. 181)

He feels that the following three principles should be borne in mind.
(Translation)
(1) The principle that the teacher should be an expert;

(2) The principle of breadth of treatment;

(3) The principle of laying the foundations, taking as long a time of preparation as necessary, especially for difficult topics." (11, p. 182)

In the Netherlands, very high standards are set for the schoolmaster, the high school teacher, in regard to his competence.
(Translation)
According to "denkpsychologie": The principle of breadth of treatment holds that the pupil who has arrived at a phase of equilibrium must have the opportunity to pause there for some time.. (11, p. 183)

The principle of fundamental preparation finds support in the consideration that the attainment of a complicated insight demands a long series of phases of development. Thus this generally necessitates a long period of preparation with which one should therefore begin early. If one does not allow the necessary time, one will not be able to achieve the proposed objective. (11, p. 183)

He distinguishes between the following four kinds of Mathematics: (Translation)

(1) Naïve mathematics represented by Euclidean geometry, has a perceptible and elementary character; the formation of concepts is not at all exact and on this account, the proofs cannot be very strict.

(2) The first aim of Critical mathematics is to find the answer to the questions which have been raised by the development of infinitesimal calculus; the less elementary character of this material makes it necessary to pay more attention to the formation of more precise concepts.

(3) Abstract mathematics is the product of the unification of pure mathematics which necessarily occurred through her complete emancipation from the empirical and the perceptual; this emancipation proceeds from the development of critical mathematics and from the discovery of non-Euclidean geometry. Besides more exact forming of concepts abstract mathematics demands greater strictness of proof. (11, p. 179)

For the sake of completeness in my argument, I should naturally mention still one more phase as follows:

(a) Empiric mathematics ... abstract mathematics cannot be understood without critical, critical not without naïve, naïve not without empirical.

I would like to express the best that one, under extremely favourable circumstances, will be able to reach in the following scheme for secondary schools:

3 years' naïve mathematics, 2 years critical mathematics and one year abstract mathematics. (11, p. 184)

4.4 THE NECESSITY OF THIS REFORM IN THE SUBJECT MATTER OF MATHEMATICS

Are all these things really necessary? Is it not perhaps best to continue along the old lines in the teaching of Mathematics? The old well tried method of teaching Mathematics may admittedly have its shortcomings, but after all it nevertheless produced very famous mathematicians. And total dislocation will moreover result if any attempt is now made to introduce something so radically new .....

Those who cannot adapt themselves to the new demands set today, will probably be able to multiply the existing arguments. Can we manage without this reform? A. S. Mowat answers this question as follows:

"It is no longer possible to defend the teaching of the traditional mathematics in high school on grounds of custom. To anyone who attempts such a defence one might well answer in the words of Bernard Shaw's St. Joan: 'Thou art a rare noodle, master! Doing what was done last time is thy rule.'" (65, p. 14)

Those who consider that they are now treading upon very uncertain ground may well ask in despair whether Mathematics should not completely disappear from the school curriculum.

"I am not objecting to the presence of mathematics in the high school curriculum; I believe there are valid reasons for its inclusion. But I do

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object to the failure of teachers of mathematics to ask themselves what those reasons are and to adjust their methods according to their findings. I do object to the failure of curriculum makers to ask why mathematics should be part of the high school curriculum. I do object to the failure of the writers of text books to define their purposes and the failure of examiners to give serious thought to what they should be examining." (65, p. 15)

From the foregoing it will therefore be seen that a very dark picture of Mathematics at school is drawn. A. S. Mowat now deals with a second subject:

"I now address myself to the second question, 'What has made mathematics teaching in the high schools so useless and so uninspiring?' I suggest that the reason is that the curriculum makers, the textbook writers and the teachers of mathematics taken collectively have ignored the reasons .... for the teaching of mathematics or have failed to seek them out. As a result teaching has lost its sparkle and textbooks have become dull." (65, p. 16)

Before these matters can be rectified, the necessary data will first have to be collected.

"However, more details are required in order to understand the special problems that are connected with the teaching of mathematics in secondary schools, as compared with the teaching of the natural sciences such as physics and chemistry." (82, p. 34)

It will in the first place be necessary to review the development of Mathematics and then to determine what specialist fields contain subject matter which can with advantage be offered to high school pupils. The next step can be to proceed to discussion of the syllabi for mathematical subjects at the high school.

This must however merely be regarded as a preliminary step. After that there will be the opportunity for extensive and intensive research by expert committees.

(Translation) It appears to me that the problem of modernizing of mathematics teaching and related problems can only be undertaken successfully by team work. (24, p. 146)

4.5 THE DEVELOPMENT OF MATHEMATICS

4.5.1 The history of Mathematics

It is well known how geometry originated as an empirical science among the ancient Egyptians and how the ancient Greeks combined geometrical phenomena into a logical system. Algebra had an Arabian origin.

"During the seventeenth century, the efforts of Fermat, Descartes, Newton, Leibniz, and others, advanced mathematics for the first time substantially beyond the level of its Greek and Moslem origins. By the end of the eighteenth century the subject had increased greatly in volume, a disproportionate part of the new material having been contributed by the incredible Euler. At the same time, little progress was made in securing the foundations of the subject, i.e. in maintaining its character as a deductive science. This fault was remedied, during the nineteenth century, by Cauchy and others and great progress was made by a relatively small body of mathematicians who were still able to spread their efforts over most of the subject, including some of its applications. While the theories and methods developed during that period were of remarkable depth, the problems dealt with were, for the most part, still sufficiently close to intuition and to (apparent) physical reality to be comprehensible to a practitioner of a related art or science such as physics or engineering. All this changed in the course of the following sixty years. During that time, mathematics acquired an increasingly abstract character. The basic idea of an algebraic operation such as addition and multiplication, was subordinated to the notion of an algebraic structure. That is to say, the focus of interest was shifted from the individual objects, such as numbers, to the contemplation of various kinds of algebraic systems as a whole. Simultaneously, the traditional notions of analysis, such as continuity and the notion of a limit, were subordinated to a general (topological) theory which is concerned with the structure of space as we (seem to) know it, and of more general spaces. At the same time the volume of mathematics research increased enormously and much of the subject now is not beyond the grasp of a physicist or engineer but even of the average mathematician, who may well be an expert in his own field but know little of most other branches of mathematics. That is not to say that the subject has split into a number of entirely disconnected parts; on the contrary, some of the most impressive successes of modern mathematics depend on the application of the notions and intuition of one field to the problems of another field which appears to be unrelated to the first. For example, a mathematician may now regard entire functions (e g. $y = x^n$, $y = \sin x$, $y = \log x$) as points in an abstract space, may talk of the distance between two functions, and in this way may bring his geometrical intuition to bear on problems in the theory of functions. Moreover, the fact that most branches of mathematics have become more abstract does not mean that they have lost contact with physics and technology. On the contrary, there are several modern branches of the subject which have turned out to be of great importance in physics such as the theory of function spaces, mentioned above, and the theory of continuous groups which depends on a combination of algebra and analysis." (82, p. 34-5)

From the foregoing it is therefore clear that the subject Mathematics not only has a very long history but that the course of its development during the 20th century has greatly accelerated, while its function has taken place in numerous directions. In addition, further deepening has continued to occur.

4.5.2 The nature of Mathematics

In Mathematics it is so easy to become ensnared in all the techniques and skills that the importance of concept development is frequently lost sight of.

"Mathematics is much more than a collection of interesting facts and useful tools for solving special problems. Mathematics is primarily concerned with the development of important ideas. Frequently mathematicians develop an interesting mathematical theory that is apparently useless so far as the solution of practical problems is concerned. Later someone finds that this mathematics fits very nicely into the pattern of some complicated problem. The mathematics theory then becomes a useful tool." (15, p. ix)

Today all who study Mathematics stand in the midst of a very interesting period and we must agree with a speaker who asserted:

"... that mathematics today has become one of the most exciting intellectual adventures in history. It is the fastest growing and the most radically changing of the sciences. Never before has there been such a flood of new ideas and new theories. New branches of mathematics like the game theory, are being applied to human relationships, and old branches like probability are being applied to fresh areas like traffic flow and communication." (87, p. 52)

4.5.3 The characteristic features of modern Mathematics can be summarised as follows:

"I. Contemporary mathematics is classical mathematics grown mature.

II. Contemporary mathematics is classical mathematics grown selfconscious and self-critical.

III. It is also modern mathematics which developed as a more efficient way of dealing with the content of classical mathematics.

IV. Finally, it is mathematics that is more and more intimately related to man's activities in industry, social life, science and philosophy." (1, p. 441)

4.5.4 The various fields of Mathematics

Sherman distinguishes between the following fields:

"There are three major fields of mathematical work: pure or theoretical mathematics, applied mathematics, and interpretative mathematics or statistics." (86, p. 147-8)

4.6 MODERN PROBLEMS TO WHICH MATHEMATICS IS APPLIED

In 1955 a list of problems which have to be solved by mathematicians was published in the Netherlands. Today this list could be greatly lengthened. It is given here to show how essential it is that pupils should learn Mathematics. These problems were investigated by the Mathematics Centre in Amsterdam.

A. Statistical applications in medicine, biology and pharmacology

1. An epidemiological investigation of tuberculosis in Idonesia.
2. Biological standardization of insulin by experiments on rabbits.
3. The number of leucocytes and eosinophil leucocytes in blood samples from women during pregnancy, delivery and childbed.
4. Errors in counting the number of eosinophils in blood.
5. Measurements on eggs of black-headed gulls.
6. The augmentation effect of hypophysis-extract and adrenal-extract on the preputial glands of rats.
7. Scheme for diagnosing rheumatism species based on serological tests.
8. Investigation of the nutritive value of food taken by pregnant women.
9. Regeneration of rat-livers.
10. A comparison of the vitamin B^1 content of blood in old and young men.
11. Investigation of the public health of two rural districts in Holland.
12. Medicines for yaws.
13. The thickness of the layer of blubber of whales.
14. The number of times bats awake during the hibernation. Capture and recapture of bats for determining the death rate.
15. The influence of light on the growth of tadpoles.

B. Statistical application in other fields

1. Delays in the landing of aircraft.
2. Experiments on laundry cleaning methods.
3. A design of experiments in steel rolling.
4. The frequency of different types of monosyllabic words in the Dutch language.
5. Frequency of delays in a transport system.
6. Comparison of the performance of different types of instruments for repairing broken threads in a spinning mill.
7. Statistical analysis of psychological tests.
8. Comparison of practical work in elementary physics required for students in various Dutch universities.
9. Regression-analysis of the power absorbed by a ship's propeller.
10. Statistical analysis of an investigation of the so-called "earth rays" and dowsing rods.
11. Statistical work for the Flame Radiation Research Joint Committee.
12. A design for a quality control system for an electrotechnical factory.

13. Sociological research on the flood disaster in the south of the Netherlands in 1953.
14. Statistics of mixing solid particles.
15. The life-term of jet planes.
16. Research on a time-scheme for glassgrinders.

C. Problems treated by the Computation Department

1. The investigation of the shape of a fresh-water body under the dunes near Amsterdam. The investigation was carried out for the benefit of the water supply of the city.
2. Computation of zeros of polynomials in connection with vibrations in railway cars.
3. The temperature of gas particles in a hot-air engine.
4. Calculations of the tides on a river on behalf of the government.
5. Integrals of scattering factors occuring in crystallography.
6. The computation and the expansion of triple integrals originating from the theory of cosmic rays.
7. Design of ships-propellers to prevent cavitation of the propeller-blade.
8. Solution of Schrödinger equations.
9. Computation of the form of ships.
10. Radiation-functions occuring in astrophysics.
11. Wavefront in connection with soundings for geological exploration.
12. Computation of coefficients in connection with vibrating airfoils.
13. Integrals in connection with temperature distribution in the human skin.
14. Redesigning a road-system to ensure easy transport of sugarbeets in a rural district that has been flooded.
15. Computation of horoscopes.
16. The upheaval of Fenno Scandia.
17. Flutter computations for wings of aircraft.
18. Computation of the production of oil-wells.
19. Design and computation of filters for carrierwave telephony.
20. Radiation of cobalt bomb in cancer therapy.
21. Fields of radio transmitters.
22. Forces occurring in certain molecules.
23. Inversion of matrices of a high rank.
24. Flow in homogeneous porous media in connection with water-supply.
25. Boundary-layer computation for aircraft."(103, p. 100-102)

4.7 SUBJECTS WHICH SHOULD BE IN THE MATHEMATICS CURRICULA OF HIGH SCHOOLS

4.7.1 Teachers

Completed questionnaires were received from 1718 high school teachers of mathematical subjects. Approximately half of them replied to the question asking whether certain subjects should be included in the Mathematics curricula of high schools. It was also asked whether any shortcomings exist in the Mathematics curricula of high schools. The replies of the teachers are analysed in Table 4.1

TABLE 4.1

SHORTCOMINGS IN EXISTING CURRICULA FOR HIGH SCHOOL MATHEMATICS

	Teachers	
	Number	Percentage
(a) Shortcomings do exist in the Mathematics curricula	614	67.4
(b) No such shortcomings exist	297	32.6
Number of teachers who replied	911	100.0
Number of teachers who did not reply	807	
Completed questionnaires received	1718	

Approximately two-thirds (67.4%) of the teachers who submitted replies nevertheless consider that shortcomings do exist in the Mathematics curricula. If it is borne in mind that more than half of the teachers are not qualified in Mathematics, the percentage must be regarded as high.

The attitude adopted by the teachers in regard to the proposed subjects is reflected in Table 4.2

TABLE 4.2

THE OPINIONS OF TEACHERS IN REGARD TO THE PROPOSED SUBJECTS FOR INCLUSION IN THE HIGH SCHOOL SYLLABI

	Teachers						
	In favour of subjects		Not in favour of subjects		Who submitted replies	Who did not reply	Total
	Number	%	Number	%	Number	Number	
(a) Arithmetical and geometrical series	884	88.0	121	12.0	1005	713	1718
(b) Analytical Geometry	758	74.8	256	25.2	1014	704	1718
(c) The trigonometrical ratios of the sum and the difference of two angles	657	69.7	285	30.3	942	776	1718
(d) Elementary Statistical Mathematics	527	56.7	402	43.3	929	789	1718
(e) Inequalities	423	48.3	453	51.7	876	842	1718
(f) Elementary Sets	354	39.3	547	60.7	901	817	1718
(g) Elementary Theory of probability	275	32.2	580	67.8	855	863	1718

Not many teachers replied to the question asking which subjects could be omitted from the curricula. Table 4.3 shows the subjects mentioned by ten or more teachers.

TABLE 4.3

SUBJECTS WHICH CAN BE OMITTED FROM THE CURRICULUM

	Subject	Number of teachers
1.	Geometry	42
2.	Arithmetic	40
3.	Negative angles in Trigonometry	23
4.	Theoretical problems and proofs in Geometry	18
5.	The remainder theorem	17
6.	Series	16
7.	The trigonometrical ratios of the sum and the difference of two angles	12

Table 4.4 shows the subjects which can be included in the curricula according to the opinion of the given number of teachers.

TABLE 4.4

SUBJECTS WHICH CAN BE INCLUDED IN THE CURRICULUM

	Subject	Number of teachers
1.	Elementary infinitesimal calculus	132
2.	None	44
3.	More formal geometrical proofs	20
4.	The binomial theorem	18

4.7.2 Conclusion

Only a small minority of the teachers availed themselves of the opportunity to put forward suggestions.

The data appearing in Table 4.2 are valuable. In the first place it is clear that the subjects for which the minimum syllabus of the Joint Matriculation Board makes provision is acceptable to the majority of the teachers.

The syllabus for Arithmetic can possibly be extended by giving attention to elementary Statistical Mathematics, and this subject can gain new importance in the high school.

Before this matter is dealt with in greater detail, however, it is necessary to analyse the opinions of the fifty university lecturers in Mathematics and related subjects.

4.7.3 The university lecturers

Table 4.5 shows how many university lecturers are in favour of the introduction of the proposed subjects in the Mathematics curricula of high schools.

TABLE 4. 5

SUBJECTS WHICH MAY POSSIBLY BE INCLUDED IN THE HIGH SCHOOL CURRICULUM ACCORDING TO UNIVERSITY LECTURERS

Number of lecturers	30	20	50	Percentage
Specialist field of the lecturers	Physics, Engineering	Mathematics, Applied Mathematics, Statistics	Combined	Combined
Subjects:				
(a) Arithmetical and geometrical series	24	18	42	84
(b) Analytical Geometry	24	19	43	86
(c) The trigonometrical ratios of the sum and the difference of two angles	26	16	42	84
(d) Elementary Statistics	16	7	23	46
(e) Inequalities	12	16	28	56
(f) Elementary Sets	8	10	18	36
(g) Elementary Theory of Probability	17	10	27	54

According to the above-mentioned university lecturers, the specialist fields listed in Table 4. 6 provide material which may with advantage be included in the high school curricula. In some cases it will also be possible to deal with the subject matter without its being necessary for the existing syllabi to be changed; this procedure can serve to enrich the subject matter.

TABLE 4.6

SPECIALIST FIELDS PROVIDING MATERIAL FOR HIGH SCHOOL MATHEMATICS

Specialist field	Number of lecturers in favour of such a procedure
1. Analytical Geometry	34
2. Probability	25
3. Classical Mechanics	21
4. History of Mathematics	20
5. Linear Algebra	17
6. Mathematical Logic	14
7. Theory of real functions	14
8. Numerical Analysis	13
9. Statistical methods	13
10. Sets	13

4.7.4 Conclusion

It is a noteworthy fact that the teachers and the lecturers are to a large extent unanimous in their opinion regarding subjects which can be added to the Mathematics syllabi of the high schools (if these have not already been included). Not much doubt exist in the minds of both groups in regard to the desirability of arithmetical and geometrical series, analytical geometry and trigonometrical relations of the sum and the difference of two angles. Inequalities are very strongly emphasised by the lecturers in the mathematical subjects (80%), while the lecturers in Physics and Engineering (40%) are divided on this point. When the matter is viewed in this light, it is not strange that only 48.3% of the teachers would like to see inequalities enjoying a more prominent place in high school syllabi.

It is really not necessary that "sets" should be mentioned in high school syllabi, as long as teachers know how to do justice to this concept and how to make it the central idea throughout the teaching of Mathematics.

4.8 DISCUSSION OF POSSIBLE MATHEMATICAL SUBJECT MATTER FOR HIGH SCHOOLS

4.8.1 Introduction

Subject matter for the following three mathematical fields will now be discussed:

Mathematics,
Applied Mathematics, and
Interpretative Mathematics or Statistics.

After that, attention will also be given to certain aspects of Commercial Mathematics.

4.8.2 Mathematics

Consideration is given to Sets, Algebra, Linear Algebra, Geometry and Analytical Geometry, Topology, Arithmetic and Trigonometry.

4.8.3 Sets

As a result of an investigation conducted in twenty-one different countries in connection with the teaching of Mathematics, J.G. Kemeny was instructed to report on an international Mathematics congress. He reported as follows in connection with possible new curricula:

> "The most striking feature of the 21 reports is the degree of similarity in the proposals for including new topics of mathematics.
>
> "There are four areas of modern mathematics that are recommended by a majority of the reports. These are elementary set theory, and introduction to logic, some topics from modern algebra, and an introduction to probability and statistics. Equally frequent is a mention of the necessity for modernizing the language and conceptual structure of high schools mathematics." [1)]
>
> "Perhaps the most frequently mentioned topic is that of elementary set theory. The concept of a set, as well as the operation of forming unions, intersections, and complements, constitute a common conceptual foundation for all of modern mathematics. It is therefore not surprising that almost all nations favouring any modernization of the high school curriculum have advocated an early introduction to these simple, basic ideas. An attractive feature of this topic is that in a relatively short time a student may be given a feeling of the spirit of modern mathematics without involving him in undue abstraction.
>
> "It should, however, be noted that in most cases only an elementary introduction of this topic is recommended. For example, the usual 'next' topic in developing set theory is that of cardinality. Only three nations have suggested this as a possible topic for inclusion in the secondary curriculum.
>
> "Most of the reports contained frequent mentions of traditional topics whose teaching would be improved by the adoption of a more modern point of view. As one example, I shall use a unit discussed in the report from the United States. This is the treatment of equations, simultaneous and inequalities. An equation or inequality is treated as an 'open sentence'. That is, it is a mathematical assertion which in itself is neither true nor false, but becomes true or false when its variables are replaced by names of numbers or points (or more abstract objects, in advanced subjects).

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1) Underlined by the writer.

"Therefore, the solution of an equation is the search for the set for which the assertion is true This set is commonly referred to as the 'truth set' or the 'solution set'

"Thinking of solutions of equations as sets has the advantage that a student is more likely to think of the possibilities of the solution having more than one element in it or, for that matter, being the empty set. Simultaneous equations may be thought of as conjunctions of several open sentences; hence their solution consists of the intersection of the individual truth sets. This point of view makes it much easier to explain the usual algorithms for solving of simultaneous equations. The attempt in any such algorithm is to replace a set of sentences by an equivalent set, i.e., one having the same truth set, but the latter being of a form in which the nature of the solution is obvious. The approach also has the advantage that equations and inequalities may be treated in exactly the same manner. The graphing of equations and inequalities, then, simply becomes a matter of graphical representation of truth sets. In this case, the meaning of 'intersection' of solution sets becomes particularly clear."(52, p. 204-5)

As has been mentioned earlier, the School Mathematics Study Group wrote experimental textbooks in which high school Mathematics with sets as the basic concept was outlined in a very practical manner. This study group is led by Prof. E.G. Begle of Yale University, U.S.A.

"The Begle group can be characterized by its writing groups in which actual materials for classroom use are prepared, which are and will continue to be used on an experimental basis in the schools. The use of the set idea appears frequently in these materials, as well as in all of the other programs. In fact, the word 'set' has become for many people a symbol for modern mathematics. Actually, the idea of set is not new - it has been used for a long time in mathematics; it is only a convenient way of expressing oneself. We speak of the set of natural numbers, the set of even numbers, and so on. Actually, an interesting algebra of sets can be developed which may have some value for the gifted students in the high school, but it is not essential." (87, p. 25-6)

The concept of sets is nevertheless a very useful means of lifting the subject matter of Mathematics out of its present groove and of placing the entire teaching of Mathematics on a higher level.

Cockcroft and Land also sought a suitable means which teachers could use in effecting a reform. They came to the following conclusion:

"The language and notation of set theory are obvious candidates, at secondary as well as primary level, for consideration by teachers. We take a very simple example of this. In co-ordinate geometry, particularly, it is the most trivial of exercises to replace the words 'locus of a point (x, y) which moves so that' by 'set of points (x, y) such that'. Intersections and union of sets then occur naturally here, and one (perhaps minor) dividend is immediately obtained, namely students are able to draw the graph of an inequality, in which case equalities and inequalities can be treated geometrically and algebraically on an equal footing. Considerations of even such a minor kind have led interested teachers naturally to include in their courses the simple ideas of linear programming which is of course a new, and to students a very up to date, source of examples which apply the methods of algebra and geometry in setting up and solving systems of linear equations and inequalities. At primary level we obtained a parallelism of ideas, at the secondary level we naturally begin to obtain material and new approaches which can be more explicitly used in class, by applying our principle." (21, p. 316)

4.8.4 Algebra

"Algebra is one of the oldest branches of mathematics, there being historical evidence that the Babylonians of 4,000 years ago were versed in its methods. From ancient times until the end of the 18th century A.D., algebra could be roughly described as that part of mathematics which dealt with the solution of equations. The long overdue axiomatization of the real and complex number systems in the 19th century marked the beginning of modern algebra. In addition to its concern with solving equations, modern algebra supplies a language and patterns of reasoning to the rest of mathematics." (48, p. iii)

That Algebra should be learned at school will certainly not readily be denied. The reform that is necessary lies in the approach to the subject.

The stress should be laid on the fact that Algebra is a language and that a formula such as

"$a^2 - b^2 = (a+b)(a-b)$"

is a way of saying something in a complete sentence. The matter may be stated even more strongly: Algebra is a field of study written in a notation adapted to the particular kind of things which we say in ordinary unpretentious English. So for example

"$a^2 - b^2 = (a+b)(a-b)$"

is nothing but an abbreviated way of writing the following English sentence: The difference in any sequence between the squares of any two numbers is equal to the product of the sum of the two numbers and the difference, in the same sequence, of the two numbers.

Simple techniques can be used in the solution of problems of a mathematical nature. For example: What must be done to $x^{2/5}$ in order to change it into x?

Pupils must moreover also learn how quantities are changed in order to suit our purposes and how further modifications are effected in order to bring these into line with reality, as for example when a negative mantissa is dealt with by addition and subtraction. 1)

4.8.5 Linear Algebra

According to Tables 4.2 and 4.5 there is indeed room for arithmetical and geometrical series and also for inequalities in the syllabi of the high school. The question which must now be asked is whether the development should be left to take its own course or whether the more modern Algebra should also not provide material for the high schools.

It would appear that it is generally accepted that linear Algebra should be studied by the future Mathematics teachers. As a university course it is a success.

"It is generally agreed that linear algebra is an excellent subject with which to introduce students to the notion of a mathematical proof, and that it is possible to tell which students should major in mathematics by observing their reactions when they are exposed to a beginning course in linear algebra." (48, p. vii)

Pedoe, however, recommends his book, which offers a geometric introduction to Linear Algebra, for high school pupils:

"With the growth of interest in mathematics in high schools, there is every reason why this book should be studied in the higher grades, since the amount of mathematics preparation for its study is slight. The subject of linear algebra is of utmost importance today, not only in pure mathematics but also in many branches of applied mathematics, including the science of computing. Thus as with differential and integral calculus,

1) Contribution by a member of the committee

the sooner a student is exposed to linear algebra, the better for his mathematical future." (48, p. viii)

As supplementary study material for the gifted pupil who wishes to continue his studies in Mathematics, such an introduction will undoubtedly be of great value.

Kemeny reported as follows in connection with the investigation into topics from modern algebra, as applied in overseas countries:

"While a majority of reports contained a suggestion that some topics from modern algebra should be chosen, there was considerably less agreement as to what this choice should be. Basically, there seems to be a split between the advocates of teaching topics from algebraic systems (groups, rings, and fields) and those who advocate linear algebra. In a few cases, both types of topics were suggested, but usually the lack of time in high school curricula prevents the introduction of a very sizable amount of modern algebra.

"It seems to me that the motivations for these two types of topics have many common features. The introduction, on an axiomatic basis, of any modern algebra has the very healthy feature of removing the common misconception that axiomatics is somewhat closely tied with geometry. I recall once having a student who told me that, in his experience, the difference between algebra and geometry was that 'in geometry you proved things, while in algebra somebody just told you what to do'. Certainly, this objective can be equally well achieved by introducing as one's basic axiomatic system either that of a group or that of a vector space.[1]

"In addition to this, either linear algebra or algebraic systems have the advantage of giving deeper insight into certain structures known to the students for other reasons. Linear algebra, of course, has many applications to geometry, while algebraic structures arise as generalizations of one's experience with numbers.

"The usual argument given for the introduction of groups, rings, and fields is that this is the only way one can bring about a true understanding of the nature of our number system. Attempts to prove to the student simple rules, such as those governing the operations with fractions, often fail because both the basic assumptions and the results to be proven are too familiar to the student. However, by moving to an abstract axiomatic system, the student is forced to abandon his intuition and rely on mathematical rigor in his proof.

"It may certainly be said, if one wishes to introduce one example of an axiomatic system in modern algebra, that the simplest and most universally useful one is that for a group. It also has the attractive feature that, in addition to being applicable to many groups of numbers well known to the student, one can introduce such simple and interesting examples as the symmetries of a simple geometric object (e.g. a square)." (52, p. 199-200)

4.8.6 Geometry and Analytical Geometry

Geometry is a subject upon which the limelight is being focussed to an increasing extent. According to Table 4.3 there are a number of teachers who consider that this subject should be completely omitted from the high school curriculum.

Since Geometry has to do with spatial relations, it is not clear whether such teachers are of the opinion that spatial relations need to be studied.

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[1] Underlined by the writer.

Or is the objection against Euclidean Geometry as such?

"It is a common belief that plane geometry was completed by Euclid 2,000 years ago and that nothing has been added to it or taken from it since. This is simply not true. That there are logical gaps in Euclid's presentation has been known for a long time. Means to remedy these deficiencies have been known for about sixty years, but strangely enough a mathematically adequate and yet elementary treatment of plane geometry in the spirit of Eudid has not appeared in print ..... The current interest in improvement of the secondary school curriculum makes this an appropriate time for such an attempt." (15, p. ix)

In addition attention must also be given to the question whether the Geometry of Euclid is the only geometry which should be taught. O.G. Sutton replies to this question as follows:

"One of the most 'pure' of all branches of mathematics is geometry, which sprang from the prosaic problem of fixing the boundaries of fields in ancient Egypt. The geometry which has been derived from such simple consideration is called Euclidean ..... To the mathematician it is only one of the geometries (and a rather dull one at that), all of which deal with the interrelations of complexes of abstract conceptions called, mainly for convenience, 'points', 'lines', 'figures', and 'surfaces'." (94, p. 1-2)

This leaves the problem still far from solution and it will be worthwhile to see what Kemeny reported about the present position in other countries:

"There seems to be general agreement that the teaching of high school geometry must be modernized, but there is a certain lack of ideas as to how this should be achieved. I recall the detailed debate at the 1958 International Congress on this particular topic, and I am under the impression that this problem is still far from settled.

"For example, the School Mathematics Study Group in the United States wrote single textbooks for each of six years for junior high school and high school mathematics. However, in the case of the tenth year, there are already two different versions of geometry available, and there may very well be a third version. This is a clearcut indication of the lack of agreement amongst leading mathematicians in the United States as to the 'right' way of teaching geometry.

"The most constructive suggestions on this topic seem to be contained in the report from Germany ..... I share the astonishment expressed by the German reporter that high school geometry has remained so terribly tradition-bound, even in the face of many changes in the teaching of algebra, and the introduction of more advanced topics. We must choose between a 2,000-year-old tradition of teaching synthetic geometry in the manner of Euclid, or oi destroying the 'purity' of geometry by the introduction of algebraic ideas. Of course, Felix Klein established a very important trend in Germany, which spread throughout the world, to attempt to build a classification of geometries by means of the transformations which leave certain geometric properties invariant. This points to the importance of the study of geometric transformations, even within high school geometry. There is also an increasing tendency to introduce metric ideas early into synthetic geometry and in many countries even an introduction to analytic geometry is part of the first year's geometry course.

"The introduction of vectors is quite generally advocated. In Germany vectors are introduced in the context of metric (as opposed to affine) geometry. However, this does not mean that vectors are tied to analytic geometry, since vector methods are used as a substitute for the introduction of a coordinate system. This approach is particularly useful in bringing out the analogy between the geometries of two, three, and more dimensions.

"A conference sponsored by ICMI at Aarhus, in Denmark, in 1960, advocated the development of a 'pure' vector geometry, in which affine geometry is built up in terms of vector ideas. While the concept of vectors free of coordinate systems may be somewhat more difficult for the beginning student to understand, many geometric proofs actually become much simpler if vectors are treated as coordinate-free. For example, this is by far the easiest way to prove that medians of a triangle meet at one point and divide each other in a 2:1 ratio.

"While there are still many advocates of treating a full axiomatic system of Euclidean geometry purely synthetically, it is becoming increasingly clear that one must either 'cheat' or demand more of the student than can be expected of him in his high school years. Even Euclid's original axiom system is a great deal more complex than is ideal for the high school student's first introduction to axiomatic mathematics. In addition, it is well known that Euclid in many places substituted intuition or the drawing of a diagram for mathematical rigor. Indeed, many of Euclid's propositions do not follow from his axioms. While several outstandingly fine axiom systems have been constructed that make Euclidean synthetic geometry rigorous (notably the system of Hilbert), these require a degree of mathematical maturity not to be expected of the secondary school student.

"The report from Israel feels that the axiomatic treatment of geometry in high school is as unrealistic as using Peano's postulates in elementary school. The report from the United States, in contrast, advocates that certain segments of Euclidean geometry be taught rigorously to give the student experience in proving theorems from axioms, but that the gaps in between be filled in by a more intuitive presentation, in which the emphasis should be in teaching students the 'facts of geometry'. An alternative to this is the much heavier reliance on the properties of real numbers to fill in gaps in Euclid's axiom system.

"Three reports advocated the inclusion of non-Euclidean geometry as part of the first treatment of Euclid. The argument for this is similar to the argument for teaching algebraic systems to improve the students' understanding of number systems. That is, if the student is forced to reason in a geometric framework other than the one he is used to, he is more likely to understand the power of the deductive system and to appreciate proofs he has seen in Euclidean geometry. I should like to add a plea that even in courses where no actual non-Euclidean geometry is taught, the student should at least be informed that such geometries do exist, and perhaps a day or two be spent discussing them. It seems to me to be a major cultural crime of most mathematical educational systems that 130 years after an invention on non-Euclidean geometry, most students(and I am afraid many teachers) are not aware of the possibility of a non-Euclidean geometry. Indeed, the statement that our universe is only approximately Euclidean, according to relativity theory - it may both in the small and the large be non-Euclidean - comes as a great shock to many pedagogues." (52, p. 201-203)

4.8.7 Topology

There is however still another specialist field of Mathematics in which space is studied. Whereas pupils in Geometry measure spatial objects and determine their corresponding relations, the students of topology are interested in other spatial aspects.

"Today mathematics investigates patterns and structures in such topics as quantity (number), measurement (metric geometry), operations (e.g. group theory), relations (e.g. order) and even logic (in the shape of form and logic).

"One of the best examples, however, is topology, itself a creation of modern mathematics. Of course, like most other conceptual edifices of any magnitude, it has roots in antiquity, and like most of the rest of mathematics can be traced to 'applications.' Except for the wellknown 'polyhedral formula' (usually called the 'Euler polyhedral formula', but apparently known to Descartes a hundred years earlier than Euler's discovery) and the 'Königsberg bridge problem' (Euler, 1735) most of the identifiable beginnings of topology are to be found in the nineteenth century." (114, p. 462)

"From its history ..... one would probably conclude that topology must be a geometry. And so it was, at least during the formative years.

"Just as in elementary Euclidean geometry we consider all figures equivalent that can be moved into coincidence with one another by rigid motion, so in topology one considers as equivalent any two figures which can be made to coincide by a motion that is quite arbitrary so long as it involves neither tearing nor folding; in particular stretching is allowed (and its converse slackening) hence, the name 'rubber sheet geometry'." (114, p. 464)

That the idea of teaching topology at school has not yet gained much ground is clear from the following report of Kemeny:

"Suggestions of a brief introduction to topology are contained in four reports. The French report proposes that an intuitive notion of neighbourhoods be given to students and on this one should base the concept of the convergence of a sequence (or the failure of convergence) and that these ideas should be used to lead in a natural way to the concept of limits and continuity. These can in turn be used to explain such geometric ideas as that of a tangent or of an asymptote. Germany and Israel make similar suggestions.

"A more ambitious program is outlined in the Polish report. The proposal is that most of the treatment be restricted to the topology of Euclidean space of one, two, and three dimensions. Starting with these well-known spaces, the concept of a metric space should be developed, and, in turn, illustrated on such examples as n-dimensional space, the space of continuous functions, and Hilbert space. The Polish programme would start with the same concepts as mentioned above from the French report. More concretely, it is suggested that discussions without proofs should be given of the Jordan-curve theorem, classification of polyhedral surfaces, and some examples of non-orientability of surfaces. The unit would terminate with a discussion of Euler's theorem.

"Your reporter would like to add his support to this suggestion, even though it may sound quite extreme. While these topics may be too difficult for the average high school student, I know from personal experience that the really bright student, in his last year of high school is fascinated by elementary topological ideas. Such as unit should be entirely practical as long as it is closely tied to concrete examples familiar to the student." (52, p. 203-4)

4.8.8 Arithmetic

Arithmetic is a very important subject in the primary school where considerable time is devoted to it, whereas in the high school a tendency exists to relegate it to the background and to effect its faily rapid disappearance from the scene.

"Karl Menger noted that the fundamental operations of arithmetic usually have been explored by pupils through the first six grades of elementary school and that in grades seven and eight boredom often sets in because the lessons are essentially repetition and not new discovery.

"It was E.G. Begle's conviction that the average student can learn and perform well in mathematics if he is allowed to advance at a slower pace and is provided with less sophisticated presentations than the very capable learner." (58, p. 81)

Just as Arithmetic is tackled gradually in the primary school, so the other parts of Mathematics must also be approached with the necessary respect in the initial stages. Quite possibly the slow learner will in due course also make good progress in the subject.

The linking up of Arithmetic with the rest of Mathematics received the special attention of Brumfiel and his co-writers of the book "Introduction to Mathematics". They concentrated on writing a mathematical arithmetic. In the introduction to the accompanying manual for teachers they write as follows:

"You may be somewhat surprised at the conspicuous omission of social arithmetic. One of the purposes of this book is to prepare the student for later courses in the curriculum. Therefore, as a pre-algebra and pre-geometry text, it does not explore topics that would be unrelated to the students' school experience. Furthermore, we are also convinced that this material gives even the student for whom this is almost a terminal mathematics course the best possible preparation for the applications of mathematics that he will face. Some mathematics must be learned before it can be applied." (17, p. iv)

"In many sections the student is expected neither to gain great skill nor to learn particular facts. Some examples of these are the work on number bases, modular arithmetic, and number theory. These topics are inserted in the text for several reasons. First and foremost, it has been our experience that students become excited about these things, and many begin for the first time to enjoy mathematics. This alone justifies inclusion of these topics. In most cases this enrichment material sheds new light upon important basic ideas. For example, the study of number bases illuminates the role of symbols in mathematics, and modular arithmetic provides an excellent environment for emphasizing the importance of the basic commutative, associative etc., laws." (17, p. iii)

In order to make Arithmetic at the high school an interesting and vital subject it is not necessary to become increasingly involved in the applications of arithmetical processes. The talented pupil in particular can spend many enjoyable and constructive hours with higher arithmetic.

What is "Higher Arithmetic"?

"The higher arithmetic, or the theory of numbers, is concerned with the properties of the natural numbers 1, 2, 3 ..... These numbers must have exercised human curiosity from a very early period; and in all the records of ancient civilization there is evidence of some pre-occupation with arithmetic over and above the needs of everyday life. But as a systematic and independent science, the higher arithmetic is entirely a creation of modern times, and can be said to date from the discoveries of Fermat (1601-1665).

"'It is just this,' said Gauss, 'which gives the higher arithmetic that magical charm which has made it the favourite science of the greatest mathematicians, not to mention its inexhaustible wealth, wherein it so greatly surpasses other parts of mathematics.'

"The theory of numbers is generally considered to be the 'purest' branch of pure mathematics. It certainly has very few direct applications to other sciences, but it has one feature in common with them, namely the inspiration which it derives from experiment, which takes the form of testing possible general theorems by numerical examples. Such experiment, though necessary in some form to progress in every part of mathematics, has played a greater part in the development of the theory of numbers than elsewhere; for in other branches of mathematics the evidence found in this way is too often fragmentary and misleading." (27, p. vii)

"A careful study of the use of arithmetic has shown that it is imperative for the student to understand that problems manifest themselves in four 'faces'. The normal task in applied mathematics is to translate from one face to another. These four faces are:

1. The physical model or situation where no numbers are attached. Thus the student soon learns that there are two ways of 'attaching' numbers; counting and measuring.

2. The table of ordered pairs. The student learns that most physical situations can be described by ordered pairs.

3. A graph of the table of ordered pairs. This is a graphical representation of the physical situation. The student must learn to translate this face into its physical meaning and appreciate the information a graph can convey.

4. A formula or equation. This is the algebraic face which can be manipulated by observing simple underlying principles to produce new facts. The results can be reinterpreted into the physical setting." (113, p. vi)

4.8.9 Trigonometry

In the university entrance course more emphasis should be laid on the analytical aspects than on the manipulation. If the students really know the basic definitions and identities and have had sufficient exercise in the application of algebraic techniques in Trigonometry, and much less exercise in the solution of obtuse-angled triangles through the use of four-figure tables of logarithms' with interpolations, they will be in a much better position to master infinitesimal calculus.[1)] In order to determine what place Trigonometry should occupy in the high school curriculum and in the Mathematics syllabi, attention should first be given to the idea of a mathematical structure.

4.9 THE IDEA OF A MATHEMATICAL STRUCTURE

4.9.1 The idea of the whole of mathematics as a unit

That pupils should learn to think has become a general demand. An attempt must be made to find the secret of reasoning. By that is meant scientific reasoning. Since science is properly marshalled knowledge, scientific reasoning can develop best where a definite scheme of thinking can lead the reasoning in the right direction and provide the design for the correct content.

"The emphasis upon mathematical reasoning and abstract thought in general, which the genius of Pythagoras had instigated, reached its climax in the theoretical acumen of Plato (428-348/7 B. C.), who rejected the experimental method with ardour and contempt. In his view, no precise study of the everchaning phenomena in the natural universe was possible, and it was only in the philosophic theory of forms and in the science of pure mathematics that absolute knowledge could be attained. These contemplative disciplines of the intellect dealt with objects timeless and invariant, known independently of experience and existing logically prior to the material world, which could at most be merely an approximation to eternal forms or ideas." (117, p. 3)

What must therefore be sought is an idea which can give stature to the mathematical line of reasoning. This is a matter of great importance for the methodology of the teaching of this subject.

"The classroom philosophy of the teacher should encourage students to think and to discover. Unfortunately, the early mathematical experiences .... (of) so many students have convinced them that in mathematics they must be told what to do. They feel that the only thing of importance is to

[1)] Contribution of a member of the Committee.

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find the right answers."

It demands the utmost exertion and effort on the part of a teacher to protect his teaching against this petrification.

> "However, learning is most significant when the student understands the march of ideas and shares in the discovery of basic concepts." (17, p. iii)

In order to ensure this the teacher should have the necessary far-sighted outlook. He should be able to indicate clear guiding lines among all the subject material.

> "In general, we do not want students to exert great effort to remember a mass of material. Rather, we want students to expend their energy thinking about concepts and discovering mathematical relationships that are new to them." (17, p. iii)

The key for the right approach to the subject must now be sought.

4.9.2 The axiomatic approach

> "Twentieth-century mathematics has been, and is, growing with unprecedented speed and vigor in a number of directions. It is of course important for well-informed people in general, and teachers of mathematics in particular, to know something of these various new developments. However, if we had to pick out a single development which is most nearly the key to the modern spirit in mathematics, it would be the widespread application of the axiomatic approach. The axiomatic approach itself is far from new: it goes back to the Greeks of the fourth century B. C. and earlier. But the effective application of this approach to literally all of mathematics is largely a development of the twentieth century.
>
> "Everyone has a rough idea of what is meant by the axiomatic approach to a subject. In this approach we clearly set forth at the outset (1) the basic notions to be dealt with and (2) the properties these notions are to be assumed to possess. The statements asserting these assumed properties are usually called axioms. We then proceed, defining new notions as needed in terms of the basic ones and deducing new assertions, called theorems, from the axioms and from previously proved theorems, by the methods of logical deduction. The basic notions in the axiomatic approach to a subject are sometimes called undefined notions to emphasize our intention not to take for granted any properties of these notions other than those specifically assumed in the axioms. Nothing, however, prevents our choice of axioms from being guided or motivated by our intuitive ideas about the subject.
>
> "This will all be clearer if we consider an example. Probably for most people the nearest thing to a familiar example of the axiomatic approach is the ordinary plane geometry of high school. There is an additional reason for choosing this particular example. It stems from Euclid's Elements of Geometry (300 B. C.), which was the first, and for many centuries virtually the only, sustained exposition of a subject from the axiomatic point of view." (12, p. 498)

4.9.3 A Mathematical Structure

> "It has sometimes been suggested that the essential feature of modern mathematics is the employment of the axiomatic method. This is, I think, an oversimplification. Systems of axioms have been a familiar feature of mathematics since the time of Euclid. However, most of the older systems of axioms had one thing in common; they were attempts to give, in axiomatic form, a complete characterisation of one particular mathematics structure, e.g. of Euclidean geometry or of the positive integers or of the real numbers. In other words, they were univalent systems; two structures satisfying such a system were automatically

isomorphic, and therefore from an abstract point of view indistinguishable from one another. On the other hand, most of the axiomatic systems used today are multivalent; there are, for example, an infinity of different mathematical structures that satisfy the axioms of a group or those of a topological space. The system of axioms thus characterises not a single mathematical structure, but a type or 'category' of such structures." (58, p. 296-7)

"When mathematics is seen as the study of logical structure, there is far less temptation to fall into recipe teaching; the fundamental business of mathematics is then seen as the disentangling of a simple logical structure from an apparently complicated situation, and the employment of the properties of that structure to obtain better command of the situation." (58, p. 297-8)

When the idea of a mathematical structure becomes clear, the importance of Euclidean geometry comes strongly to the fore again.

"Mathematicians seem to have forgotton that for over two thousand years Euclid's geometry was a model of vigor. Not even the greatest mathematicians observed its deficiencies. Similarly the foundation of the real number system was not created until the latter part of the nineteenth century.

"Neither Euler nor Gauss could have defined a real number, and it is unlikely that they would have enjoyed the gory details." (53, p. vii)

It now also becomes obvious why an attempt must be made to seek the development of a clear number structure in Algebra.

"A development of the structure of a system of numbers should begin with undefined elements (as few as possible), postulates concerning use of them (as few as possible) definitions of addition and multiplication of them, and, finally, theorems that express derived conclusions concerning operations with them. Complete discussion of the system of whole numbers is too difficult for elementary algebra." (40, p. vii)

It remains the teacher's task, however, to continue giving attention to this idea of a whole mathematical structure, irrespective of the specialist field of mathematics in which he may be engaged.

"On every mathematics teacher in the country today falls a responsibility to prepare himself for substantial changes in his subject. New curriculums are requiring him to emphasize the unifying principles and structure of mathematics, the study of number systems and of numeration system to bases other than 10, logic and deductive thinking, the language of sets and elementary set theory, and a more precise mathematics vocabulary." (85, p. 29)

"The mathematician sees the number system as a complex of interrelated structures. He studies these structures separately, and in their relationships to each other. The exploration of these structures has revealed that we have, not a number system, but number systems; not algebra but algebras; not geometry, but geometries; not space, but spaces. While the properties of numbers and space have been generalized the subject matter of mathematics has been pluralized." (3, p. 9)

Algebra

"Any newcomer to the subject, reading a book on modern algebra, will soon come to realize that this subject is one in which the construction and structure of algebraic structures play the major role. Manipulative skills and conceptional understanding play equal roles. We take this, probably the most important lesson to be learnt from algebra at the higher level, as an example of what a teacher may find in studying this subject." (21, p. 316-7)

"Not all these concepts have been found to be of explicit use as formulated in the abstract, but all have in one way or another led to new thinking and the introduction of concrete examples at a lower level." (21, p. 317)

"Thus, if we consider three familiar ideas - the addition of real numbers, the multiplication of the numbers in a finite field, and the result of performing in succession two displacements in Euclidean space - and, for all three, study only the skeleton remaining when each is thought of as set of abstract elements with an appropriate law of combination, we quickly see that each can be described as a group. And properties of the three may be studied together by the axiomatic theory of groups.

"The group is an example of one of the three basic mathematical structures that we now recognize. It is one kind of so-called 'algebraic' structure. The other two basic structures are called 'ordered' and 'topological' and each can be described abstractly, the first concerning itself with a generalization of the usual 'less than or equal to' relation, the second with the notion of continuity. Modern mathematics is increasingly concerned with systems that satisfy at once the axioms for two different kinds of structure." (81, p. 436)

4.9.4 The aims of the teaching of Mathematics seen in the light of the structure idea

An international symposium on school Mathematics put the purpose of teaching of Mathematics as follows:

(Translation)

Concerning the aims of the teaching of Mathematics there is general agreement that pupils should:

(1) master mathematical structures,
(2) make themselves acquainted with the valid relations therein,
(3) be able to express these properties in different ways (schematic representation, ordinary language, symbolic notations),
(4) recognize and determine the logical connection between these relations ..... (46, p. 113-4)

4.9.5 The teaching of Mathematics in the future

H. G. Brinkman summarised his view of the form which the high school teaching of Mathematics would assume in the future, as follows (Translation):

In order to obtain good overall picture it is desirable to devide it into three:

A. changes which are chiefly in didactics in the narrow sense, and in method,
B. expansion of the subject matter in breadth,
C. extension of the subject matter in depth,
A1. suiting the notation to that of modern mathematics,
A2. improving the nomenclature,
A3. improving the discipline (accuracy) in the use of the vernacular in Mathematics (13, p. 308-9),
A4. stating the central concepts as sets, variable, function and relation and visualization of structures (13, p. 310),
A5. more emphasis on the deductive method
A6. using formal logic (13, p. 311)
A7. giving attention to the question as to why mathematics is applicable
A8. improvement of the preparation for independent study at a university or high school (13, p. 313),

B1. treatment of vectors
B2. complex numbers

B3. determinants
B4. theory of errors
B5. graphical solution of equations
B6. use of slide rule and logarithmic paper, etc.
B7. finer differences applied to the summation of sequences and the treatment of methods of interpolation
B8. combinatories
B9. statistics

C1. modern algebra
C2. axiomatics
C3. topology (13, p. 315)

4.10 APPLIED MATHEMATICS

4.10.1 The present position in South Africa

In Questionnaire N.B.375 school principals were asked whether there were pupils who took Mechanics as one of the school subjects in their schools. Table 4.7 reflects the details furnished in their replies.

TABLE 4.7

THE NUMBER OF SCHOOLS AT WHICH MECHANICS IS OFFERED AS A SCHOOL SUBJECT

Department	Mechanics	
	offered as a subject	not offered as a subject
Cape of Good Hope	1	188
Natal	0	34
Orange Free State	0	63
Transvaal	3	123
South West Africa	0	8
Education, Arts and Science	16	30
TOTAL	20	446

4.10.2 Conclusion

With the exception of some schools of the Department of Education, Arts and Science, only four public schools (of those from which completed questionnaires were received) offer Mechanics as a school subject.

4.10.3 Extramural Mechanics

The question was also asked whether any pupils take Mechanics extramurally. The replies are shown in the date given in Table 4.8.

In the schools in the Cape Province some pupils take Mechanics instead of Chemistry and Physical Science, while others take it as a seventh subject.

TABLE 4.8

THE NUMBER OF SCHOOLS WITH PUPILS TAKING MECHANICS EXTRAMURALLY

Department	Mechanics	
	taken extramurally	not taken extramurally
Cape of Good Hope	0	182
Natal	0	33
Orange Free State	0	62
Transvaal	8	118
South West Africa	0	8
Education, Arts and Science	2	41
TOTAL	10	444

4.10.4 Conclusion

Only ten schools reported that some of their pupils took Mechanics extramurally.

4.10.5 Further particulars

In 8 schools in the Transvaal Mechanics is taken by the pupils as a seventh subject as well as Physical Science and Chemistry.

4.10.6 The number of pupils taking Mechanics in Stds. 9 and 10

Table 4.9 shows how many pupils take Mechanics.

TABLE 4.9

THE NUMBER OF PUPILS TAKING MECHANICS

Standard	Cape Province	Transvaal	Education, Arts and Science	Total
9	18	36	254	308
10	7	62	287	356
TOTAL	25	98	541	664

4.10.7 The reasons why schools do not offer Mechanics

Table 4.10 gives an analysis of the number of schools which for the given reasons, do not offer Mechanics as school subject.

TABLE 4.10

REASONS WHY MECHANICS IS NOT OFFERED AS A SCHOOL SUBJECT

Reasons	Department						Total
	Cape Province	Natal	Orange Free State	Trans-vaal	South West Africa	Education, Arts and Science	
1. The curriculum is already overloaded; trained teachers are difficult to obtain; better justice is done to the work in technical schools and colleges	27	3	28	43	0	1	102
2. The subject does not occur in the curriculum	76	10	19	2	3	11	121
3. The highest class in the school is Std. 8	5	1	1	1	1	1	10
4. The school is too small or not properly equipped	18	3	4	19	1	3	48
5. The girls display no interest	9	4	0	9	0	2	24
6. No consideration as yet given to the matter	11	8	3	32	0	3	57
TOTAL	146	29	55	106	5	21	362

4.10.8 Conclusion

Although some school principals stated in their replies to the questionnaires that Mechanics does not occur in the cirriculum the answer cannot always be regarded as valid. Mechanics does for example occur in the Transvaal curriculum. Possibly the school principals concerned meant that they do not make provision for the subject in the school curriculum. This can be regarded as an evasive reply.

A large variety of reasons is given, but it is clear that there must be a more deeply seated reason why so many schools adopt a completely indifferent attitude towards Applied Mathematics.

4.10.9 Discussion

It is very obvious that Applied Mathematics or Mechanics is not a popular high school subject. Kemeny reported that this is an international phenomenon.

> "It is painfully clear, in reading the 21 national reports, that relatively little attention has been given by our reformers to the teaching of applications of mathematics. The only notable exception to this is the inclusion of statistics in a majority of the recommendations. Aside from this, only scattered suggestions are made, none of them occurring in more than two reports. Indeed, some reporters have specifically complained that, while an enormous effort has been made in their nations to improve the teaching of pure mathematics, the topic of applied mathematics has apparently been forgotten. I would like to propose to ICMI that a study of the teaching of applications of mathematics should receive high priority in its studies of the next four-year period."

"The philosophy of teaching applied mathematics is particularly well described in the report from the Netherlands."

"It is an urgent problem whether secondary education must restrict itself to pure mathematics. Applications gain more and more momentum in the social system. If these applications were only operational, one could ask whether they should be taught at all in high schools. Teaching applied mathematics, however, implies developing new habits of thinking, which in many cases differ from those in abstract mathematics. For instance, in statistics it is difficult to acquire operational skill as long as one has not really and independently understood the fundamental notions." (52, p. 205-6)

Sherman also stresses the importance of Applied Mathematics.

"The essential difference between a theoretical mathematician and an applied mathematician is that the latter desires immediate practical results from his mathematical investigations. Inasmuch as mathematics is a tool that can be used in almost every field of science and engineering, more and more government and industrial organisations are turning to the applied mathematician for the solution of complex and scientific and industrial problems." (86, p. 145)

4.11 ELECTRONIC COMPUTERS

4.11.1 A form of Applied Mathematics

A new form of Applied Mathematics has recently sprung into prominence. Possibly this development can help to create a new content, form of and interest in Applied Mathematics at school.

"Almost everyone has heard of the miraculous accomplishments of these computers - how years of work by former standards are now compressed into days or even hours. However, it is not the machine that does the thinking, it is the mathematician. It is he who defines the essential problem, determines the steps required for a solution, and then prepares the instructions which correlate the problem with a set of directions that the machine can follow." (86, p. 150)

4.11.2 Training of persons to operate electronic computers

According to Hall such training has the following implications:

(1) Teaching the use of computers implies three tasks:

(a) Deliberate cultivation of a 'feeling for numbers' and systematic teaching of computational mathematics i.e. graphical, numerical and statistical analysis.

(b) Teaching students to produce flow charts of calculation procedures and to code them for a computer

With this goes the need to teach students the distinction between creative thought, for which there is no substitute, and 'a good deal of what passes for thought in this world' but which can be done by computers.

(c) Teaching not less but more mathematical analysis

Incidentally, Forsyth's definition of numerical analysis is: Numerical analysis is the branch of applied mathematics which uses mathematical ideas to devise and evaluate numerical techniques for employing computers to solve problems, and to study their convergence and errors." (39, p. xiii-xiv)

(2) "In the second place there is the need to teach students how to set up mathematical models and how to interpret the results of mathematical analysis of such models in a useful way. Teaching students to set up mathematical models, to solve and to interpret the solutions usefully, requires:

(a) That we subject to critical analysis the concepts of a solution - the primary objective of a mathematical theory - and, in addition, the even more fundamental concept of a problem.

(b) That we show the student how to obtain the guidance in model building which computers and simulators can give him.

(c) That we assist all students, and especially those specialising in mathematics, to cultivate wide interests."

(3) "In the third place there is the need to take advantage of the positive help computing machines offer in teaching certain key mathematical concepts, such as number base, the axioms of algebra, function, limit, error, approximation, linearity and non-linearity, etc., as well as by enabling models to be examined and giving scope to inductive reasoning." (39, p. xv-xvi)

4.11.3 The electronic computer in the high school

In this connection A. Forsythe expresses the following opinion:

"There are, of course, several different ideas about where, if at all, computing should be introduced in the high school curriculum. I would like to take a firm stand in favour of teaching something about computing to most high school students. In support of this, let me quote from a report to the Advisory Board of the School Mathematics Study Group from its ad hoc Committee on Computing. 'The number of individuals who will work with or be directly affected by digital computers in the next decades will be so great that the entire high school population needs to learn something about computing'." (36, p. 6-7)

Simpson provides the following data:

"At present there are over 3,000 computers in the United States, and each computer requires ten mathematicians before it can function properly. That means that at present 30,000 mathematicians are needed for computing alone. At present, this cannot be accomplished because of the shortage of personnel in the field. We are not educating enough mathematicians to man these machines efficiently." (87, p. 32-3)

4.11.4 Programming at school

"Another question regarding training is: What kind of equipment will be needed for training codes? Dr. Yett feels that the minimum equipment is a desk calculator which can be treated as an automatic digital calculator. Then a student can program a problem. The elements of coding can be done without a calculator but the rest of the operation requires a machine." (87, p. 35)

4.11.5 The qualifications required of persons to be trained as programmers

M. Sherman, who investigated careers for mathematicians, sets the following qualifications for a programmer:

"Programmers engaged in solving scientific or engineering problems have the same basic functions as office programmers. They must have a strong background in mathematics, inasmuch as they deal chiefly with scientists and mathematicians who give them problems in complex mathematical forms which have to be simplified into arithmetic forms the computer can handle. This so called 'numerical analysis' can be applied to many types of research problems." (86, p. 152)

4.12 STATISTICS AND PROBABILITY THEORY

4.12.1 Introduction

Besides Mathematics and Applied Mathematics there is a third subject in this group, namely Interpretative Mathematics or Statistics. What does modern statistics connote? Peters and Kinsella have given the following answer to this question:

> "Essentially it is a method for solving problems which involve making decisions in the face of uncertainties due to incomplete information. This involves, of course, deciding what data are pertinent to the problem, collecting as many data as are feasible, organizing and presenting them, interpreting them, and finally, using them to make the decision which is most likely to be correct." (80, p. 272)

4.12.2 The application of Statistics

> "Modern statistics is used in the design of agricultural experiments to help us find ways of obtaining more food and better food at less cost. Medical research uses statistics to find out whether certain treatments, like polio shots, reduce or prevent the incidence of certain diseases. Business and industry use statistical methods to check the quality of their products and to decide on the most economical ways to organize their operation and processes. Engineers and production managers of the future must know the probability and statistics basic to operations research of which quality control is an example. The future psychologist must have a background in probability and statistics for research design and for understanding learning models based on probability. The biologist who wants to be competent in the field of genetics had better be acquainted with Markov chains, which in turn leans on probability concepts. The future development of the theory of games, also dependent on probability ideas, is likely to yield more applications in devising strategies in military affairs and in business competition. Statistical mechanics is already a standard course in the preparation of engineers and physicists. The astronomer is applying the theory of probability to the statistical study of the distribution of star galaxies. The Monte Carlo method, an offspring of probability, has been successfully used in the study of the neutron. Such diverse subjects as heat and information theory seem to be governed by similar laws of probability. It is well known that insurance companies depend on laws of probability in dealing with the duration of human life." (80, p. 273-4)

> "The work of the statistician can be divided into two categories - mathematical statistics and applied statistics.

> "Mathematical statisticians use mathematical techniques to devise and improve methods of analyzing statistical data.

> "The applied statistician, on the other hand, uses established statistical procedures to collect and analyze data in a specific field, whether it be economics, psychology, public health, finance, or engineering." (86, p. 154)

> "The actuary is a highly specialized statistician. He works for insurance companies, where he specializes in statistics related to the lives of people, the probabilities of living and dying at each age; or the chances of becoming disabled, hospitalized or unemployed. With this knowledge he is able to calculate insurance risks and premiums." (86, p. 155)

4.12.3 Probability Theory

Probability theory is one of the cornerstones of Statistics.

"Probability and its technological brother, statistics are used by most major industries in this country to keep check on the quality of the articles produced. Physicists develop new theories of atomic phenomena which use probability as a fundamental tool. Military experts plan the defence of our country using the results of research in probability. Since probability theory is a mathematical idealization of certain aspects of human uncertainty, it is not surprising that it has such widespread applications - so much of human activity must take place in the face of uncertainty." (77, p. 229)

Probability theory itself has recently become increasingly abstract.

"In recent years the theory of probability has become a purely mathematical topic which does not deal in any way with the happenings of real events around us." (77, p. 230)

This fact must be seriously borne in mind when the possibility of teaching Statistics at high schools is considered.

"The status of probability and statistics is entirely different from that of logic and sets. The introduction of these subjects into the high school curriculum is proposed usually on the basis of their inherent attractiveness and importance, rather than their instrumental use in other branches of mathematics. In almost all cases both probability and statistics were advocated, usually closely tied together. I shall follow the convention that under the heading of 'probability' a branch of pure mathematics is meant, while 'statistics' describes a branch of applied mathematics. If this view is accepted, we must see here both the most widely recommended subject in pure mathematics and the only widely recommended subject in applied mathematics, for inclusion in high school education.

"I would like to suggest that the extent to which probability theory is to be taught in high school should be one of the topics of discussion following this report to the Congress. Probability theory recommends itself as a very attractive branch of pure mathematics because it is so easy to give examples, from everyday experience, involving probabilistic computations. Therefore, the student is challenged to combine mathematical rigor and intuition.

"However one may consider introducing probability theory from a purely classical point of view, in which one deals with equally likely events and defines probability simply as a ratio of favourable outcomes to total number of outcomes. In this case, probability problems reduce to problems of counting or combinatories. There is no doubt that such simple combinatorial problems are well within the grasp of the average high school student and, indeed, such topics have long been included in high school algebra courses. In many of the reports sent to me it was not clear whether the probability theory advocated goes beyond such elementary computations.

"To capture any of the spirit of modern probability theory, it is necessary to introduce the concept of a measure space and to define probabilities of various events in terms of measures of subsets. While anything like a full treatment of measure theory is much too difficult for high school students, a number of experiments have shown the possibility of doing this for discrete situations, or even more restricted, for finite sets. Since the normal problems familiar to high school students deal only with a finite number of possible outcomes, this formulation of the foundations of probability theory corresponds particularly closely to the students' everyday experience. Recommendations for such a very elementary treatment of probabilistic measure theory are contained in four reports." (52, p. 198-9)

"Teachers should know that in its advanced, modern cloak, probability theory has as much claim to being a purely mathematical doctrine as any other area of mathematics.

"It is certainly not now feasible to teach probability theory as a pure mathematical subject in high school or earlier. But many of the notions important in probability theory can be grasped in a clear, intuitive way by children."

(1) To familiarize the student with some of the fundamental, intuitive ideas of probability and expose him to real and imagined experiments which should precede a thorough and vigorous study of the subject.

(2) To provide a novel and interesting context for standard mathematical ideas and thereby to deepen the student's understanding of and interest in these ideas.

(3) To give students a new and refreshing method for, in essence, drilling on mathematical ideas which are already taught as part of the standard curriculum." (77, p. 230)

The statistical method is unfortunately still something unknown at school. This state of affairs cannot be allowed to continue. Every Mathematics teacher should be fully conversant with this method. Where the individual phenomenon lacks regularity, the mass phenomenon sometimes shows a surprisingly orderly pattern and this is studied by means of the statistical method. Armed with this knowledge the Mathematics teacher will find that his field has come much closer not only to the natural sciences but also to such subjects as History and Geography according to the modern methods of approach. He will then be better equipped to inspire his pupils to explorative work. All figure material becomes a potential field of discovery. In and through the application of his mathematical knowledge the youthful researcher acquires a thorough insight into Mathematics as such and in its context with and its value in other fields of knowledge. 1)

This will result in students in the natural sciences displaying a greater interest in a course in Statistics than is today the case. If they do not become acquainted with the subject at school, the chances are that they will fail in later life as scientists in the so-called exactness of their conclusions based on observations. 1)

"If the teaching of statistics is to be formalized and become a part of the secondary school curriculum, general objectives such as the following must guide the development of a sound program in statistics:

1. Understanding of the types of problems in our society that can be solved through the use of statistics.

2. Understanding of the role of mathematics and probability in statistical reasoning.

3. Understanding of basic statistical techniques, concepts, and methods.

4. Ability to define a problem, design a simple experiment, and carry out the proper statistical analysis necessary to draw valid conclusions.

5. Appreciation of the limitations of statistical inference and an undertaking of the generalizing ability of experimental evidence." (75, p. 254)

The question is how far this acquaintance with the statistical method should extend. The answer will depend upon the time which cab be made available for this purpose. Possibly such a course should not be offered before Std. 9 and perhaps even only when Std. 10 is reached.

The following may with advantage be treated in such a course:

1)Remark of a member of the Committee.

Elementary descriptive statistics: tabulation, graphic representation, averages, standard deviation and correlation.
The concepts sampling and population.
The idea of variability in observation.
The concept of probability with a minimum probability.
The concept sampling distribution, the sample average and a brief indication of the statistical conclusion.

> "If we next turn out attention to the junior high school, we note that the ability to interpret data in tables and graphs is frequently a serious objective of instruction. However, not much attention is given to frequency graphs, such as the histogram, the frequency polygon, the cumulative frequency graph, and the graph of cumulative per cents. These would not be difficult to teach." (80, p. 275)
>
> "Symbolism is an important concept in the study of statistics in many fields of mathematics."
>
> "Approximation is also an important idea in statistics."
>
> "Discreteness and continuity are essential ideas in statistics."
>
> "In the tenth grade we concentrate on deduction as the method of mathematical proof Induction is used sparingly to arrive at hypotheses to be tested by deduction and the danger of basing conclusions on it is emphasized." (80, p. 279)
>
> "In the eleventh or twelfth grade a unit on permutations and combinations is followed by one on probability. The latter is defined in terms of relative frequency." (80, p. 280)
>
> "There are very many simple examples at hand for teachers to introduce statistical ideas. Classification, distribution, probability and sampling situations abound in the world around us.
>
> "Under this heading it is important for the general education of all pupils, whether they are potential mathematicians or not, that a statistical way of thinking about certain aspects of every day life should be introduced.
>
> "Under the heading both of algebra and statistics, the concept of a 'mathematical model' plays an important part.
>
> "Many of the mathematical ideas now being advocated lend themselves to this work and we think that the time has come for mathematics teachers to pay some attention to this interpretation of the idea of 'applicable mathematics'." (21, p. 318)

4.12.4 Summary

(Translation) At the end of his report Kemeny recommends the following problems to the C.E.I.M. for study during the four years preceding the International Mathematics Congress of 1966:

1. How can the teaching of the applications of mathematics be modernized in secondary education? This problem has hardly been considered.

2. To what extent is the 'axiomatisation' of mathematics possible in secondary education? The opinions on this problem diverge sharply in different countries.

3. In what way and to what extent is the teaching of probability theory possible at the secondary school?

This is indeed the subject that is touched upon and recommended in the various national reports, but many questions of a pedagogic or didactic nature must still be answered. (45, p. 105)

4.13 COMMERCIAL MATHEMATICS

4.13.1 Introduction

Although it was not a directive to the committee to give specific attention to Commercial Mathematics, it is nevertheless essential that cognizance should also be taken of the requirements of the business world.

Serious attention has been devoted to this matter in the U.S.A. where, for example, R.E. Simpson reported as follows at a conference held in connection with this project:

> "A proposal to study the improvement of mathematics as it relates to business was submitted to the Bureau of National Defence Education Act and approved in the spring of 1960. This proposal had three objectives:
>
> 1. To conduct a planning conference with representatives from mathematics and business education for the improvement of mathematics as it relates to business.
> 2. To conduct a production workshop based upon the recommendations of the planning conference.
> 3. To develop a teaching guide based upon the materials developed by the production workshops." (87, p. v)

4.13.2 Recommendations

The following are the most important recommendations which stemmed from this investigation:

1. "Most far-seeing educators feel that when the elementary and secondary schools are doing the kind of job in mathematics they want to do, then the needs of business mathematics will be adequately met.

2. Until that time comes when elementary and secondary schools are doing the kind of job they want to do, a few changes can be made. The principal recommendations regarding the business mathematics programs were as follows:

   (a) That remedial mathematics and business mathematics be separate courses.

   (b) That more care be exercised in selecting teachers for business mathematics. Teachers should be either mathematics teachers with an interest in business, or business teachers with a mathematics background (an unusual combination).

   (c) That help from mathematicians be given business mathematics, by the following: workshops, summer session courses, and an interchange of information between professional organizations, departments, and individuals.

   (d) That business mathematics in senior high school be organized around mathematics concepts with illustrations from business, rather than from consumer education.

   (e) that future texts in business mathematics be organized around the development of mathematics concepts rather than around business principles.

   (f) That if a course of instruction for business mathematics is to be written to fill this interim period, then writers of such

> a project be mathematicians with experience in writing and teaching, who can call upon business educators to illustrate the mathematics concepts with problems from business." (87, p. 3-4)

4.13.3 The concepts

Of particular interest is the analysis made of the concepts which occur in Commercial Mathematics.

In the first place a table in respect of the courses in the Junior High School (Stds. 5 to 7) was drawn up:

(a) The Junior High School

Topics	Concepts
Wages and salaries	Four basic processes, fractions, decimals, per cent formulas.
Banking	Basic processes, decimals.
Thrift and Savings	Basic processes, decimals, per cent, graphs, formulas.
Purchasing goods (including auto)	Basic processes, fractions, decimals, ratio and proportion, measurements, estimating.
Instalment buying	Basic processes, decimals, per cent, ratio and proportion, formulas, estimating.
Real Estate	Basic processes, decimals, measurements, ratio and proportion, formulas, estimates.
Insurance	Basic processes, fractions, decimals, per cent, graphs, formulas.
Services, Utilities, etc.	Basic processes, decimals, measurements, estimating.
Budgeting	Basic processes, fractions, decimals, per cent, graphs, estimating. (87, p. 74)

(b) The Senior High School

> "The following course outline is a composite of courses in business mathematics given in the senior high schools of the state.
>
> "Opposite the topics are some of the basic mathematical concepts sometimes taught in conjunction with the business subject matter. The four basic concepts (addition, subtraction, multiplication and division) are involved in each of these topics, and have not been repeated each time.

Topic	Basic Concept
Earning money	
Pay checks Social Security, withholding etc. Finance records, budgets Commission	Formulas, per cent, fractions, decimals, estimating.
Transactions with a bank	
Deposit tickets, check stubs Bank statements and reconciliation	Estimating
Arithmetic of buying	
Sales tickets, involving aliquot parts Determining unit prices Determining average prices	Estimating Ratio and proportion Estimating, ratio and proportion
Borrowing money	
Promissory notes Bank loans Credit unions, loan companies Collateral Instalment buying	Decimals, per cent, formulas (i.e. third case of percentage, etc.)
Savings and investments	
Savings accounts Life insurance Stocks and bonds Real estate	Decimals and per cent per cent graphs, statistics per cent signed numbers per cent
Expenditures for the home	
Home ownership Real estate taxes Property insurance Water Gas	Decimals, per cent, formulas, statistical graphs, polls, correlations
Taxes	
Property, sales Old-age benefit Income	Decimals, per cent, graphs, correlations, formulas
Problems of the retailer	
Store location Building, layout Display	Computations with measurements, use instruments and scales, indirect measurement scale drawing, etc.
Balance sheet	Decimals
Profit and loss statement	Decimals
Purchases and purchase records	Decimals
Trade discount	Decimals and per cent
Buying expense	
Cash discount	Decimals and per cent
Sales and sales records	Decimals and per cent, Graphs
Retail discount	Decimals and per cent correlations

Topic	Basic Concept
Profit and loss	Decimals and per cent formulas
Pricing goods	
Payrolls	Decimals and per cent formulas"

4.13.4 Conclusion

Most of the concepts in Commercial Mathematics already occur in the Arithmetic of the primary school, while virtually all the other concepts can be reduced to the concept of ratio.

The taking of Mathematics as well as Commercial Mathematics in the high school results in a large measure of overlapping. If a future business manager has to choose between Mathematics and Commercial Mathematics, the following quotation merits very serious consideration:

> "What specific knowledge of mathematics does the manager of the future need? Some knowledge of algebra, trigonometry, analytical geometry, calculus ..... In order to understand the logical, theoretical, and computational aspects of modern statistics ..... business students will require in their mathematical background some knowledge of matrices and determinants; vectors and vector spaces; lines, planes, and convex sets in higher dimensions; linear inequalities; systems of linear equations; and, of course, probability." (87, p. 38)

This quotation is taken from a lecture given by Dr. Frank Williams, Professor of Business, San Francisco State College. In the light of this pronouncement a reassessment of Commercial Mathematics in the high school is a matter of paramount urgency.

4.14 TOPICS FOR VACATION COURSES

Now that cognizance has been taken of this possible Mathematics subject material for high school pupils, the question naturally arises how all this material can ever be included in the high school curriculum.

There will always be more suitable material than can be compressed within the limited framework of a syllabus. On the other hand it is the gifted pupil in particular who frequently needs a wider field of study. Possibly the long vacations offer talented pupils with the right opportunity to keep themselves occupied with a little extra work. Those matriculants who have to wait for the start of the academic year after having completed nine months military service may also find a vacation course an enjoyable and refreshing experience.

During the 1961 summer vacation a course lasting five weeks was offered in Oklahoma, U.S.A. The general aim was to give talented high school pupils a training in Mathematics which did not occur in either the high school curriculum or in the beginners' courses at the university. The following were further aims of the course:

"1. to introduce the students to the modern approach to mathematics and provide challenging experiences in mathematics at a level and technique not encountered in high school;

2. to make students aware of the fundamental structure of mathematics on which the skills and mechanical procedures of elementary mathematics are based;

3. to give students an introduction to rigorous reasoning, abstract, and postulational systems;

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4. to aid students in learning how to study mathematics;

5. to develop critical thinking and creativeness in mathematics;

6. to develop a desire to pursue the study of mathematics for research or teaching through contact with research mathematicians and college professors."

The pupils who attended the course were carefully selected. Twenty seven pupils with I.Q.'s ranging from 120 to 137 (15 boys and 12 girls) were the final choice. The following courses were offered:

"1. An Introduction to the Foundations of Mathematics Content: the nature of mathematics, introduction to propositional logic, the meaning and use of proof, algebraic structures, sets and Boolean algebras, partitions and permutations.
Text: notes by the instructor and Introduction to Finite Mathematics by Kemeny, Snell, and Thompson.

2. Selected Topics in Mathematics
Content: matrices, vectors, linear programming, modular systems, mathematical induction.
Text: notes by the instructor and Introduction to Finite Mathematics by Kemeny, Snell, and Thompson.

3. The Real Number System.
Content: the meaning of an axiomatic system, binary operations, the field properties, the continuity property, and other properties of the real number system which may be deduced from the basic structure.
Text: notes by the instructor.
Technique: the students were given the undefined terms and a few fundamental properties (assumptions).
Then they deduced all the possible properties of real numbers from the given basic structure. Next a few more assumptions were given and again they deduced more properties of real numbers. This method was continued until the real number system was completely described. From the basic structure of the real number system the students deduced all the skills and rules of procedures of elementary mathematics." (33, p. 180)

The lecturers were university professors with considerable experience of high school work.

The pupils were also given the opportunity to submit independent work on a subject of their own choice. Assignments on the following subjects were handed in: prime numbers, Ancient Greek methods of calculation, congruency and modulus systems, counting systems, recurring decimals, inversion, Pythagorean triads, the abacus, vector analysis, non-Euclidean geometry, trisection of the general angle, lattices, the symbolic logic of Lewis Carroll."(33, p. 180-1)

At the end of the course every pupil had to complete a questionnaire. From their replies it appeared that the study of the real number system had given them a new insight into the nature of Mathematics. The other topics also led them to realise that Mathematics is more than a collection of rules and techniques.

When the Mathematics subject material is judged with a view to determining whether it is suitable for high school pupils, the matter must be approached with an open mind. The immediate adoption of a negative attitude by saying that there is no time for such material in the high school curriculum, will lead to complete stagnation.

Even if no time can be found in the high school curriculum for certain subjects, there is after all still always the possibility of using booklets for additional reading and learning work, vacation courses and competitions in Mathematics as in the case of the Mathematics Olympiads. When a school has to show what achievements it is capable of in a competition, school time is generally available.

4.15 CONCLUSION

In this chapter the Mathematics subject material has been discussed at length with particular reference to overseas literature. In the following chapter the existing syllabi will be dealt with in greater detail and an attempt will also be made to show how these syllabi can be adapted to make provision for the material treated in this chapter.

CHAPTER 5

THE MATHEMATICS SYLLABI

5.1 THE OBJECT OF THE INVESTIGATION

The object of this part of the investigation is an analysis of the Mathematics, General Mathematics and Arithmetic syllabi of the various departments of education with a view to determining the following:

Whether the syllabi are adapted to the latest developments in the specialist fields of these subjects;

whether obsolete subject matter occurs in the syllabi and to what extent uniformity exists between the syllabi of the various departments of education;

whether it is possible to change the syllabi in such a way that the gulf between the high school and university can be reduced;

whether the syllabi of the respective departments of education are of such a nature that the aims and envisaged educational value of the subject can be achieved.

It is clear that a very comprehensive investigation was necessary to meet the foregoing requirements. This chapter links up with the previous chapters since the aim and value of the teaching of Mathematics in connection with the subject matter have to be considered very thoroughly.

5.2 THE SIMILARITIES AND DIFFERENCES BETWEEN THE SYLLABI OF THE VARIOUS DEPARTMENTS OF EDUCATION AND THOSE OF THE JOINT MATRICULATION BOARD

5.2.1 The Mathematics syllabus of the Joint Matriculation Board

The Joint Matriculation Board has drawn up a Mathematics syllabus which is regarded as a "minimum curriculum". The syllabi of the various departments of education must at least meet the requirements of this syllabus, if candidates wished to qualify for entrance to a university. (38, p. 123-133)

The syllabus of the Joint Matriculation Board is given in Appendix 7.

5.2.2 Cape of Good Hope

The syllabus for the Senior Secondary Course of the Cape Province Department of Education came into effect on 1st January, 1964. The new syllabus was adopted in Std. 9 at the beginning of 1964 and the first senior certificate examination on this syllabus will be written in November, 1965. This syllabus departs from that of the Joint Matriculation Board in the following respects:

Graphs:

The use of stencils is permissible in the drawing of curves.

In regard to the graph of the function $x^2 + y^2 = k$ it is laid down that where k is not a perfect square, k must be found without the use of square root tables or logarithms.

Geometry:

The following items which do not occur in the syllabus of the Joint Matriculation Board, have been included in that of the Cape Province:

(1) Practical

The following geometrical constructions:

A tangent to a circle from a point on the circumference or from a point outside the circumference of a circle.

The escribed, inscribed and circumscribed circles of given triangles.

A segment of a circle on a given chord subtending an angle equal to a given angle at the circumference.

The fourth proportional of three given straight lines.

The third proportional of two given straight lines.

Applications of the foregoing for the construction of rectilinear figures. (28, p. 188)

(2) Theoretical

This section is called "Synthetic Mathematics" in the syllabus of the Joint Matriculation Board.

The following two propositions occur in the Cape Province curriculum but not in that of the Joint Matriculation Board. They are, however, included in the syllabus for Additional Mathematics of the Joint Matriculation Board.

> "29. Definition of similarity.
>
> Any straight line drawn parallel to a side of a triangle divides the other two sides proportionally, and the converse.
>
> "30. Equiangular triangles have their corresponding sides proportional, and the converse."

The following two propositions occur in the syllabus of the Joint Matriculation Board but not in that of the Cape Province:

> "31. If two chords of a circle intersect at a point either inside or outside a circle, the rectangle contained by the segments of one is equal to the rectangle contained by segments of the other, and its converse.
>
> "32. If from a point outside a circle a secant and a tangent to the circle are drawn, the rectangle contained by the whole secant and the segment of it outside the circle is equal to the square on the tangent, and its converse."

5.2.3 Natal

Algebra:

Arithmetical and geometrical series occur in the syllabus of the Joint Matriculation Board but not in that of Natal. Aids are allowed in the drawing of curves.

Geometry:

The following proposition occurs in the Natal curriculum but not in that of the Joint Matriculation Board:

> "If two circles touch each other, their centres and the point of contact are collinear." (70, p. 35)

The following constructionsalso occur in the Natal syllabus but not in that of the Joint Matriculation Board:

"To draw the external common tangent to two circles." (70, p. 35)

The following also occur in the Natal syllabus but not in that of the Joint Matriculation Board:

"Constructions on ratio and proportion

(i) The division of a straight line, internally and externally, in a given ratio. (70, p. 35)

(ii) Construction of the fourth proportional to three given straight lines.

(iii) Construction of the third proportional to two given straight lines.

(iv) Construction of the mean proportional to two given straight lines." (70, p. 36)

Trigonometry

The following are also additional in the Natal syllabus: Constructions based on trigonometrical ratios of angles ranging from 0^o to 360^o.

An elementary knowledge of the trigonometrical ratios of negative angles within the range 0^o to -90^o. (70, p. 36)

The following occurs in the syllabus of the Joint Matriculation Board:

"Addition formulae for sine and cosine, where proofs are restricted to the cases when the angles are acute. (It is recommended that the sin (A+B) formula should be derived by the use of the formula for the area of a triangle."

The Cape and the Natal Departments of Education include in the addition formulae both the sum and the difference of the following: Sin $(A \pm B)$ and Cos $(A \pm B)$.

5.2.4 Orange Free State

Arithmetic does not occur as a separate subject: it is either disposed of prior to Std. 9 or covered by the Algebra syllabus. The work in connection with similarity is set out more fully in this syllabus than in that of the Joint Matriculation Board.

5.2.5 Transvaal

In 1959, with a view to coming into full operation in the November/ December 1961 examinations, the Transvaal syllabus differentiates in detail between the Std. 9 and Std. 10 courses and the university entrance course (B stream and A stream respectively).

The university entrance course contains a considerable amount of material which does not occur in the syllabus of the Joint Matriculation Board, namely the following:

(a) Proof of the remainder theorem (Function methods). (101, p. 6)

(b) Imaginary roots of a quadratic equation. (101, p. 12)

(c) For what value of k are the roots of $(k+1)x^2 - 3kx+2k = 0$ equal?

Determine the maximum value of c for which $2x^2 - 4x + c = 0$ will have real roots.

N. B. The connection between this work and the related work on graphs must continue to be emphasized. (101, p. 14)

(d) The solution of equations of the third degree with one unknown and only with numerical coefficients of which at least one root can be determined by means of the Remainder theorem. (101, p. 10)

(e) The sketch graph of $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers. (101, p. 18)

(f) In the syllabus of the Joint Matriculation Board mention is merely made of:

"Simple ratio and proportion."

In the university entrance course of the Transvaal Department of Education the following is found:

"(a) The use of the k method for

(i) The proofs of the equivalence of ratios;
(ii) the determinations of the ratios of unknowns from equations of the first and the second degree;
(iii) the solution of equations with not more than three unknowns of the type

$$ax + by + cz = d$$

$$\frac{x}{e} = \frac{y}{f} = \frac{z}{g},$$ where a, b, ..., g represent whole numbers.

"(b) The following propositions (no proofs required) and their application to simple examples, including solution of suitable equations.

Invertendo; alternando; componendo; dividendo; componendo et dividendo; addendo." (101, p. 14)

In addition the Transvaal syllabus also contains the following which do not occur in the syllabus of the Joint Matriculation Board:

"Positive and negative increments and the calculation of the increment in the value of the dependent variable for a given change in the value of the independent variable.

"The average gradient of a curve between two given points on it.

"Discussion of the gradient of a curve at any point on the curve by the treatment of simple cases such as:

x^2, $2x^2 + 3$, etc.

"Determination of the gradient function of x^2, $2x^2 + 3$ etc., by means of the h-method, where h represents a very small increase in the value of x.

"Use of the "D" notation for the "gradient function of" for example,

$$Dx^2 = 2x \text{ and } D(2x^2 + 3) = 4x$$

"The gradient function of a, ax, ax + b, x^2, ax^2, $ax^2 + b$, $ax^2 + bx + c$, x^3 and $ax^3 + bx^2 + cx + d$, where a, b, c and d have numerical values.

"Calculation of the gradient of a curve at any point on any of the above-mentioned curves.

"The calculation of the equations of the tangent to a curve at any point on it.

"Explanation of the geometrical fact that the gradient of a curve at the point where the function assumes maximum or minimum value, is equal to zero.

"Determination of the co-ordinates of the point where a quadratic function assumes its maximum or minimum value, the X ordinate being equated to 0 through the gradient function and the Y ordinate through substitution (confined to cases which work out exactly).

"Maximum or minimum values of $ax^3 + bx^2 + cx + d$ (where a, b, c and d are numerical coefficients) confined to cases which work out exactly. More detailed sketches of third grade functions." (101, p. 18 and 20)

Analytical Mathematics:

The Transvaal syllabus for admission to the university contains the following item which does not occur in the syllabus of the Joint Matriculation Board:

"Derivation of the formula $\tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$" (101, p. 22)

Geometry:

The following items occur in the Transvaal curriculum for admission to a university but not in that of the Joint Matriculation Board:

"The construction of a rectangle with a given side, equal in area to a given rectangle." (101, p. 34)

"If one pair of sides of a triangle is proportional to one pair of sides of another triangle and the included angles are equal, the two triangles are similar."

"The perpendicular drawn from the right angle of a right-angled triangle to the hypotenuse, divides the triangle into two triangles which are similar to each other and to the given triangle."

"The areas of similar triangles are proportional to the squares on the corresponding sides." (101, p. 38)

"The division (internally only) of a given straight line in a given ratio (defined arithmetically or geometrically)." (101, p. 34)

Trigonometry:

The following items are encountered in the Transvaal syllabus for admission to a university but not in the syllabus of the Joint Matriculation Board:

The derivation of the formula

$1 + \cos A = \frac{(b + c + a)(b + c - a)}{2bc}$ from the formula

$a^2 = b^2 + c^2 - 2bc \cos A$.

The application of the properties of a triangle in the determination of elements of acute angled triangles and the calculation of simple altitudes and distances.

"Heights and distances may now occur in more than one plane but the work is restricted to cases where not more than two triangles are concerned, sufficient data being furnished in respect of one of these in order to be able to calculate all the required elements." (101, p. 40)

(The part between quotation marks must be regarded as the additional portions.)

The following is further additional material in the Transvaal curriculum for entrance to a university:

"Value determinations and constructions of trigonometric functions with regard to angles within the range from -90° to 360°."

5.2.6 National Senior Certificate and Senior Certificate of the Department of Bantu Education

Complete agreement exists between the syllabus for the above-mentioned certificates and the syllabus of the Joint Matriculation Board.

5.2.7 Summary

The following subject matter does not occur in the syllabus of the Joint Matriculation Board but is found in the other syllabi:

(a) Similar triangles,
(b) the trigonometric ratios of the differences between two angles, e.g. sin (A-B) and cos (A-B).

The intention is possibly that the theorems for the differences between A and B should be included in the theorems for the algebraic sum of A and B.

The principal differences between the provincial syllabi and the syllabus of the Joint Matriculation Board are that small omissions sometimes occur and that extra subject matter is included.

The Cape Senior Certificate syllabus mentions constructions under "Practical Geometry" and includes a number of constructions which are not required by the Joint Matriculation Board. On the other hand, the Cape syllabus omits two propositions which are prescribed by the Joint Matriculation Board.

The Natal syllabus includes many more geometrical constructions than in the case of other syllabi, but omits Progressions.

The Transvaal syllabus for university admission differs considerably from the other syllabi, since it prescribes more subject matter. The principal difference is the fact that the Transvaal syllabus makes provision for differential calculus. To this is also added the concept of imaginary roots. Although the Transvaal syllabus is more comprehensive than the other syllabi it does not include all the subject matter of the courses in Additional Mathematics offered elsewhere.

5.3 THE SYLLABUS FOR ADDITIONAL MATHEMATICS

5.3.1 The syllabi of the Joint Matriculation Board and the Natal Department of Education

In Appendix 8 the syllabus of the Joint Matriculation Board is compared with the syllabi for Additional Mathematics and the Advanced Course of the Natal Department of Education, Natal is the only province which offers Additional Mathematics and for this purpose abandoned their own Mathematics Higher Course after December, 1962.

5.3.2 Discussion of the syllabi for Additional Mathematics

The Natal syllabus is less comprehensive than that of the Joint Matriculation Board, this being attributable mainly to the fact that a large part of the work is treated in the advanced courses. There are a few items which occur in the syllabus of the Joint Matriculation Board but not in the syllabus for Additional Mathematics of the Natal Department of Education. The most important of these are Ratio, proportion and variation. Here certain basic principles are very much in evidence, and the Natal Department of Education should give strong consideration to their inclusion in the Algebra syllabus.

Since Natal has been offering the advanced course for Matriculation Exemption, the importance of the Additional Mathematics course has declined, since a large part of the work is now in any case dealt with in the Mathematics course (Advanced grade).

Algebra:

The following are also treated in the Natal course for the advanced grade:

Theory of quadratic equations

The nature of roots as determined by the discriminant, the sum and difference of roots, and the formation of equations if the roots, or the sum and product of roots, are given.

Also occurring in the advanced course are the following: the quadratic expression (function) $ax^2 + bx + c$

(i) Conversion into perfect squares
(ii) Maximum and minimum of this expression - algebraic and graphic methods
(iii) Graphic representation of this function
(iv) Algebraic and graphic treatment of the sign of a quadratic function.

5.4 MATHEMATICS SYLLABI FOR PUPILS WHO DO NOT WISH TO QUALIFY FOR MATRICULATION OR FOR MATRICULATION EXEMPTION

5.4.1 Existing courses

Two provincial departments of education at present offer courses to pupils who wish to complete a school-leaving examination without any intention of obtaining admission to a university. The two provinces are Natal and the Transvaal. In Natal the course is known as the Ordinary grade and in the Transvaal as the Std. 10 course.

Arithmetic and Algebra

Appendix 7 shows which subjects occur in the two courses, namely those marked only with asterisks.

The following differences occur:

Logarithms

Natal syllabus

Logarithms: application to

(a) mechanical evaluations;
(b) arithmetical processes;
(c) compound interest (including calculations of rate of interest and time);
(d) given formulas;

(e) Mensuration.

Transvaal syllabus

Application of logarithms:

(a) straightforward calculations and the calculation of an unknown from adequate data, including the changing of the subject of the formula, but excluding examples in which the log notation occurs;

(b) the formula for simple interest: $I = \frac{K \times R \times T}{100}$ (with attention to $A = K + I$)

(c) the formula for compound interest: $A = KR^n$

(i) to calculate A, K and the rate of interest, where n expressed in the compounded period is a whole number;

(ii) to calculate n.

In regard to the remainder theorem the following is specified for the ordinary grade in the Natal course:

The Remainder Theorem: application of the remainder theorem restricted to

(a) the remainder;
(b) easy linear factors;
(c) one or two unknown coefficients in a polynomial.

N.B. Proof of the remainder theorem is not required.

In the Transvaal the Std. 10 syllabus makes more detailed mention of functional notation and the remainder theorem:

"2.(a) Functional notation restricted to exercises based exclusively on substitution, e.g.:

(i) Given $f(x) = x^2 - 13x - 12$, determine $f(x-1) + f(x+1)$

(ii) Given $f(m) = 3m^2 - m + 1$, prove that $f(m+1) - f(m) - 2f(0) = 6m$

(b) The Remainder Theorem:

(i) The division of a polynomial with numerical coefficients by $x - a$ in order to introduce the remainder theorem.

(ii) Division by a linear factor

(iii) The determination of the factors of an expression not higher than the third degree.

(iv) The determination of not more than three undetermined coefficients in a given function, being given that their division by certain divisor(s) leaves no remainder or a given remainder.

5.4.2 Discussion of the Natal and Transvaal curricula

Arithmetic

The Natal curriculum offers the possibility of a thorough study of mathematical Arithmetic. A manual in which integers, fractions, (ordinary and decimal), rational and irrational numbers, real numbers and the representation of real numbers on a straight line are treated in a very thorough manner, is that of Brumfiel, Eicholz and Shanks: "Introduction to Mathematics."

In this book Arithmetic is offered in such a manner that it forms a good introduction to further study in Algebra and Geometry. This is done by stressing the mathematical concept. "Regardless of the level of student attainment, in the future more attention must be given to mathematical ideas than has been the case in the past." (16, p. v)

Since integers occur in the Natal syllabus, the first four chapters of the manual concerned can be used to great advantage. Together they form an entity under the title: "Symbols and numerals." The headings and contents of the chapter are as follows:

"1. Symbols: Introduction, new and old symbols, numbers and numerals, punctuation symbols.

2. History of numerals: Introduction, Rome, Egypt, Sumer and Babylon, Greece, the Hindu-Arabic numerals.

3. Place value and bases: Place value in base ten, bases other than ten, base two, base twelve.

4. Base ten: Symbols for rational numbers, arithmetic skills." (16, p. vii)

Since rational numbers are mentioned in the syllabus, cognizance may also be taken of the second unit of the above-mentioned book with advantage. This unit is called "Rational numbers" and is divided up as follows:

"5. Definitions: Whole numbers or counting numbers, fractions and rational numbers.

6. Basic principles of addition and multiplication: Introduction, the existence and uniqueness principle, the commutative principle, the associative principle, the distributive principle, properties of zero and one, other systems.

7. Factors and prime numbers: Evens and odds, prime numbers, greatest common divisor, least common multiple.

8. Number pairs, fractions, and rational numbers: Number pairs, fractions, rational numbers.

9. Subtraction and division: Subtraction of rational numbers, division.

10. Inequalities and the number line: Inequalities, the number line, measurement, rounding off.

11. Applications: Introduction, ratio and number pairs, measurement, miscellaneous." (16, p. vii-ix)

Mensuration (which also occurs in the Natal syllabus) and the main processes are thoroughly dealt with in the foregoing chapters.

This syllabus furthermore also makes provision for real numbers and the representation of real numbers by points on a straight line. It can be strongly recommended that the following subjects should be included in the work schemes of the teachers concerned.

"12. Decimals: Decimals as symbols, repeating decimals and multiplication, repeating decimals represent rational numbers.

13. Irrational numbers: Non-repeating decimals, the number $\sqrt{2}$ is not rational, definition of irrational numbers.

14. Real numbers: Definition of real numbers, basic principles, approximations, roots.

15. The real line: The rational line, the real line, completeness." (16, p. ix)

The roots of equations, which are also mentioned in the Natal syllabus, are dealt with in Chapter 14 of the above-mentioned book. It is clear that the Natal syllabus for Mathematics (Ordinary grade) offers the possibility of making something special of the parts on Arithmetic. The fundamentals of the subject can be treated in such a manner that proper justice is done to the most important mathematical concepts. For future teachers of Arithmetic in the primary school and even also for teachers of Arithmetic in the Junior classes, this part of the teaching of Mathematics is so important that it may not be given perfunctory treatment merely for the sake of a little more advanced teaching in Algebra, Geometry, etc.

The Transvaal Std. 10 course shows a very serious shortcoming in so far as the treatment of the arithmetical fundamentals and concepts are concerned. Attention is concentrated on logarithms which, with all the computers available today, are being used to an very much smaller extent. Logarithms are resorted to for the calculation of compound interest and in most cases interest tables are available for this purpose. There is no doubt that the Natal syllabus offers much greater possibilities in respect of the teaching of Arithmetic than does the Transvaal syllabus.

Algebra and Differential Calculus

The Transvaal Department of Education includes Differential Calculus in the curriculum of pupils who do not obtain matriculation exemption. The following do not occur in the course prescribed by that Department:

> "The calculation of the equations of the tangent to a curve at any point on it."
>
> "Maximum or minimum values of $ax^3 + bx^2 + cx + d$ (where a, b, c and d are numerical coefficients) are restricted to cases which work out exactly. More detailed sketches of third degree functions."

The differential calculus taught at high schools is so limited in its scope that serious consideration should be given to the question whether pupils who do not proceed with Mathematics after Std. 10 should not rather receive more instruction in Arithmetic.

Present-day writers display a striking lack of enthusiasm for differential calculus in the high school. Serious misgivings are expressed by C. B. Allendoerfer on this matter:

> "My thesis to-day is that both the secondary schools and colleges are heading for serious trouble in their teaching of calculus, and that a full discussion of this matter is of great importance to all of us. I shall organize my remarks around three propositions:
>
> 1. Although calculus is an essential part of a mathematical education, its importance relative to algebra, geometry, and other subjects has been overemphasized;
>
> 2. Calculus is now being taught too early to students who could be far better off studying other mathematics subjects;
>
> 3. When it is taught, calculus is presented in the wrong way." (5, p. 482)
>
> "My thesis is that calculus is frequently taught at the wrong time, by the wrong people, and in the wrong way. It is high time that we gave this

matter our urgent attention " (5, p. 485)

Mathematical Structure:

Both syllabi show little of the development of a clear mathematical structure as is, for example, the case in Euclidean Geometry. The impression obtained is that the Algebra is the ordinary matric syllabus minus a little. There is little question here of introduction of a structure which is important to these pupils and also to the development of their reasoning ability. They must admittedly know how to apply the remainder theorem, but there is no necessity for them to know its origin. And they must likewise know about the functional notation, but they need learn about the functional concept only after equations have been solved. The only link with Arithmetic is provided by means of logarithms.

Since these pupils do not in any case continue their studies of Mathematics at a university, it can be strongly recommended that they should be offered a course with some other content. Certain mathematical concepts should be given considerable prominence with a view to the development of the faculty of comprehension. In addition, a modern approach should be aimed at. Since many of these pupils later become primary school teachers, it is essential that their training in the mathematical subjects at the high school should prepare them for a reform in primary school teaching.

Geometry:

There is not much difference between the Transvaal and the Natal syllabi. Basically they are the same, but there is indeed a difference in the outline of the work which has to be done.

It is very clear that the terminology leaves much to be desired. In Natal the Afrikaans word for "walk" is used for a circle passing through a particular point, and words such as "konsiklies" (concyclic) and "konkurrent" (concurrent) require considerable explanation before the pupils will understand what they mean. It is incomprehensible why the clear, descriptive Afrikaans terms should make way for these foreign terms which are difficult to understand. Sometimes lines "ontmoet" each other sometimes they "sny" each other and at other times they "kruis" whereas in all cases the same process of intersection is meant.

In the Transvaal Analytical Geometry is not offered to pupils but differential calculus is. Why the position is not the other way around, is a matter which still has to be clarified.

Trigonometry:

The Natal syllabus gives more satisfaction since the pupils are sometimes afforded the opportunity to see the trigonometric ratios in their full context from 0° to 360°.

In the Transvaal the horizon is artificially restricted to 180°, thereby removing the possibility of full insight. Perhaps this has been done merely to obviate the risk of some examination questions being too difficult.

5.5 THE SYLLABI FOR STDS. 6, 7 AND 8

5.5.1 Comparison of the Arithmetic Syllabi of the different departments of education

In Appendices 4, 5 and 6 the Arithmetic syllabi of the various departments of education are compared. The Orange Free State syllabus for Std. 7 has been included in that for Std. 8.

The syllabus for the so-called A-stream in the Transvaal does not offer much more subject matter than that for the B-stream. The idea is possibly that this subject matter should be treated with somewhat greater depth in the case of pupils in the A-stream.

5.5.2 Comparison of the syllabi for Algebra and Geometry of the various departments

Algebra for Std. 6

Transvaal

The syllabus for Algebra is not given separately but as part of the syllabus for General Mathematics. For that reason the Algebra to be dealt with is given between Arithmetic and Geometry.

The work is split up for the three courses, namely the university entrance course, the school-leaving examination and the Std. 8 group which are briefly indicated here as the A, B and C groups respectively. The work to be done by each group will be indicated by means of asterisks.

This syllabus is found in the syllabus for General Mathematics, Stds. 6-8, which was issued by the Transvaal Education Department in October, 1959.

		A	B	C
1.	Algebra as a new language The conversion of simple problems into algebraic equations and their solution	x	x	x
2.	Symbolic expressions in work which does not lead to equations	x	x	x
3.	Like and unlike terms The sum and difference of algebraic expressions in columns (excluding indices)	x	x	x
4.	Brackets	x	x	x
5.	Further exercise in substitutions also with reference to formulas	x	x	
6.	Multiplication and division in Algebra			
	(a) monomials only; powers introduced	x	x	
	(b) Binomials and trinomials with binomials	x	x	
	(c) Polynomials with binomials	x		
7.	More difficult equations	x	x	

Orange Free State

In this province the subject General Mathematics is also given but consists of the common three divisions. The curriculum for Algebra is set out separately below.

1. Introduction to the symbolic language of Algebra by

   (a) simple problems from Arithmetic where literal numbers are used instead of numerical figures;

   (b) processes with literal numbers with reference to simple formulas:

      (i) Addition and subtraction, e.g., perimeter of given figures, etc.

(ii) multiplication and division, e.g. areas of given figures and volumes of given bodies;

(c) simple cases of the change of the subject in easy formulas, e.g.

$$A = lb$$

$$l = \frac{A}{b}, \text{ etc.}$$

(d) exercise in the substitution of numerical numbers for literal numbers in such formulas.

2. The tracing and formulation of formulas in regard to, for example, the perimeter and area of simple geometrical figures such as, inter alia, the right angle triangle, T and H figures.

3. Straightforward problems, the solution of which form the introduction to the solution of simple equations with one unknown.

4. Introduction to the concept of negative numbers (this syllabus comes from the pamphlet: Syllabus for the Junior High School (Stds. 6, 7 and 8), General Mathematics, which is issued by the O.F.S. Department of Education).

Natal

The syllabus for Algebra is given here separately as part of the Mathematics syllabus for Std. 6.

This is to be found in the pamphlet: Syllabuses for Arithmetic Std. 6 and Mathematics Std. 6 which is issued by the Natal Department of Education.

1. The use of algebraic symbols.

2. Terms and algebraic expressions.

3. Like and unlike terms.

4. The sum and difference of like terms. (Compound addition and subtraction are excluded).

5. Multiplication and division of monomials.

6. Substitution.

7. Symbolic expression.

8. Removal and insertion of brackets. Brackets within brackets will be excluded.

9. Equations. Easy examples with one unknown.

10. The solution of problems. Checking of the results by substitution.

Cape Province

According to the manual: Junior Secondary Course issued by the Department of Public Education of the Cape of Good Hope in 1953, no Algebra is taught in Std. 6.

The other Mathematics courses for Stds. 6, 7 and 8

In the following appendices comparisons are drawn between the Algebra and Geometry syllabi of the various departments of education.

Appendix 1: Algebra for Stds. 7 and 8
Appendix 2: Geometry for Std. 6
Appendix 3: Geometry for Stds. 7 and 8

5.5.3 Discussion

In the foregoing chapter the possible reform of the Mathematics subject matter for high schools was discussed in detail. The question is now whether these ideas of reform are also applicable to the lower classes.

The well-known educationist, Dr. A. L. Behr, writes as follows on this subject: (Translation)

The fundamental principles of new tendencies

The privilege recently fell to my lot to travel through Western Europe and the United States of America to make a study of the teaching of Arithmetic and Mathematics in the countries which I visited.

The revolutionary nature of the changes which were observed is striking. The traditional subject, Arithmetic, is being replaced by a subject known as Elementary Mathematics at the primary school. This subject is really, after the home language, the core subject at primary school. The subject Elementary Mathematics includes Arithmetic as well as Algebra and Geometry.

That primary school children are quite capable of mastering the basic concepts of Algebra and Geometry is undoubtedly true. I myself have experienced the fact that pupils nine or ten years old did work which is prescribed for secondary schools here.

In order to understand what its intended by these new tendencies in instruction in Mathematics and Arithmetic at the primary school, it is necessary to view the purpose and presentation of the subject.

In the subject "Elementary Mathematics" the accent falls on thinking and the systematic arrangement of data. Pure Arithmetic, especially with respect to the basic operations, is not as extended now as it was in the past. Drill and mechanical calculations are limited throughout to small numbers. As far as multiplication and division are concerned, multipliers and divisors of more than two figures are seldom, if ever, used. Unnecessary routine work is reduced and subject matter which does not fit in with everyday life is eliminated.

Aids are used to stimulate reasoning, and learning by intuition rather than simply by repetition is insisted on. In the United States of America I saw how primary school children used a desk calculator to do the calculations involved in a problem which they had to solve. This was naturally only done after the basic concepts with respect to the main operations had already been established.

A syllabus is tried out in an experimental manner in a number of schools before it is made generally applicable.

The importation of Elementary Mathematics into the primary school has consequently brought about the modernization and extension of the scope of Mathematics at the secondary school. There is now room for new up-to-date concepts such as group theory, topology and the binary number system, etc. (10, p. 57)

That the revision of the syllabi should of necessity take due account of the present trends of thought clearly emerges from the following pronouncement of A.N. Whitehead:

> "Any serious fundamental change in the intellectual outlook of human society must necessarily be followed by an educational revolution. It may be delayed for a generation by vested interrests or by the passionate attachment of some leaders of thought to the cycle of ideas within which

they received their own mental stimulus at an impressionable age. But the law is inexorable that education to be living and effective must be directed to informing pupils with those ideas, and to creating for them those capacities which will enable them to appreciate the current thought of their epoch." (112, p. 116)

Any reform in teaching must be tackled in a positive and determined manner otherwise the results can be detrimental.

"Education which is not modern shares the fate of all organic things which are kept too long." (112, p. 117)

When syllabi are drawn up, the material must be selected very carefully, due account being taken of the purpose of the instruction and the requirements of the pupils concerned. "The conclusion at which we arrive is, that mathematics, if it is to be used in general education, must be subjected to a rigorous process of selection and adaptation. It must, on the face of it, deal directly and simply with a few general ideas of far-reaching importance." (112, p. 119)

On what basis should the concepts which have to receive attention be selected?

"Our courses of instruction should be planned to illustrate simply a succession of ideas of obvious importance." (112, p. 119)

"The goal to be aimed at is that the pupil should acquire familiarity with abstract thought, should realise how to apply general methods to its logical investigation." (112, p. 120)

5.5.4 Observation

There is only a partial agreement between the syllabi of the various departments of education, as is clearly apparent from Appendices 1 to 8.

5.6 THE AIMS OF THE SYLLABI

5.6.1 The teachers

According to their replies to the relevant question in Questionnaire N.B.377, the following numbers of teachers give instruction mainly in the subject concerned. The details appear in Table 5.1.

TABLE 5.1

THE NUMBER OF TEACHERS WHO GIVE INSTRUCTION MAINLY IN THE MATHEMATICAL SUBJECT MENTIONED

Subject	Cape Province	Natal	Orange Free State	Transvaal	South West Africa	Education, Arts and Science	Total
Arithmetic	72	52	43	145	3	28	343
General Mathematics	168	7	4	138	10	20	347
Mathematics Stds. 6-8	83	42	39	118	0	56	338
Mathematics Std. 9 & 10	148	51	66	198	5	56	524
No reply received	43	3	11	37	1	18	113
TOTAL	514	155	163	636	19	178	1665

To every teacher the question was put whether he regards as satisfactory the aims of the department's syllabus for the mathematical subject for which he is mainly responsible or whether it is feasible to carry these aims into effect and, if not, what the principal reason is why this is not possible.

5.6.2 The Mathematics Syllabi

Cape of Good Hope

According to the school-leaving syllabus which has been in operation since 1956, the aim of the Mathematics syllabus of the Cape Department of Education is as follows:

(1) To provide the pupils with a fundamental mathematical proficiency and knowledge without which they would be excluded from most of the skilled professions with a scientific or technical basis.

(2) To make the pupils proficient in the use of mathematical methods in the solution of problems.

(3) To cultivate a logical way of thinking in the pupils and to develop the ability to reason systematically.

(4) To enable the pupils to obtain a thorough insight into the concept and knowledge of mathematical principles and to make it possible for them:

(i) to solve problems which occur in every-day life;

(ii) to promote their study of other subjects;

(iii) to pursue further study with ease;

(iv) to display an intelligent interest in the subject matter offered in periodicals and daily papers on matters of a mathematical nature such as those of local government, transport, electrotechnical development, etc. (51, p. 880)

Orange Free State

In the syllabus in use for Stds. 6 to 8 in 1956, the following is recorded under the heading: Introduction and Aim:

"(1) General

The teaching of Mathematics must at all times take due account of the subject's inherent and unique significance for:

(a) the pupil's spiritual and mental development,
(b) the demands made by every-day life on every civilised person,

AND

(c) the requirements of related subjects.

Mathematics calls for the ability to work and think neatly, carefully and systematically and offers the opportunity to develop these valuable characteristics in the pupil. The Mathematics teacher must make full use of this opportunity.

Although every subdivision, namely Arithmetic, Algebra, Geometry, Graphs and Trigonometry, has its own object and function within the whole, all remain part of the single subject however, which can take its rightful place only when each of these divisions is integrated with the others.

Through such integration the various aspects of the problem are illuminated, abstract concepts are graphically represented, geometrical problems are solved algebraically in a simpler manner and vice versa, and the insight and understanding of the pupil are deepened.

It is the task of the teacher to cultivate in the pupil an interest in and love for Mathematics.

"(2) Algebra and Graphs

In Algebra the symbolic language and the representative nature of concepts is an important source of the pupil's difficulties. By making use from the outset of well-known arithmetical problems, the pupils become acquainted with algebraic symbol and formula in a natural manner. Operations with formulas then automatically lead to the introduction and expansion of concepts without formal definitions and rules first having to be laid down.

Conversance with the various processes is acquired through repeated changing of the subject of the formula. The formula should consequently occupy the central position in the teaching of Algebra. If this is done, the function concept continues to assume the greatest prominence. In this connection the graph can be of inestimable value because it is indispensable for the graphical representation of functions and the elucidation of fundamental concepts relating to it, examples being gradient, continuity, etc.

After the initial explanation of axes of co-ordinates and the graphical representation of statistical data the graphic and algebraic treatment of formulas should go hand in hand. The two should form an entity.

"(3) Geometry and Trigonometry

Geometry has a special fascination and formative value owing to:

(a) its aesthetic beauty in form and balance, and
(b) the logical inevitability of its deductive reasoning.

At the beginning however the pupil displays more interest in the practical significance of Geometry and its intimate connection with the world in which he lives. It is the task of the teacher to ensure this link. By stressing the practical meaning of the proposition he can stimulate the interest of the pupil and create the need for formal proof. Without this the proof will at first appear artificial and senseless to the pupil.

The utilitarian aspect and practical significance of Geometry link up in a natural way with Trigonometry. So, for example, a geometric proposition on proportionality in a triangle enables the pupil to solve interesting problems on heights and distances. Straightforward practical problems of this nature make teaching fruitful and rouse the interest of the pupil. And they help to integrate the various sections of Mathematics into a meaningful whole."

School-leaving Certificate

No general aims are mentioned in the curriculum.

Transvaal

Geometry:

The only aim mentioned is "exercise in pure reasoning and exposition."

A.L. Kotzee explains the aims as follows:

(Translation) The civilized person's sucessful participation in modern economic life demands advanced knowledge of the theory of numbers and the application thereof. The success of the great number of people known as research workers who form the core of the public administration of domestic affairs throughout the world, depends to a great extent on the knowledge of the theories and application of specialized sections of Mathematics. (56, p. 56)

Natal

According to the Education Gazette of 7th April, 1960, the aims of the Junior Secondary Course are as follows:

"(i) To develop further the pupil's skill in calculation.

(ii) To train pupils to calculate and measure accurately.

(iii) To develop in pupils an insight into and a concept of mathematical principles, and thereby to assist them in solving mathematical problems which confront them in everyday life.

(iv) To prepare them for making calculations which they may require in the study of other school subjects, as well as in connection with the work which they may undertake if they should leave school at this stage.

(v) To inculcate in pupils the 'concept of proof' and to augment it.

(vi) To enable them to cultivate methods of logical thinking." (67, p. 579)

According to the edition of 25th August, 1960, the Senior Secondary Course has the following aims:

"1. To enable pupils to acquire the fundamental mathematical knowledge and ability without which they would be unable to follow a skilled vocation of a scientific or technical nature.

2. To enable to develop skill in solving problems mathematically.

3. To train pupils in habits of logical thinking and to develop systematic reasoning ability.

4. To provide pupils with a thorough understanding and appreciation of mathematical principles so that these may be used -

   (i) to enable them to solve problems arising in everyday life;
   (ii) to help them to proceed with more advanced mathematical studies;
   (iii) to help them in the study of other subjects;
   (iv) to enable them to read with intelligent interest articles of a mathematical nature regarding local government, transport, electro-technical development, etc., which they may encounter in newspapers and magazines." (68, p. 1319)

5.6.3 The opinion of the teachers (senior classes)

The teachers who take Mathematics mainly in Stds. 9 and 10 were asked whether the aims of their departmental syllabus are satisfactory. Table 5.2 indicates the number of teachers who replied in the affirmative and those who gave negative answers.

TABLE 5.2

THE NUMBER OF TEACHERS' FINDINGS ON THE AIMS OF THE SYLLABI FOR MATHEMATICS IN STDS. 9 AND 10

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Opinions	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
Satisfactory	129	88	39	76	53	80	166	84	3	60	36	64	426	81
Unsatisfactory	13	9	10	20	11	17	25	12	1	20	17	30	77	15
No reply received	5	3	2	4	2	3	7	4	1	20	3	6	20	4
TOTAL	147	100	51	100	66	100	198	100	5	100	56	100	523	100

The replies to the question whether these aims are feasible in practice are shown in Table 5.3.

TABLE 5.3

THE NUMBER OF TEACHERS' OPINIONS ON THE PRACTICABILITY OF THE AIMS OF THE MATHEMATICS SYLLABI FOR STDS. 9 AND 10

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Opinions	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
Practicable	92	63	35	70	44	67	94	47	2	40	28	50	295	56
Impracticable	49	33	13	25	19	29	99	50	3	60	25	44	208	40
No reply received	6	4	3	5	3	4	5	3	0	0	3	6	20	4
TOTAL	147	100	51	100	66	100	198	100	5	100	56	100	523	100

In the Transvaal half of the teachers regard the aims of the syllabi for Stds. 9 and 10 as impracticable. In the other provinces and South West Africa, as well as in the Department of Education, Arts and Science, there is also a fairly high percentage of teachers who apparently have serious misgivings. Their opinions why the aims cannot be achieved are shown in Table 5.4.

TABLE 5.4

THE NUMBER OF TEACHERS' REASONS WHY THE AIMS OF THE MATHEMATICS SYLLABI FOR STDS. 9 AND 10 CANNOT BE ACHIEVED

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Opinions of teachers	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
The contents of the curriculum	5	10	4	31	3	16	7	7	1	33	1	4	21	10
Lack of time	43	88	8	62	14	74	85	86	2	67	20	80	172	83
No reply received	1	2	1	7	2	10	7	7	0	0	4	16	15	7
TOTAL	49	100	13	100	19	100	99	100	3	100	25	100	208	100

Lack of time appears to be the principal cause of concern. In the Transvaal in particular the pinch is felt rather severely because the curricula of the Transvaal Department of Education contain the most subject matter.

5.6.4 The opinions of the teachers (junior classes)

The teachers taking Mathematics mainly in Std. 6 to 8 were asked the question whether the aims of their department's Mathematics syllabus are satisfactory. Table 5.5 indicates the number of teachers who replied in the affirmative and those who answered in the negative.

TABLE 5.5

THE NUMBER OF TEACHERS' OPINIONS ON THE AIMS OF THE SYLLABI FOR MATHEMATICS IN STDS. 6 TO 8

Department	Cape Province		Natal		Orange Free State		Transvaal		Education, Arts and Science		Total	
Opinions of teachers	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
Satisfactory	74	89	32	76	33	85	102	86	48	86	289	85
Unsatisfactory	7	8	8	19	4	10	8	7	7	12	34	11
No replies received	2	3	2	5	2	5	8	7	1	2	15	4
TOTAL	83	100	42	100	39	100	118	100	56	100	338	100

The overwhelming majority (85%) of the teachers are satisfied with the syllabi of their department in so far as the Mathematics for Std. 6 to Std. 8 is concerned. Natal is the only province where less than 80% of the teachers are satisfied and even there the percentage is still 76%.

The teachers were asked the question whether it is possible to achieve these aims in practice. The replies of the teachers taking Mathematics mainly in Stds. 6 to 8 are shown in Table 5.6.

TABLE 5.6

THE NUMBER OF TEACHERS' OPINIONS ON THE PRACTICABILITY OF THE AIMS OF THE MATHEMATICS SYLLABI FOR STDS. 6 TO 8

Department	Cape Province		Natal		Orange Free State		Transvaal		Education, Arts and Science		Total	
Opinions of teachers	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
Practicable	52	63	29	69	24	61	63	53	36	64	204	60
Impracticable	27	32	11	26	10	26	49	42	20	36	117	35
No reply received	4	5	2	5	5	13	6	5	0	0	17	5
TOTAL	83	100	42	100	39	100	118	100	56	100	338	100

Those teachers who stated in their replies that the aims are impracticable were asked why the aims of the Mathematics syllabi for Stds. 6 to 8 cannot be achieved. Their replies are shown in Table 5.7.

TABLE 5.7

THE NUMBERS OF TEACHERS' REASONS WHY THE AIMS OF THE SYLLABI CANNOT BE ACHIEVED

Department	Cape Province		Natal		Orange Free State		Transvaal		Education, Arts and Science		Total	
Opinions of teachers	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
The contents of the syllabus	5	19	1	9	1	10	5	10	3	15	15	13
Lack of time	20	74	9	82	9	90	42	86	13	65	93	80
No replies received	2	7	1	9	0	0	2	4	4	20	9	7
TOTAL	27	100	11	100	10	100	49	100	20	100	117	100

Eighty per cent of teachers who experienced problems in the realisation of the aims of the syllabi attribute this fact to a lack of time. The contents of the syllabi do not present any real problem.

5.6.5 The Arithmetic Syllabus

Orange Free State

The Arithmetical demands of everyday life and especially commerce and industry are that workers should be at home with figures and be able to deal with them accurately, and the Arithmetic syllabus has therefore a highly practical bias, aiming more especially at stabilising the already acquired arithmetical shill. In his daily affairs as well as in every department of Mathematics and kindred subjects, the pupil must be able to calculate quickly and accurately.

For the pupil to become acquainted with the arithmetical interpretation and solution of practical problems, it is essential that examples should be drawn from situations which are within the practical life of the pupil.

In general, calculations should be of a direct character and more complicated examples should be avoided, although these can occasionally be given to pupils with greater initiative. (75(a), p. 3-4)

Natal

For Stds. 1 to 8 the following aims are envisaged: The teaching of Arithmetic should encourage neatness and orderly arrangement of work; endeavour to apply all principles to practical conditions arising out of the experiences of the pupils; encourage individual work; afford a basis for a possible wider study of Mathematics. (71(a), p. 2)

Transvaal

The aim is to train the pupils to work accurately and neatly with figures, to introduce them to calculations which they may be required to do at one time or another in adult life, and to impart knowledge of certain practices obtaining in the world of commerce with which they should be conversant on leaving school.

Considerable latitude is allowed to the schools to give more attention to certain parts of the work than to the others according to their own particular circumstances.

There will be overlapping of the work done in certain other subjects but it must be remembered that these subjects may not be taken by all pupils and that Arithmetic is compulsory for the first three standards of the high school.

Thus it will be only during the arithmetic lessons, while still at school, that many pupils will have an opportunity of acquiring knowledge of certain commercial practices. (100, p. 1)

The object of the teaching of Arithmetic is explained further as follows by A. L. Kotzee (translation from Afrikaans):

> Arithmetic is included in the curriculum not only for the sake of the mental development of the pupils but also with a view to their preparation for intelligent participation in all aspects of modern life in which figures play an important role.
>
> In regard to primary education, the aim is that (translation from Afrikaans)
>
> (i) the number concept should be developed in pupils in such a way that they will understand and master number situations which they encounter in life, and
>
> (ii) the pupils should know certain arithmetical facts, concepts and processes and be able to apply them rapidly and accurately. (56, p. 49)

The aim in teaching Arithmetic is therefore "to use figures effectively and accurately in situations which occur in life."

5.6.6 The opinion of the teachers

Those teachers who are responsible mainly for Arithmetic were asked the question whether the aims envisaged by the Arithmetic syllabus of their department are satisfactory. Table 5.8 reflects the number of teachers who replied in the affirmative, and those who gave negative answers.

TABLE 5.8

THE NUMBER OF TEACHERS' OPINIONS ON THE AIMS OF THE ARITHMETIC SYLLABI

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Opinions	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
Satisfactory	68	95	45	87	39	91	131	90	1	25	25	89	309	90
Unsatisfactory	1	1	2	4	1	2	6	4	0	0	3	11	13	4
No reply received	3	4	5	9	3	7	8	6	3	75	0	0	22	6
TOTAL	72	100	52	100	43	100	145	100	4	100	28	100	344	100

The replies given to the question whether these aims are feasible in practice are reflected in Table 5.9.

TABLE 5.9

THE NUMBER OF TEACHERS' OPINIONS ON THE IMPRACTICABILITY OF THE AIMS OF THE ARITHMETIC SYLLABI

Department	Cape Province		Natal		Orange Free State		Trans-vaal		South West Africa		Education, Arts and Science		Total	
Opinion	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
Practicable	63	60	27	52	27	63	78	54	0	0	20	71	215	57
Impracticable	17	16	16	31	11	26	57	39	2	67	8	29	111	30
No reply given	25	9	9	17	5	11	10	7	1	33	0	0	50	13
TOTAL	105	100	52	100	43	100	145	100	3	100	28	100	376	100

The percentage of teachers finding the aims practicable is somewhat on the low side, namely 57%.

Those teachers who regard the aims as impracticable were asked why the aims of the Arithmetic syllabus cannot be achieved. The replies of the teachers are reflected in Table 5. 10.

TABLE 5.10

THE PRINCIPAL REASONS WHY THE TEACHERS' AIMS CANNOT BE ACHIEVED IN THE MATHEMATICS CURRICULA

Department	Cape Province		Natal		Orange Free State		Trans-vaal		South West Africa		Education, Arts and Science		Total	
Reasons	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
The contents of the curriculum	1	6	2	13	0	0	1	2	0	0	3	38	7	6
Lack of time	14	82	13	81	10	91	56	98	2	100	4	50	99	90
No reply given	2	12	1	6	1	9	0	0	0	0	1	12	5	4
TOTAL	17	100	16	100	11	100	57	100	2	100	8	100	111	100

It is clear that the contents of the syllabi are too comprehensive. The pupils must learn so much that there is apparently no time to halt for a moment in order to ascertain whether the pupils' reasoning is being developed. For that reason it is interesting to observe that 90% of the teachers who consider that they cannot realise the aims of the Arithmetic syllabi, attributed this to a lack of time.

5.6.7 Discussion

L. J. Brueckner formulated the following aims for the teaching of Arithmetic:

"(1) Developing meaningful concepts of numbers and of the number system.

(2) Becoming increasingly skilful in the fundamental operations and in ability to apply them in social situations.

(3) Developing competence in utilizing systems and instruments of measurement and quantitative procedures in dealing with problems of daily living.

(4) Developing desirable interests and attitudes towards arithmetic.

(5) Developing effective methods of studying and learning arithmetic.

(6) Developing desirable behaviour patterns and good citizen traits as a result of group activities in the arithmetic program." (13a, p. 10)

Here the arithmetical concept and the development of concepts are given special prominence, not so much stress being laid on the completeness of the knowledge of the business world. Arithmetic as a subject is also strongly emphasised, whereas in the syllabi of some of the other departments of education Arithmetic itself is relegated to the background, the stress falling on general knowledge.

5.6.8 The Syllabi for General Mathematics

Cape of Good Hope

The aims of the syllabus are:

(i) To develop further the pupils' skill in calculation.

(ii) To train pupils in accuracy in calculation and measurement.

(iii) To develop in them an insight into and a conception of mathematical principles, and thereby to assist in solving mathematical problems which confront them in everyday life.

(iv) To prepare them for the making of calculations which they may require in the study of other school subjects, as well as in connection with the work which they may undertake if they should leave school at this stage.

(v) To enable them to cultivate a logical way of thinking. (29. p. 124)

At a later stage the following aims were added:

"To inculcate the 'concepts of proof' in pupils." (51, p. 887)

Transvaal

It is difficult to ascertain precisely what the aims of the course are. The principal aim given is "integration."

"The division of mathematics into sections is largely artificial although there are certain practical advantages. The sections overlap extensively and arithmetic is the thread which weaves its way through the mathematics.

The integration of arithmetic with mathematics brings the Transvaal into line with mathematics syllabuses both in this country and overseas.

Complete integration cannot be achieved through a change in the syllabus. Teachers have opportunities in their classes for integration with smaller sections than these covered by the main headings of a syllabus."

In connection with Algebra the following aim is put forward:

"Algebra flows easily out of arithmetic. The pupil must be brought to realise the value of algebra." (99, p. 6)

The following aim is given in connection with the teaching of Geometry:

"The chief aims of the course are to enable pupils to become acquainted with geometric terminology and the properties of figures and to develop skill in the use of geometric instruments." (99, p. 10)

At the Conference of Transvaal Inspectors of Education held on 29/30 September, 1960, the following reasons were given for the introduction of General Mathematics in Std. 6:

(Translation) There is no sphere of life where numbers do not find an application. Even the nomads had to count their cattle, regulate their financial affairs and make elementary calculations. The effective employment of figures in the broadest sense of the word is of fundamental importance to any individual who wishes to take a successful and intelligent interest in the community. There is no mode of life or career (in the modern world) which can be carried on without a reasonable knowledge of fundamental methods of Arithmetic. General Mathematics and its branches form the basis for a successful practice in farming, commerce, industry, science, political economy and housekeeping. Planning, progress and success are expressed in figures to a great extent..... A study of the subject General Mathematics forms a part of the general moulding of the child. It is, in particular, the subject which demands broad concentration, observations, accuracy and perseverance from pupils.

It has great educational value in the development of intellectual discipline, an objective attitude and habits of hard work....

Through this, many pupils are placed in a position to acquire a knowledge of elementary mathematical principles which may lead to many juniors losing the traditional fear of Mathematics which they had entertained.

Extension of the programme for General Mathematics to Std. 8 as opposed to the separate subjects Mathematics and Arithmetic ought, just as in General Mathematics, to prevent the presentation of the subjects in water-tight compartments.

A properly integrated syllabus will clear difficulties away which have arisen through the treatment of Arithmetic and Mathematics as separate subjects (compare for example, logarithms and the laws of indices) and will eliminate unnecessary overlapping. It will help the pupil to apply knowledge which he has acquired in one field to other fields. (91, p. 232)

5.6.9 <u>The opinions of the teachers on the syllabi for General Mathematics</u>

All the teachers were asked whether their department has a prescribed syllabus for General Mathematics. Their response to the question is shown in Table 5.11.

In the Orange Free State complete clarity in regard to the General Mathematics syllabus apparently did not as yet exist in 1962. Those teachers who were fully conversant with the syllabus were asked whether the syllabus for General Mathematics satisfied the aims set out in it. The replies are shown in Table 5.12.

The majority of the teachers are of the opinion that the syllabus serves its purpose.

Whereas the foregoing questions were put to <u>all</u> the teachers, those teachers who indicated that they were responsible mainly for General Mathematics were asked whether they regard the aims of the syllabus as satisfactory. Table 5.13 shows the number of teachers who gave an affirmative answer and those who replied in the negative.

As in the case of Arithmetic, the overwhelming majority of the teachers regard the aims of the curriculum for General Mathematics as wholly satisfactory (89%).

TABLE 5.11

REPLIES OF TEACHERS TO THE QUESTION WHETHER THEIR DEPARTMENT HAS A SYLLABUS FOR GENERAL MATHEMATICS

Department	Number of teachers in the various departments						
	Cape Province	Natal	Orange Free State	Transvaal	South West Africa	Education, Arts and Science	Total
Yes	458	105	85	558	18	126	1350
No	5	11	42	6	0	14	78
No answer	51	39	36	72	1	38	237
TOTAL	514	155	163	636	19	178	1655

TABLE 5.12

THE OPINIONS OF TEACHERS ON THE QUESTION WHETHER OR NOT THE SYLLABI FOR GENERAL MATHEMATICS FULFILLED THEIR PURPOSE

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Reply	Num-ber	%	Num-ber	%	Num-ber	%	Num-ber	%	Num-ber	%	Num-ber	%	Num-ber	%
Yes	413	90.2	70	66.7	69	81.2	415	74.4	17	94.4	101	80.2	1085	80.4
No	21	4.6	14	13.3	8	9.4	97	17.4	1	5.6	13	10.3	154	11.4
No answer	24	100.0	21	20.0	8	9.4	46	8.2	0	0	12	9.5	111	8.2
TOTAL	458	100.0	105	100.0	85	100.0	558	100.0	18	100.0	126	100.0	1350	100.0

TABLE 5.13

THE OPINIONS OF THE TEACHERS MAINLY RESPONSIBLE FOR THIS SUBJECT IN REGARD TO THE AIMS OF THE SYLLABI FOR GENERAL MATHEMATICS

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Opinions	Num-ber	%	Num-ber	%	Num-ber	%	Num-ber	%	Num-ber	%	Num-ber	%	Num-ber	%
Satisfactory	153	91	7	100	3	75	119	86	9	90	18	90	309	89
Unsatisfactory	7	4	0	0	0	0	12	9	1	10	1	5	21	6
No reply given	8	5	0	0	1	25	7	5	0	0	1	5	17	5
TOTAL	168	100	7	100	4	100	138	100	10	100	20	100	347	100

The teachers were also asked whether it is possible to carry out these aims in practice. The way in which they replied is reflected in Table 5.14.

TABLE 5.14

THE NUMBER OF TEACHERS' OPINIONS ON THE PRACTICABILITY OF THE AIMS OF THE SYLLABI FOR GENERAL MATHEMATICS

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Opinions	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
Practicable	112	67	3	43	3	75	66	48	9	90	16	80	209	60
Impracticable	46	27	4	57	0	0	65	47	1	10	4	20	120	35
No reply given	10	6	0	0	1	25	7	5	0	0	0	0	18	5
TOTAL	168	100	7	100	4	100	138	100	10	100	20	100	347	100

Those teachers who replied that the aims of the syllabi for General Mathematics are impracticable, were also asked why this is the case. The replies of the teachers mainly responsible for this subject are furnished in Table 5.15.

TABLE 5.15

THE NUMBER OF TEACHERS' REASONS WHY THE AIMS OF THE SYLLABI FOR GENERAL MATHEMATICS CANNOT BE REALISED

Department	Cape Province		Natal		Transvaal		South West Africa		Education, Arts and Science		Total	
Opinions of teachers	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
The contents of the syllabus	4	9	0	0	11	17	1	100	0	0	16	13
Lack of time	38	82	3	75	43	66	0	0	3	75	87	72
No reply given	4	9	1	25	11	17	0	0	1	25	17	15
TOTAL	46	100	4	100	65	100	1	100	4	100	120	100

More than a third (35%) of the teachers mainly responsible for General Mathematics regard the aims as not completely practicable. Almost three quarters (72%) of them attribute this to a lack of time.

5.6.10 Discussion

Particular aims

In their book entitled "Fundamental Concepts of Elementary Mathematics", Brumfiel, Eicholz and Shanks give an aim for each chapter:

"Chapter I. Symbols

(1) To distinguish clearly between the symbols of mathematics and the ideas that the symbols represent.

(2) To introduce some new symbols that will be important in later work.

(3) To explain the use of parentheses.

Chapter 2. History of Numerals

(1) To arouse student interest.

(2) To focus attention upon the fact that symbols are invented by man for specific purposes.

(3) To give students some appreciation for our simple, effective, modern symbolism for numbers.

(4) To improve reading ability.

The last objective is, of course, an aim of each chapter.

Chapter 3. Place Value and Bases

(1) To develop a clear understanding of place value in our base 10 system.

(2) To arouse student interest.

(3) To encourage students to think of grouping objects together.

Chapter 4. Base Ten

(1) To point out the existence of many names for each number and to stress in particular that each number has three special "canonical" names - a fraction name, a decimal name, and a per cent name.

(2) To strengthen understanding of base notation.

(3) To maintain and improve computational skills in base ten.

(4) To give practice in mental arithmetic. (14, p.2)

Chapter 5. Definitions

(1) To help students see numbers as abstract concepts, i.e. as ideas we draw out of the physical world by examining various sets of objects.

(2) To set the stage for the study of properties of rational numbers.

Chapter 6. Basic principles of addition and multiplication

(1) To introduce the basic commutative, associative, distributive, etc. principles.

(2) To emphasize the importance of the basic principles by exhibiting their validity in the unfamiliar setting of modular arithmetic.

Chapter 7. Factors and prime numbers

(1) To stimulate student interest by presenting some elementary but useful concepts of number theory.

(2) To introduce the terms "factor", "divisor", "gcd" and "lcm" in order to clarify the whole number properties used in working with fractions.

Chapter 8. Number pairs, fractions, and rational numbers

(1) To develop carefully the concept of rational number.

(2) To emphasize the importance of the basic principles by showing that the techniques for computing with rational numbers are determined by these principles.

Chapter 9. Subtraction and division

(1) To make it clear to students that the operations subtraction and division are defined in terms of addition and multiplication, emphasizing the similarity of the patterns that accompany these definitions.

(2) To improve computational skill.

(3) To introduce the key concepts that certain pairs of operations are inverses of each other and that certain pairs of rational numbers are reciprocals of each other.

Chapter 10. Inequalities and the number line

(1) To introduce number line concepts.

(2) To give students a feeling for the denseness of the rational numbers; that is, to enable them to see that between any two rational numbers there is a third.

(3) To strengthen understanding of the concepts of measurement and rounding off.

(4) To develop understanding and techniques useful in comparing the sizes of rational numbers.

Chapter 11. Applications

(1) To develop skill in the solution of story problems that involve rational numbers and require reasoning closely related to the representation of number on the number line.

(2) To present more difficult concepts of approximate measurement.

(3) To emphasize the importance of rational numbers. (14, p. 62)

Chapter 12. Decimals

(1) To make clear the relationship between repeating decimals and rational numbers.

(2) To develop skill in computing with fractions and decimals.

(3) To present a technique for determining the rational number represented by an infinite decimal.

Chapter 13. Irrational numbers

(1) To introduce irrational numbers

(2) To give some practice in the approximation of square roots.

Chapter 14. Real numbers

(1) To introduce the real numbers.

(2) To review the basic principles of arithmetic, emphasizing that these principles are the same for real numbers as for rational and whole numbers.

(3) To use square- and cube-root tables and to work with approximations for irrational numbers.

Chapter 15. The real line

(1) To strengthen real-number concepts.

(2) To re-emphasize number line concepts. (14, p. 140)

Chapter 16. Sets and variables

(1) To develop those concepts of set theory that seem necessary for the study of algebra, for example, subset of a set, union of two sets, intersection of two sets.

(2) To make it clear how variables are used in working with sets.

(3) To introduce the concept of solution set of a sentence, and to consider in particular solution sets for equations.

(4) To introduce the idea of generalization.

Chapter 17. Two variables and graphs

(1) To introduce sentences in two variables and the concept of the solution set of such a sentence.

(2) To develop the concept of simultaneous equations without using the customary formal language.

(3) To teach concepts of graphing by using simple lattice point graphs.

Chapter 18. Negative numbers

(1) To introduce the negative integers.

(2) To focus attention again upon the basic principles by showing how they determine the rules for computing with integers.

Chapter 19. Inequalities, the number line, and infinite sets

(1) To give precise meaning to $>$ and $<$ for integers.

(2) To "complete" the real number line by filling in the "left" half.

(3) To examine carefully the concept of an infinite set in preparation for the solving of problems in Chapter 20.

(4) To expand upon graphing concept.

Chapter 20. "Story" problems

(1) To learn to write appropriate equations for "Story" problems.

(2) To help students recognize the "sensible" solution sets that belong to particular problems. (14, p. 168)

Chapter 21. General principles

(1) To give students substantial intuitive understanding of geometrical concepts. (This is the principal objective of the entire geometry unit.)

(2) To call attention to the fact that the objects of geometry are <u>ideas</u>, just as numbers are ideas. We write <u>symbols</u> to help us <u>think</u> about numbers, and so we write <u>symbols</u> and draw <u>pictures</u> to help <u>as think</u> about geometry.

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(3) To call attention to some of the basic principles from which the study of geometry develops. The "discovery" technique is used heavily in leading up to the formulations of these principles. (A precise verbal formulation of these geometric principles and subsequent student memorization is undesirable. But it is important that the student intuitively understands the significance of these basic concepts.)

(4) To learn may facts and definitions that will provide useful background for the systematic study of plane geometry.

Chapter 22. Measurement

(1) To emphasize the two basic steps in measurement, namely the choice of a unit and the subsequent counting process.

(2) To give the student simple intuitive experiences that will enable him to see clearly that the choice of unit is arbitraty.

(3) To indicate the role of the abstract concepts of congruence of segments and congruence of angles in describing measurement of segments and measurement of angles.

Chapter 23. Plane and space figures

(1) To give students experiences with ruler-compass constructions.

(2) To provide students with many opportunities for visualizing three-dimensional figures.

(3) To introduce much of the terminology associated with the familiar two- and three-dimensional figures.

Chapter 24. Perimeter, area, and volume

(1) To become acquainted with some of the simple perimeter, area, and volume formulas, and to develop some facility in their use.

(2) To strengthen measurement concepts.

(3) To guide students through the kind of geometric reasoning that is essential for the development of certain geometric formulas.

Chapter 25. Similar triangles and trigonometry

(1) To introduce the Pythagorean theorem and call attention to a few of its applications.

(2) To develop the concept similarity of triangles and to present some of its applications.

(3) To strengthen concepts of ratio and proportion.

(4) To introduce the trigonometric ratios as an application of the concept of similarity." (14, p. 236)

It is worthwhile following this example by motivating each subject in a syllabus just as clearly as has been done above. Another noteworthy point is the frequency with which the concept is stressed.

In the following paragraph the spotlight will be focussed on the mathematical concept.

5.7 A CENTRAL CONCEPT IN THE TEACHING OF MATHEMATICS

5.7.1 The meaning of a central concept

Considerable research on the mathematical concept has been done in this country in recent years.

The number concept

Pioneer work has been done by J. Spoelstra in die field of the number concept. His study deals with the development of the number concept of the child during his first school year. "As immediate object of the teaching of numbers, the following may be mentioned (translation):

(a) to teach the child to learn the number names in the right order;

(b) to lead the pupils in such a way that they can pair off the number names correctly with the object of a series;

(c) to teach the child to associate small groups correctly with the group name;

(d) to enable the child to acquire a knowledge of the coins and measures in daily use;

(e) to make it possible for the child, armed with this knowledge, to solve problems of an arithmetical nature which he encounters in life, this instruction serving as preparation for further knowledge in the field of numbers." (122, p. 113)

Spoelstra brings a large number of important concepts forward: in (a) the concept of order; (b) the concept of one-to-one correspondence; (c) the set concept. On this solid and firm foundation the conceptual ability of a pupil can be developed along the right lines from his first school year.

The ratio concept

The work of A. J. van Rooy on the mathematical ratio concept links with that of Spoelstra and is directly applicable to the syllabus for the secondary school. He analyses the ratio concept and relates it to human reasoning. (Translation) We can therefore expect that a person's ratio concept will be an indication of his ability to solve any problem in which an insight into relation is important. In reasoning, the ratio concept therefore occupies a key position. The ability to see internal relations, the ratio concept in its wider sense, is merely a necessary condition for the solution of problems. (107, p. 33) After an empirical study he comes to the following conclusion:

(Translation)

(1) At the end of the pupil's primary school career his ratio concept has not yet fully developed.

(2) The task of the primary school in the sphere of arithmetic is to bring the pupil to the stage where he can do ratio sums according to the unit method.

(3) The task of the secondary school is to combine ratios with fractions so that pupils will learn to do ratio sums according to the fraction method.

(4) Algebra and Geometry must be introduced not later than Std. 6. Since they must link up with primary school arithmetic in the closest manner possible the material must be chosen in such a way that the ratio concept is also the central theme here.

(5) In the secondary school the ratio concept should not only be the central concept of arithmetic. It must enjoy the necessary attention in the entire teaching of mathematics. (107, p. 58)

When the mathematical concept is accepted as the starting point in the compilation of a syllabus, it is possible to ensure a unit structure containing all the various parts of Mathematics. This is equally true not only of the ratio concept, but also of all the other concepts such as, for example, the set concept. For educational purposes the relationship between the mathematical concept and the reasoning of the pupils must be borne in mind.

The concept of a variable

A further contribution to the study of the mathematical concept and the reasoning of pupils is the work of J. J. de Wet, He analysed the concept of a variable and came to the following conclusion: (Translation) The variable is one of those concepts in mathematics which we must first experience intuitively and which we then subsequently use and define in mathematics. (120, p. 13)

The variable is not the most important concept in mathematics but it is indispensable for further study and also essential in school mathematics.

And yet it was found that the concept receives only casual attention in the syllabus, the textbook and examination questions for all standards. It was established that the logical order in school mathematics demands an early introduction of the concept. (120, p. 117-8)

An empirical investigation led the writer to the following findings:

(Translation)

The relationship existing between the I.Q. and mathematical concepts (in this case the variable) has been confirmed. The variable cannot develop as desired if the pupil is not mentally mature or not endowed with sufficient intellect. Intelligent pupils form a better concept of the variable.

The variable develops gradually but shows certain shortcomings in the process of development. So, for example, the variable is confused with the unknown.

The shortcomings which exist in the development of the concept and the irregular development of the various aspects of the concept in contrast with the general aspect of the concept, is attributed by us to a teaching of mathematics which judging from the syllabus, textbooks and examination questions, does not allow justice to be done to an important concept in Mathematics. (120, p. 118-9)

The function concept

In a second study A. J. van Rooy considered the function concept. (Translation) The mathematical ratio and variation concepts are united in the function concept. (107, p. 7) He analyses the development of the psychology of thinking and comes to the conclusion that the mechanistic approach has become obsolete. (Translation) It is realised that the earlier approach was very one-sidedly materialistic - mechanistic. If the atom is no longer the real substance, it is also no longer necessary that thinking should be explained in terms of matter. In the field of thinking, too, the real, the perceptual, must occupy a less important posistion, and more attention will have to be given to the non-perceptive. (107, p. 37-8)

After analysing "functional thinking" he applies it as follows to teaching:

(Translation) The pupil must therefore be able to discover a relation in varied circumstances. Here relation and variation are intimately connected with one another and consequently present the opportunity for functional thinking. The same but nevertheless changed structure must be sought; in other words, a constant must also be present in this variable. A pupil who can still see the constant elements in every changed problem, can move from the unknown to the known and thereafter delight in the results of his thinking.

For that reason the function concept must be sought in every part of mathematics so that the development of functional thinking can permeate the whole of the teaching of Mathematics. (107, p. 99)

With reference to the foregoing views it will be interesting to see what happens to the idea of a central concept in the teaching of Mathematics in South African high schools.

5.7.2 A central concept in high school Mathematics

The teachers were asked whether, according to their opinion, there is a central concept in high school Mathematics. In a second question it was asked whether, according to the official curriculum, any such central concept existed. The replies are reflected in Table 5.16.

TABLE 5.16

THE NUMBER OF TEACHERS' OPINIONS ON THE EXISTENCE OF A CENTRAL CONCEPT IN HIGH SCHOOL MATHEMATICS

Opinion of the Teachers	Teachers					
	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
According to their own opinions:						
There is a central concept	307	43.4	292	29.7	599	35.4
There is no central concept	192	27.1	118	12.0	310	18.3
No reply given	209	29.5	573	58.3	782	46.3
TOTAL	708	100.0	983	100.0	1691	100.0
According to the official syllabus:						
There is a central concept	258	36.4	275	28.0	533	31.5
There is no central concept	179	25.3	102	10.4	281	16.6
No reply given	271	38.3	606	61.6	877	51.9
TOTAL	708	100.0	983	100.0	1691	100.0

Conclusion

Slightly more than a third (35.4%) of the teachers are of the opinion that there is a central concept in high school Mathematics. Less than half of the teachers replied to the question whether, according to the official curriculum, there is a central concept in high school Mathematics.

In the teaching of Mathematics not much interest is shown in the mathematical concept as such. Complaints that concept formation does not receive the necessary attention are therefore not unfounded. If the majority of the teachers do not even know whether the official syllabus makes provision for a central concept, they certainly also do not display much interest in concept formation.

5.7.3 What concept may be regarded as the central concept

The teachers were asked the question what in their opinion and according to the official curriculum, the central concept in high school Mathematics is. The replies given by the teachers are shown in Tables 5.17 and 5.18. In Table 5.18 a distinction is drawn between the teachers in the various provinces and in Table 5.17 between teachers who are qualified in Mathematics and those who are not. The numbers do not tally exactly since in one section use was made of the opinions of 1691 teachers and in the other of 1665 teachers.

TABLE 5.17

THE CENTRAL CONCEPT IN HIGH SCHOOL MATHEMATICS ACCORDING TO THE OPINIONS OF THE TEACHERS

	Central concept	Teachers					
		Qualified in Mathematics		Unqualified in Mathematics		Total	
1.	Logical reasoning	105	14.8	91	9.3	196	11.6
2.	Fundamental skill, knowledge, accuracy	50	7.1	35	3.6	85	5.0
3.	Function	44	6.2	19	1.9	63	3.7
4.	Equations	36	5.1	22	2.2	58	3.4
5.	Ratio	13	1.8	16	1.6	29	1.7
6.	Function and equation	12	1.7	5	0.5	17	1.0
7.	Relation	9	1.3	6	0.6	15	0.9
8.	Quantity, number	10	1.4	8	0.8	18	1.1
9.	Proceeding from concrete to abstract concept	8	1.1	4	0.4	12	0.7
10.	Symbolic method of statement	5	0.7	0	0	5	0.3
11.	Proof	3	0.4	2	0.2	5	0.3
12.	Sets	2	0.3	2	0.2	4	0.3
13.	No reply given	411	58.1	773	78.7	1184	70.0
	TOTAL	708	100.0	983	100.0	1691	100.0

Conclusion

Seventy per cent of the teachers found themselves unable to say what the central concept in high school Mathematics is. Of those who did express an opinion on the subject, the largest groups indicated "logical reasoning" and "fundamental skill, knowledge, accuracy", as the central concepts. But these are not mathematical concepts. A negligible minority admittedly mentioned concepts such as function, ratio, equations, relations, quantity and number. The important concept of sets was indicated by only four teachers as the central concept.

That 70% of the teachers mentioned in Table 5.18 found themselves unable to answer this question probably shows that they have never yet properly asked themselves what really is the sense of teaching Mathematics. This fact is eloquent evidence of a very great lack of direction in the teaching of Mathematics. It is difficult to see how an enthusiasm for this subject can be roused in pupils under the present confused circumstances.

5.7.4 The central concept according to the official syllabi

Table 5.19 indicates what, according to the opinions of the teachers in the various departments, the central concept in the Mathematics syllabus is.

TABLE 5.18

WHAT THE CENTRAL CONCEPT IN HIGH SCHOOL MATHEMATICS SHOULD BE ACCORDING TO OPINIONS OF THE TEACHERS IN THE VARIOUS DEPARTMENTS OF EDUCATION

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Concept	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. Logical reasoning	90	17.5	20	13.0	14	8.6	45	7.1	2	10.5	20	11.2	191	11.5
2. Fundamental skill, knowledge, accuracy	38	7.4	2	1.3	7	4.3	25	3.9	1	5.2	12	6.7	85	5.1
3. Function	10	1.9	2	1.3	10	6.1	34	5.3	2	10.5	5	2.8	63	3.8
4. Equation	14	2.9	2	1.3	7	4.3	29	4.6	1	5.2	5	2.8	58	3.5
5. Ratio	5	1.0	6	3.9	1	0.6	13	2.0	0	-	3	1.7	28	1.7
6. Function and equation	4	0.8	2	1.3	0	-	11	1.7	0	-	0	-	17	1.0
7. Relation	6	1.2	3	1.9	1	0.6	3	0.5	0	-	2	1.1	15	0.9
8. Quantity, number	1	0.2	2	1.3	2	1.2	9	1.4	1	5.2	3	1.7	18	1.1
9. Proceeding from concrete to abstract concept	2	0.4	3	1.9	0	-	1	0.2	0	-	6	3.4	12	0.7
10. Symbolic method of statement	2	0.4	0	-	0	-	1	0.2	0	-	2	1.1	5	0.3
11. Proof	3	0.6	1	0.6	0	-	1	0.2	0	-	0	-	5	0.3
12. Sets	0	-	1	0.6	2	1.2	1	0.2	0	-	0	-	4	0.2
13. No reply given	339	66.0	111	71.6	119	73.1	463	72.7	12	63.4	120	67.5	1164	69.9
TOTAL	514	100.0	155	100.0	163	100.0	636	100.0	19	100.0	178	100.0	1665	100.0

TABLE 5.19

THE CENTRAL CONCEPT IN THE MATHEMATICS SYLLABI OF THE VARIOUS DEPARTMENTS OF EDUCATION ACCORDING TO THE NUMBER OF TEACHERS' OPINIONS

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Central Concept	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. Logical reasoning	54	10.5	5	3.3	7	4.3	17	2.7	1	5.3	10	5.6	94	5.6
2. Fundamental skill, knowledge, accuracy	49	9.5	0	-	5	3.1	12	1.9	2	10.5	10	5.6	78	4.7
3. Function	8	1.6	0	-	6	3.7	22	3.5	1	5.3	2	1.1	39	2.3
4. Equation	11	2.0	2	1.3	4	2.5	25	3.9	0	-	2	1.1	44	2.6
5. Ratio	2	0.4	2	1.3	0	-	3	0.5	0	-	1	0.6	8	0.5
6. Function and ratio	2	0.4	0	-	0	-	9	1.4	0	-	1	0.6	12	0.7
7. Relation	0	-	0	-	1	0.6	2	0.3	0	-	1	0.6	4	0.2
8. Quantity, number	1	0.2	1	0.6	2	1.2	7	1.1	1	5.3	2	1.1	14	0.8
9. Proceeding from concrete to abstract concept	3	0.6	2	1.3	0	-	0	-	0	-	3	1.7	8	0.5
10. Symbolical method of statement	2	0.4	0	-	0	-	1	0.2	0	-	1	0.6	4	0.2
11. Proof	5	1.0	1	0.6	0	-	0	-	1	5.3	0	-	7	0.4
12. Sets	2	0.4	1	0.6	1	0.6	1	0.2	0	-	2	1.1	7	0.4
13. No reply given	375	73.0	141	91.0	137	84.0	537	84.3	13	68.3	143	80.3	1346	81.1
TOTAL	514	100.0	155	100.0	163	100.0	636	100.0	19	100.0	178	100.0	1665	100.0

Table 5.20 shows what, in the opinion of the qualified and the unqualified Mathematics teachers, the central concept in high school Mathematics is.

TABLE 5.20

THE CENTRAL CONCEPT IN THE HIGH SCHOOL SYLLABI FOR MATHEMATICS ACCORDING TO THE OPINIONS OF THE TEACHERS

Central concepts according to the official curricula	Teachers					
	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
1. Logical reasoning	46	6.5	49	5.0	95	5.6
2. Fundamental skill, knowledge,accuracy	50	7.1	29	3.0	79	4.7
3. Function	24	3.4	15	1.5	39	2.3
4. Equation	24	3.4	20	2.0	44	2.6
5. Ratio	6	0.8	2	0.2	8	0.5
6. Function and equation	10	1.4	2	0.2	12	0.7
7. Relation	2	0.3	2	0.2	4	0.2
8. Quantity, number	6	0.8	8	0.8	14	0.8
9. Proceeding from concrete to abstract concepts	6	0.8	2	0.2	8	0.5
10. Symbolic method of statement	2	0.3	2	0.2	4	0.2
11. Proof	6	0.8	1	0.1	7	0.4
12. Sets	5	0.7	2	0.2	7	0.4
13. No reply given	521	73.6	849	86.4	1370	81.1
TOTAL	708	100.0	983	100.0	1691	100.0

Conclusion

More than 80% of the teachers find themselves unable to state with certainty what the central concept in high school Mathematics is. If to this figure is added the percentage of persons who give "logical reasoning", "fundamental skill, etc.", "proceeding from concrete to abstract concepts" (which are not really mathematical concepts at all) as the central concept, 91.9% of the teachers do not know what a central concept is according to the official syllabi.

If there really are any such concepts, no clarity exists in regard to them. Possibly a central concept must be sought for each separate section of this subject.

5.7.5 The central concept in Arithmetic

Table 5.21 shows what, in the opinion of the teachers, the central concept in Arithmetic is.

Conclusion

Almost half (47.2%) of the teachers were prepared to give their own opinion in regard to the central concept in the teaching of Arithmetic. Approximately a third (33.8%) of them are of the opinion that there is no central concept in Arithmetic. Of those who are aware of such a concept, the majority (26.8%) indicated the ratio concept as the central concept.

In so far as the official syllabi are concerned, less than a third (32.1%) of the teachers found it possible to express an opinion. Of these, 44.3% could discover no central concept in the curricula, while 17.7% also stipulated the ratio concept.

All this indicates that the inherent object of Mathematics as a subject at school is not recognised. This relative aimlessness is without doubt one of the principal reasons why so many pupils assert that Mathematics is a difficult subject. Much more trouble should be taken to show what an important role the subject Mathematics plays in a community. If this were done, interest would be roused and the study of the subject made more purposeful.

"Arithmetic can be learnt, and learnt quite effectively, with very little understanding of the mathematical concepts involved. What is often not learnt - or learnt only by accident - is the basic mental operation that underlies all later mathematics as well - the recognition of mathematical relationship in the environment." (23, p. 697)

Table 5.22 shows what, in the opinion of the teachers in the various departments, the central concept of the Arithmetic syllabi should be.

Table 5.23 reflects the impressions of the teachers in regard to what the central concept in the teaching of Arithmetic is according to the official syllabi of the various departments of education.

5.7.6 The central concept in Algebra

Table 5.24 indicates what, according to the opinion of the teachers, the central concept in Algebra is.

Condusion

It is a striking fact that the teachers are more outspoken in stating their own opinions in regard to the central concept than in saying what stands in the official syllabi. Slightly less than half (48%) of the teachers expressed their personal opinion, while approximately a third (33.2%) recorded their interpretation of the syllabi. The qualified teachers in Mathematics also replied more readily than their colleagues without the necessary qualifications in Mathematics (69.4% as against 32.7% and 48.6% as against 22.2%).

A third (33.4%) of the qualified teachers of Mathematics see the function concept as the central concept in the teaching of Algebra. A smaller group (30.3%) accord this place of honour to equations, while 27.5% are of the opinion that there is no such central concept. In so far as the interpretation of the official curricula is concerned, the ratio is exactly the reverse. Just over a third (34.6%) of the teachers with the necessary qualifications in Mathematics consider that the syllabi were not drawn up round a central concept, while 29.1% give pride of place to equations and 26.7% to function. This order is maintained when the opinions of the unqualified teachers of Mathematics are examined more closely.

An urgent need of greater clarity therefore exists in regard to what the central concept in the teaching of Algebra should be.

In Table 5.25 the opinion of the teachers in the various education departments on what the central concept in Algebra ought to be, appears.

Table 5.26 reflects the interpretation which the teachers attach to the departmental syllabi in respect of the central concept in the teaching of Algebra.

In Table 5.27 the number of teachers who consider that the function concept should be the central concept in the teaching of Algebra is compared with the number of teachers who interpret the function concept as the central concept in the official syllabi.

It is only in the Orange Free State where the percentage of teachers indicating the function concept as the central concept in the syllabus exceeds the percentage of teachers who personally consider that it should be the central concept.

TABLE 5.21

THE CENTRAL CONCEPT IN ARITHMETIC ACCORDING TO THE OPINION OF THE TEACHERS AND ACCORDING TO THEIR INTERPRETATION OF THE SYLLABI

Central Concept	Personal Opinion						Official syllabi					
	Qualified in Mathematics		Unqualified in Mathematics		Total		Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. None	121	31.2	149	36.3	270	33.8	121	44.5	119	44.0	240	44.3
2. Ratio	97	25.0	117	28.5	214	26.8	42	15.4	54	20.0	96	17.7
3. Quantity	77	19.8	59	14.4	136	17.0	45	16.5	36	13.3	81	14.9
4. Function	10	2.6	21	5.1	31	3.9	10	3.7	17	6.3	27	5.0
5. Equations	9	2.3	20	4.9	29	3.6	4	1.5	11	4.1	15	2.8
6. Relation	19	4.9	17	4.1	36	4.5	15	5.5	10	3.7	25	4.6
7. Magnitude	27	7.0	10	2.4	37	4.6	14	5.1	9	3.3	23	4.2
8. Sets	8	2.1	5	1.2	13	1.6	6	2.2	4	1.5	10	1.8
9. Similarity	13	3.4	5	1.2	18	2.3	8	2.9	3	1.1	11	2.0
10. Other	7	1.7	7	1.9	14	1.9	7	2.7	7	2.7	14	2.7
TOTAL	338	100.0	410	100.0	798	100.0	272	100.0	270	100.0	542	100.0
Reply to question	338	54.8	410	41.7	798	47.2	272	38.4	270	27.5	542	32.1
No reply given	320	45.2	573	58.3	893	52.8	436	61.6	713	72.5	1149	67.9
GRAND TOTAL	708	100.0	983	100.0	1691	100.0	708	100.0	983	100.0	1691	100.0

TABLE 5.22

THE CENTRAL CONCEPT IN THE TEACHING OF ARITHMETIC ACCORDING TO THE VIEWS OF THE TEACHERS IN THE VARIOUS DEPARTMENTS

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Central Concept	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. No central concept	75	32.1	20	29.0	24	32.4	117	36.1	3	37.5	25	32.1	264	33.5
2. Ratio	67	28.6	22	31.9	13	17.6	91	28.1	2	25.0	17	21.8	212	26.9
3. Quantity	42	17.9	6	8.7	18	24.3	48	14.8	2	25.0	18	23.1	134	17.0
4. Function	14	6.0	4	5.8	4	5.4	5	1.5	0	-	4	5.1	31	3.9
5. Equations	11	4.7	0	-	5	6.8	6	1.9	1	12.5	6	7.7	29	3.7
6. Relation	10	4.3	10	14.5	3	4.0	10	3.1	0	-	3	3.8	36	4.6
7. Magnitude	3	1.3	4	5.8	1	1.4	25	7.7	0	-	3	3.8	36	4.6
8. Sets	2	0.9	2	2.9	2	2.7	6	1.9	0	-	1	1.3	13	1.7
9. Similarity	5	2.1	0	-	2	2.7	11	3.4	0	-	0	-	18	2.3
10. Other	5	2.1	1	1.4	2	2.7	5	1.5	0	-	1	1.3	14	1.8
TOTAL	234	100.0	69	100.0	74	100.0	324	100.0	8	100.0	78	100.0	787	100.0
Replied to question	234	45.5	69	44.5	74	45.4	324	50.9	8	42.1	78	43.8	787	47.3
No reply given	280	54.5	86	55.5	89	54.6	312	49.1	11	57.9	100	56.2	878	52.7
GRAND TOTAL	514	100.0	155	100.0	163	100.0	636	100.0	19	100.0	178	100.0	1665	100.0

TABLE 5.23

THE CENTRAL CONCEPT IN THE OFFICIAL ARITHMETIC SYLLABI AS INTERPRETED BY THE NUMBER OF TEACHERS

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Concept	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. No central concept	73	46.2	19	38.0	13	28.3	107	48.4	6	75.0	20	37.0	238	44.3
2. Ratio	26	16.4	11	22.0	10	21.7	41	18.6	0	-	7	13.0	95	17.7
3. Quantity	27	17.1	5	10.0	10	21.7	28	12.7	1	12.5	9	16.7	80	14.9
4. Function	12	7.6	1	2.0	3	6.5	7	3.2	0	-	3	5.6	26	4.8
5. Equations	6	3.8	0	-	1	2.2	4	1.8	1	12.5	3	5.6	15	2.8
6. Relation	3	1.9	8	16.0	1	2.2	7	3.2	0	-	6	11.0	25	4.7
7. Magnitude	2	1.3	3	6.0	2	4.3	12	5.4	0	-	4	7.4	23	4.3
8. Sets	2	1.3	1	2.0	2	4.3	3	1.3	0	-	2	3.7	10	1.9
9. Similarity	2	1.3	0	-	1	2.2	8	3.6	0	-	0	-	11	2.0
10. Other	5	3.1	2	4.0	3	6.5	4	1.8	0	-	0	-	14	2.6
TOTAL	158	100.0	50	100.0	46	100.0	221	100.0	8	100.0	54	100.0	537	100.0
Replied to question	158	30.8	50	32.2	46	28.2	221	34.7	8	42.2	54	30.3	537	32.3
No reply given	356	69.2	105	67.8	117	71.8	415	65.3	11	57.8	124	69.7	1128	67.7
GRAND TOTAL	514	100.0	155	100.0	163	100.0	636	100.0	19	100.0	178	100.0	1665	100.0

TABLE 5.24

THE NUMBER OF TEACHERS' OPINIONS ON THE CENTRAL CONCEPT IN ALGEBRA

Central Concept	Personal opinions of teachers						Official syllabi as interpreted by teachers					
	Qualified in Mathematics		Unqualified in Mathematics		Total		Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. Function	164	33.4	69	21.5	233	28.7	92	26.7	39	17.9	131	23.3
2. Equation	149	30.3	104	32.4	253	31.1	100	29.1	59	27.1	159	28.3
3. None	135	27.5	110	34.3	245	30.1	119	34.6	100	45.9	219	39.0
4. Similarity	20	4.1	14	4.4	34	4.2	13	3.8	7	3.2	20	3.6
5. Relation	10	2.0	10	3.1	20	2.5	7	2.0	6	2.8	13	2.3
6. Ratio	4	0.8	7	2.2	11	1.4	3	0.9	5	2.3	8	1.4
7. Sets	3	0.6	1	0.3	4	0.5	4	1.2	0	-	4	0.7
8. Magnitude	0	-	3	0.9	3	0.4	2	0.6	0	-	2	0.4
9. Other	6	1.3	3	0.9	9	1.1	4	1.1	2	0.8	6	1.0
TOTAL	491	100.0	321	100.0	812	100.0	344	100.0	218	100.0	562	100.0
Replied to question	491	69.4	321	32.7	812	48.0	344	48.6	218	22.2	562	33.2
No reply given	217	30.6	662	67.3	879	52.0	364	51.4	765	77.8	1129	66.8
GRAND TOTAL	708	100.0	983	100.0	1691	100.0	708	100.0	983	100.0	1691	100.0

TABLE 5.25

WHAT THE CENTRAL CONCEPT IN THE TEACHING OF ALGEBRA SHOULD BE ACCORDING TO THE OPINIONS OF THE NUMBER OF TEACHERS

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Concept	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. Function	63	26.0	21	29.6	28	38.8	88	28.7	4	50.0	27	27.0	231	28.9
2. Equation	80	33.1	15	21.1	15	20.8	100	32.6	2	25.0	34	34.0	246	30.8
3. No central concept	67	27.6	21	29.6	22	30.6	102	33.2	2	25.0	28	28.0	242	30.2
4. Similarity	17	7.0	3	4.2	3	4.2	5	1.6	0	-	6	6.0	34	4.2
5. Relation	5	2.1	6	8.5	2	2.8	6	2.0	0	-	1	1.0	20	2.5
6. Ratio	4	1.7	1	1.4	0	-	4	1.3	0	-	2	2.0	11	1.4
7. Sets	2	0.8	1	1.4	0	-	1	0.3	0	-	0	-	4	0.5
8. Magnitude	0	-	2	2.8	1	1.4	0	-	0	-	0	-	3	0.4
9. Other	14	1.7	1	1.4	1	1.4	1	0.3	0	-	2	2.0	9	1.1
TOTAL	242	100.0	71	100.0	72	100.0	307	100.0	8	100.0	100	100.0	800	100.0
Replied to question	242	48.1	71	45.8	72	44.2	307	48.2	8	42.1	100	56.2	800	48.0
No reply given	272	52.9	84	54.2	91	55.8	329	51.8	11	57.9	78	43.8	865	52.0
GRAND TOTAL	514	100.0	155	100.0	163	100.0	636	100.0	19	100.0	178	100.0	1665	100.0

TABLE 5.26

THE CENTRAL CONCEPT IN THE TEACHING OF ALGEBRA ACCORDING TO THE OFFICIAL SYLLABI

Department	Number of teachers who recognise a certain concept as the central concept													
	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Concept	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. Function	36	23.1	9	17.0	22	44.9	53	24.6	1	12.5	9	12.5	130	23.5
2. Equation	39	25.0	6	11.3	12	44.5	75	34.7	1	12.5	23	31.9	156	28.2
3. No central concept	63	40.4	23	43.4	11	22.5	83	38.4	6	75.0	31	43.0	217	39.2
4. Similarity	9	5.8	4	7.5	3	6.1	2	0.9	0	-	2	2.8	20	3.6
5. Relation	3	1.9	6	11.3	1	2.0	0	-	0	-	2	2.8	12	0.2
6. Ratio	3	1.9	0	-	0	-	0	-	0	-	4	5.6	7	0.4
7. Sets	2	1.3	1	1.9	0	-	0	-	0	-	1	1.4	4	0.6
8. Magnitude	0	-	1	1.9	0	-	1	0.5	0	-	0	-	2	0.4
9. Quantity	0	-	0	-	0	-	0	-	0	-	0	-	0	-
10. Other	1	0.6	3	5.7	0	-	2	0.9	0	-	0	-	6	1.0
TOTAL	156	100.0	53	100.0	49	100.0	216	100.0	8	100.0	72	100.0	554	100.0
Replied to question	156	30.4	53	34.2	49	30.1	216	34.0	8	42.1	72	40.4	554	33.3
No reply given	358	69.6	102	65.8	114	69.9	420	66.0	11	57.9	106	59.6	1111	66.7
GRAND TOTAL	514	100.0	155	100.0	163	100.0	636	100.0	19	100.0	178	100.0	1665	100.0

TABLE 5.27

NUMBER OF TEACHERS WHO CONSIDER THAT THE FUNCTION CONCEPT SHOULD BE THE CENTRAL CONCEPT IN THE TEACHING OF ALGEBRA COMPARED WITH THE NUMBER OF TEACHERS WHO CONSIDER THAT IT IS THE CENTRAL CONCEPT ACCORDING TO THE SYLLABI

Department of Education	Teachers who consider that the function concept is the central concept according to			
	their personal opinion		the official syllabi	
	Number	Percentage	Number	Percentage
Cape of Good Hope	63	26.0	36	23.1
Natal	21	29.6	9	17.0
Orange Free State	28	38.8	22	44.9
Transvaal	88	28.7	53	24.6
South West Africa	4	50.0	1	12.5
Education, Arts and Science	27	27.0	9	12.5
TOTAL	231	28.9	130	23.5

In the Orange Free State syllabus for Algebra and Graphs the following assertion is made: "The formula should consequently occupy the central place in the teaching of Algebra so that the function concept can continually assume the greatest prominence."

Possibly this clear reference to the function concept in the syllabus explains why a higher percentage of teachers in the Orange Free State than in any other department of education recognised this concept as a central concept. The extent to which teachers are influenced by the official syllabi is also apparent from the fact that it so happens that the highest percentage of teachers who stated that the function concept should be the central concept, is encountered in the case of the Orange Free State, if South West Africa is not taken into account.

5.7.7 The central concept in Geometry

Table 5.28 shows what, in the opinion of the teachers, the central concept in Geometry is.

Conclusion

No clarity exists in regard to what the central concept in the teaching of Geometry is. Almost half (43.2%) of the teachers consider that the official syllabi do not make mention of any such central concept.

Table 5.29 reflects the opinion of the teachers in regard to what the central concept in the teaching of Geometry should be. The teachers have been classified according to the various departments of education.

Table 5.30 shows how many teachers interpret the existing syllabi in such a way that the given concept is the central concept.

5.7.8. The central concept in Trigonometry

Table 5.31 shows what, according to the opinion of the teachers, the central concept in Trigonometry is.

Conclusion

The majority of the teachers who are indeed aware of a central concept in Trigonometry, distinguished the ratio concept as such.

TABLE 5.28

THE CENTRAL CONCEPT IN GEOMETRY ACCORDING TO THE NUMBER OF TEACHERS' PERSONAL OPINIONS AND ACCORDING TO THEIR INTERPRETATION OF THE SYLLABI

Central Concept	Personal opinion						Official syllabi					
	Qualified in Mathematics		Unqualified in Mathematics		Total		Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. No central concept	145	31.0	94	31.1	239	31.0	135	41.9	90	45.2	225	43.2
2. Relation	94	20.1	57	18.9	151	19.6	45	14.0	21	10.6	66	12.7
3. Similarity	76	16.2	33	10.9	109	14.2	52	16.1	21	10.6	73	14.0
4. Congruency	62	13.2	58	19.2	120	15.6	42	13.0	37	18.6	79	15.2
5. Ratio	28	6.0	23	7.6	51	6.6	14	4.3	10	5.0	24	4.6
6. Magnitude	17	3.6	13	4.3	30	3.9	9	2.8	9	4.5	18	3.5
7. Sets	13	2.8	9	3.0	22	2.9	7	2.2	4	2.0	11	2.1
8. Equation	9	1.9	4	1.3	13	1.7	1	0.3	3	1.5	4	0.8
9. Function	4	0.9	7	2.3	11	1.4	4	1.2	1	0.5	5	1.0
10. Quantity	2	0.4	1	0.3	3	0.4	5	1.6	1	0.5	6	1.2
11. Other	18	3.9	3	1.1	21	2.7	8	2.6	2	1.0	10	1.7
TOTAL	468	100.0	302	100.0	770	100.0	322	100.0	199	100.0	521	100.0
Replied to question	468	66.1	302	30.7	770	45.5	322	45.5	199	20.2	521	30.8
No reply given	240	33.9	681	69.3	921	54.5	386	54.5	784	79.8	1170	69.2
GRAND TOTAL	708	100.0	983	100.0	1691	100.0	708	100.0	983	100.0	1691	100.0

TABLE 5.29

WHAT THE CENTRAL CONCEPT IN THE TEACHING OF GEOMETRY SHOULD BE ACCORDING TO THE OPINIONS OF THE NUMBER OF TEACHERS

Department	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Concept	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. No central concept	65	27.5	19	27.5	25	35.2	101	35.3	1	12.5	26	29.6	237	31.3
2. Relation	35	16.1	27	39.1	8	11.3	55	19.2	2	25.0	17	19.3	147	19.4
3. Similarity	31	13.1	3	4.4	17	23.9	39	13.6	3	37.5	15	17.0	108	14.2
4. Congruency	39	16.5	9	12.9	8	11.3	45	15.7	1	12.5	14	15.9	116	15.3
5. Ratio	15	6.4	4	5.8	4	5.6	20	7.0	0	-	8	9.1	51	6.7
6. Magnitude	18	7.6	1	1.5	0	-	9	3.2	0	-	1	1.1	29	3.8
7. Sets	5	2.1	3	4.4	3	4.2	10	3.5	0	-	1	1.1	22	2.9
8. Equations	7	3.0	1	1.5	1	1.4	2	0.7	0	-	2	2.3	13	1.7
9. Function	9	3.8	0	-	2	2.8	0	-	0	-	0	-	11	1.5
10. Quantity	3	1.3	0	-	0	-	0	-	0	-	0	-	3	0.4
11. Other	6	2.6	2	2.9	3	4.3	5	1.8	1	12.5	4	4.6	21	2.8
TOTAL	236	100.0	69	100.0	71	100.0	286	100.0	8	100.0	88	100.0	758	100.0
Replied to question	236	45.9	69	44.5	71	43.6	286	45.0	8	42.1	88	49.4	758	45.5
No reply given	278	54.1	86	55.5	92	56.4	350	55.0	11	57.9	90	50.6	907	54.5
GRAND TOTAL	514	100.0	155	100.0	163	100.0	636	100.0	19	100.0	178	100.0	1665	100.0

TABLE 5.30

THE CENTRAL CONCEPT ACCORDING TO THE OFFICIAL GEOMETRY SYLLABI

Department	Teachers who interpreted a given concept as the central concept													
	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Concept	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. No central concept	62	41.1	22	44.0	18	39.1	90	45.5	1	16.7	28	45.2	221	43.1
2. Relation	20	13.2	12	24.0	4	8.7	20	10.1	0	-	9	14.5	65	12.7
3. Similarity	15	9.9	2	4.0	11	23.9	34	17.2	1	16.7	10	16.1	73	14.2
4. Congruency	26	17.2	6	12.0	6	13.0	28	14.1	1	16.7	10	16.1	77	15.0
5. Ratio	6	4.0	3	6.0	3	6.5	11	5.6	0	-	1	1.6	24	4.7
6. Magnitude	9	6.0	1	2.0	1	2.2	2	1.0	3	50.0	1	1.6	17	3.3
7. Sets	1	0.7	2	4.0	1	2.2	6	3.0	0	-	1	1.6	11	2.1
8. Equation	1	0.7	1	2.0	0	-	1	0.5	0	-	1	1.6	4	0.8
9. Function	3	2.0	0	-	1	2.2	1	0.5	0	-	0	-	5	1.0
10. Quantity	4	2.6	0	-	0	-	1	0.5	0	-	1	1.6	6	1.2
11. Other	4	2.6	1	2.0	1	2.2	4	2.0	0	-	0	-	10	1.9
TOTAL	151	100.0	50	100.0	46	100.0	198	100.0	6	100.0	62	100.0	513	100.0
Replied to question	151	29.4	50	32.3	46	28.2	198	31.1	6	31.6	62	34.8	513	30.8
No reply given	363	70.6	105	67.7	117	71.8	438	68.9	13	68.4	116	65.2	1152	69.2
GRAND TOTAL	514	100.0	155	100.0	163	100.0	636	100.0	19	100.0	178	100.0	1665	100.0

TABLE 5.31

THE CENTRAL CONCEPT IN TRIGONOMETRY ACCORDING TO THE TEACHERS

Central Concept	Personal opinions of teachers						Official syllabi as interpreted by teachers					
	Qualified in Mathematics		Unqualified in Mathematics		Total		Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. Ratio	215	47.2	102	42.0	317	45.4	140	44.0	56	32.7	196	40.1
2. No central concept	103	22.6	73	30.0	176	25.2	103	32.4	72	42.1	175	35.8
3. Magnitude	41	9.0	13	5.3	54	7.7	18	5.7	8	4.7	26	5.4
4. Relation	37	8.2	21	8.6	58	8.3	19	6.0	12	7.0	31	6.4
5. Function	28	6.2	12	4.9	40	5.7	14	4.4	8	4.7	22	34.5
6. Similarity	10	2.2	3	1.2	13	1.9	6	1.9	3	1.8	9	1.8
7. Equation	7	1.5	9	3.7	16	2.3	6	1.9	5	2.8	11	2.2
8. Quantity	5	0.9	2	0.8	7	0.9	5	1.6	2	1.2	7	1.4
9. Sets	3	0.7	1	0.4	4	0.6	2	0.5	0	-	2	0.4
10. Congruency	0	-	5	2.1	5	0.7	0	-	3	1.8	3	0.6
11. Other	7	1.5	2	1.0	9	1.3	5	1.6	2	1.2	7	1.4
TOTAL	456	100.0	243	100.0	699	100.0	318	100.0	171	100.0	489	100.0
Replied to question	456	64.4	243	24.7	699	41.3	318	44.9	171	17.4	489	28.9
No reply given	253	35.6	740	75.3	992	58.7	390	55.1	812	82.6	1202	71.1
GRAND TOTAL	708	100.0	983	100.0	1691	100.0	708	100.0	983	100.0	1691	100.0

Table 5.32 reflects the number of teachers who consider that the given concept should be the central concept in the teaching of Trigonometry. A distinction is drawn between the teachers in the various departments.

Table 5.33 reflects how the teachers interpret the official syllabi of their department.

5.7.9 Discussion of the syllabi in terms of the mathematical concepts

The response of the teachers to the relevant questions in the questionnaire shows that the mathematical concept is not taking its rightful place.

Algebra

The following has been written in regard to the Algebra syllabus of the Transvaal Department of Education (Translation):

> It is clear that the principal consideration in the drafting of this syllabus was not the development of functional thinking, but the liquidation of problems with the least exertion on the part of all concerned. When we consider that as long ago as 1893 in Chicago Prof. Felix Klein drew the attention of teachers in a cogent way to the possibility and desirability of functional thinking being developed, then there can be nothing but astonishment at the ultra-conservatism of the drafters of the syllabus of the Transvaal Department of Education. (107, p. 103-4)

This astonishment can be extended to all the departments of education and becomes even greater when one reflects upon the ignorance of the teachers in regard to the aims of the syllabi and the indescribable confusion which exists regarding the mathematical concepts.

> (Translation):
>
> The time has arrived when a vigorous reform should be effected in our teaching of Algebra. A search must be made for new directions in which the intellectual structure of the pupils will be carefully developed so that they will be able to an ever-increasing extent to determine relations and to distinguish between the variable and the invariable. (107, p. 104)

Geometry:

The largest group of teachers is convinced that there is no central concept in the Geometry syllabi, and a very considerable proportion of the teachers are satisfied with this state of affairs.

> (Translation):
>
> This reveals a very serious shortcoming in the teaching of the subject. In the secondary school altogether too little attention is given to the basic mathematical concepts such as area, perimeter, length, angle and the various kinds of planes. The result is that the pupils have to build up a structure of knowledge and insight on a very broad foundation. It is for that very reason that the initial classes in Geometry are of the utmost importance. (107, p. 104)

It is after all not the syllabus on paper which is the decisive factor but the idea. Once the importance of the function concept is realised and the teaching of the subject is inspired with the thought of developing functional thinking, the shortcomings which may exist in the present syllabi will not be able to impede sound teaching. Geometry in the higher classes will then also acquire a new significance; simple problem liquidation will have to be superseded by the purposeful development of the function concept. In this way the locus will occupy the place which it deserves because the position taken by a particular geometrical point is seen as the variable while certain relations

TABLE 5.32

WHAT THE CENTRAL CONCEPT IN THE TEACHING OF TRIGONOMETRY SHOULD BE

Department	Teachers who accord a central position to the given concept													
	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Concept	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. Ratio	101	47.6	20	35.1	30	47.6	113	42.6	5	62.5	42	50.6	311	45.2
2. No central concept	50	23.6	18	31.6	11	17.5	75	28.3	1	12.5	20	24.1	175	25.4
3. Magnitude	15	7.1	7	12.3	6	9.5	20	7.5	0	-	6	7.2	54	7.8
4. Relation	13	6.1	5	8.8	5	7.9	24	9.1	1	12.5	8	9.7	56	8.2
5. Function	11	5.2	4	7.0	6	9.5	11	4.2	1	12.5	6	7.2	39	5.7
6. Similarity	3	1.4	0	-	1	1.6	9	3.4	0	-	0	-	13	1.9
7. Equation	8	3.8	2	3.5	1	1.6	4	1.5	0	-	1	1.2	16	2.3
8. Quantity	2	0.9	1	1.7	1	1.6	2	0.8	0	-	0	-	6	0.9
9. Sets	3	1.4	0	-	1	1.6	0	-	0	-	0	-	4	0.6
10. Congruency	1	0.5	0	-	0	-	4	1.5	0	-	0	-	5	0.7
11. Other	5	2.4	0	-	0	1.6	3	1.1	0	-	0	-	9	1.3
TOTAL	212	100.0	57	100.0	63	100.0	265	100.0	8	100.0	83	100.0	688	100.0
Replied to question	212	41.2	57	36.8	63	38.7	265	41.7	8	42.1	83	46.6	688	41.3
No reply given	302	58.8	98	63.2	100	61.3	371	58.3	11	57.9	95	53.4	977	58.7
GRAND TOTAL	514	100.0	155	100.0	163	100.0	636	100.0	19	100.0	178	100.0	1665	100.0

TABLE 5.33

THE CENTRAL CONCEPT OF TRIGONOMETRY ACCORDING TO THE TEACHERS' INTERPRETATION OF THE SYLLABI OF THEIR DEPARTMENT

Department	Teachers who recognise a given concept as the central concept													
	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
Concept	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1. Ratio	61	44.9	7	17.0	21	47.7	72	38.1	5	62.5	25	39.6	191	39.7
2. No central concept	51	37.5	19	46.3	10	22.7	69	36.5	1	12.5	24	38.1	174	36.2
3. Magnitude	6	4.4	5	12.2	4	9.1	9	4.8	0	-	2	3.2	26	5.4
4. Relation	3	2.2	3	7.3	3	6.8	16	8.5	1	12.5	4	6.3	30	6.2
5. Function	5	3.7	4	9.8	3	6.8	6	3.2	1	12.5	3	4.8	22	4.6
6. Similarity	2	1.5	0	-	1	2.3	5	2 6	0	-	1	1.6	9	1.9
7. Equation	4	2.9	2	4.9	1	2.3	3	1.6	0	-	1	1.6	11	2.3
8. Quantity	1	0.7	0	-	0	-	4	2.1	0	-	1	1.6	6	1.2
9. Sets	0	-	0	-	1	2.3	0	-	0	-	1	1.6	2	0.4
10. Congruency	1	0.7	0	-	0	-	1	0.5	0	-	1	1.6	3	0.6
11. Other	2	1.5	1	2.5	0	-	4	2.1	0	-	0	-	7	1.5
TOTAL	136	100.0	41	100.0	44	100.0	189	100.0	8	100.0	63	100.0	481	100.0
Replied to question	136	26.5	41	26.5	44	27.0	189	29.7	8	42.1	63	35.4	481	28.9
No reply given	378	73.5	114	73.5	119	73.0	447	70.3	11	57.9	115	64.6	1184	71.1
GRAND TOTAL	514	100.0	155	100.0	163	100.0	636	100.0	19	100.0	178	100.0	1665	100.0

remain the same. (107, p. 105)

In the foregoing quotations the function concept has been stressed. The object, however, was not to enthrone the function concept in particular but to emphasise the importance of a central concept. Whether the function concept or the set concept should occupy the central place is immaterial here. What is of paramount importance is that the mathematical concept should receive attention and that the teachers should be very well acquainted with the position which the concepts should occupy in the syllabus.

Trigonometry

The teachers who are favourably disposed towards the idea of such a central concept are relatively unanimous in their view that the ratio concept should strike the keynote in Trigonometry. An experimental investigation showed however that (in the school concerned) the insight of pupils into the basic concepts underlying Trigonometry was so poorly developed that a matriculation pupil could not be distinguished from a Std. 7 pupil.

(Translation):

This finding focusses attention on the importance of Trigonometry being included in principle even in the early stages when the basic concepts are taught to the pupil in the lower classes. If the introduction of the subject is postponed too long, it is later assumed as a matter of course that pupils already have the necessary insight into the ratios whereas this is really not the case. Trigonometry then easily degenerates into nothing more than a looking up of tables, while the development of functional thinking has not benefitted in the least from the course.

When the right-angled triangle is dealt with, the pupil should be given the opportunity to discover that the doubling of one of the acute angles does not result in the doubling of the opposite side. Once he has fully acquired this basic insight, he will not so readily assert in the higher classes that tan 2A = 2 tan A. In addition he can learn that what is not true of the opposite side is indeed true of the circle segment and in this manner gain a functional insight into the nature of an angle. (107, p. 106)

Arithmetic

The largest group of teachers (44.3%) is of the opinion that the official syllabi stress no central concept in the teaching of Arithmetic. More than a quarter (26.8%) accord this position of honour to the ratio concept although only 17.7% of the teachers declare that they also see the matter in this light in the syllabi.

According to the syllabi the teaching of Arithmetic in Stds. 6, 7 and 8 is not greatly concerned with concepts and concept formation.

(Translation):

Stress is not laid on the acquisition of insight at all, let alone functional thinking, but on certain methods of calculation which the pupils may perhaps need as adults at some later stage in their lives.

The problem is that in this rapidly developing world many of the calculations which the pupil may find necessary as an adult, are unpredictable.

By giving the necessary attention to concept formation, it can be ensured that the pupil will not only be in command of the necessary methods of solution for particular problems under particular circumstances, but also learn to continue adapting himself to the rapidly changing world. (107, p. 107)

5.7.10 The pupil and the mathematical concepts

P.S. Jones discusses the mathematical concept, as seen on the part of the pupil, by means of a number of axioms and propositions:

> "Axiom 1. The best learning is that in which the learned facts, concepts, and processes are meaningful to and understood by the learner.
>
> "Axiom 2. Understanding and meaningfulness are rarely if ever 'all or none' insights in either the sense of being achieved instantaneously or in the sense of embracing the whole of a concept and its implications at any one time." (50, p. 1)
>
> "The sudden perception or flash of insight which is one of the joys of mathematics learning and teaching comes only to those who have, with thought, struggled to extend or apply concepts which have been partially understood earlier.
>
> "THEOREM 1. Teachers must plan so that pupils continually have recurring but varied contacts with the fundamental ideas and processes of mathematics." (50, p. 2)
>
> "THEOREM 2. Teachers in all grades should view their tasks in the light of the idea that the understanding of mathematics is a continuum, that understandings grow within children throughout their school career." (50, p. 3)
>
> "Teachers should look to the future and teach some concepts and understandings even if complete mastery cannot be expected. Teachers should do this in order to build readiness for new topics." (50, p. 4)
>
> "Careful consideration for the vertical organization and continuity of our mathematics program not only in its content but even in its presentation, especially in its emphasis on the understanding of basic ideas, will not only tend to create readiness for new learnings, but will tend to eliminate possible sources of later interference." (50, p. 4)

5.8 THE STANDARDS IN WHICH CERTAIN SUBJECTS SHOULD BE TACKLED

5.8.1 The drafting of syllabi

An important factor which should be taken into account in compiling syllabi for the different standards is the maturity of the pupils to master certain concepts. The opinions of teachers was obtained in connection with two algebraic subjects, namely positive and negative numbers and graphs; two aspects of Geometry, namely Practical Geometry and Geometry as a logical structure; and finally in regard to the standard in which Trigonometry should be introduced.

The teachers' views according to the data obtained become clear in Table 5.34.

5.8.2 Conclusion

Algebra

The majority of teachers (57.2%) are in favour of positive and negative numbers being introduced as early as Std. 6, but then only numerical numbers. They therefore prefer that

$$(+3) + (+4) = +7$$
$$(+3) - (+4) = -1$$
$$(+3) - (-4) = +7 \text{ etc.}$$

should be treated in Std. 6.

TABLE 5.34

STANDARDS IN WHICH THE GIVEN SUBJECTS SHOULD BE TAUGHT FOR THE FIRST TIME

Standard	Teachers who indicated the given standard											
	5		6		7		8		9		Total	
	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
(a) Positive and negative numbers (numerals only as opposed to natural numbers)	139	11.9	666	57.2	330	28.4	25	2.1	4	0.4	1164	100.0
(b) Positive and negative numbers (letters and numerals)	29	2.5	504	43.9	556	48.4	59	5.2	0	-	1148	100.0
(c) Graphs	73	6.6	394	35.8	331	30.1	302	27.5	0	-	1100	100.0
(d) Practical Geometry	164	15.4	675	63.3	224	21.0	4	0.3	0	-	1067	100.0
(e) Geometry as a logical structure	0	-	109	10.5	613	59.1	231	22.3	85	8.1	1038	100.0
(f) Trigonometry	5	0.5	37	3.7	201	20.2	744	74.5	11	1.1	998	100.0

In Std. 7 a start can then be made with

$(+3a) + (+4a) = +7a$
$(+3y) - (+4y) = -y$
$(+3z) - (-4z) = +7z$ etc.

Although the majority of the teachers (48.4%) are in favour of directed numbers (positive and negative numbers) with letters and figures being introduced for the first time in Std. 7, a strong minority (43.9%) consider that the pupils should become acquainted with such numbers as early as Std. 6. A very small minority (6.6%) of the teachers would like to see graphs being tackled even in Std. 5. The percentages in respect of the teachers who are in favour of the treatment of graphs for the first time in Std. 6, Std. 7 and Std. 8 are 35.8%, 30.1% and 27.5% respectively. The point of view in regard to the stage at which graphs should be introduced is therefore very uncertain. It is apparently bound up with the doubt whether Graphs should be regarded as an integral part of Algebra or as a section to be treated separately.

Geometry

The majority of the teachers (63.3%) are in favour of a start being made with practical Geometry in Std. 6. A smaller majority (59.1%) would like to start with Geometry as a logical structure only in Std. 7. It is noteworthy that the number of teachers (22.3%) in favour of starting in Std. 8 with Geometry as a logical structure exceeds those who think (10.5%) that it should be introduced as early as Std. 6.

Trigonometry

Approximately three quarters (74.5%) of the teachers are in favour of making a start with Trigonometry only in Std. 8.

5.8.3 Discussion

When a syllabus is drawn up, there will often be a difference of opinion in regard to what subject matter should be offered in certain standards for the first time. A distinction must, however, be drawn between the subject matter and the basic concepts. The basic concepts must precede in a very subtle manner the subject matter based on them.

If it is borne in mind that the American "grade six" indicates the sixth school year and that it therefore corresponds to the South African Std. 4, note may with advantage be taken of the following statements of Brumfiel and others:

> "Current thinking concerning a modernized mathematics curriculum has made it abundantly clear that basic mathematical concepts must be introduced as early as possible.
>
> "Regardless of the level of student attainment, in the future more attention must be given to mathematical ideas than has been the case in the past.
>
> "The natural place to begin such a program in the junior high school, for many good students have an excellent understanding of rational arithmetic by the end of grade six.
>
> "It is our experience that even very young students, both strong and weak, are attracted to and fascinated by relations between numbers. There is a strong natural tendency to be interested in abstract relationships for the sake of the patterns that present themselves." (16, p. v)

Not much agreement exists among teachers in regard to the most suitable standards in which certain parts of the work can be tackled.

5.9 DIFFERENTIATION

5.9.1 The pupils who should take Mathematics

G. H. A. Steyn writes that the question "What pupils should take the subject?" is best answered with reference to the conference of inspectors mentioned earlier in this report, namely -

> "It is absolutely essential that every pupil who is in any way capable of doing so, should choose Mathematics as a subject." (91, p. 233)

On the other hand it is obvious that not all pupils can learn the subject at the same rate and that some mature at an earlier stage than others in regard to the acquisition of certain insights. In addition account must be taken of the fact that pupils take Mathematics at school with different aims in mind.

5.9.2 The reasons why pupils take Mathematics

I. Adler deals with this subject in an article which appeared in "The Mathematics Teacher" in 1963.

> "I shall confine my remarks to two questions about the movement for curriculum revision: (1) What are the goals towards which the changes in the curriculum are directed? (2) For which pupils is the revised curriculum appropriate?
>
> "I find that considerable stress is given to three aims in particular:
>
> 1. Training mathematicians .....
> 2. Training members of the professions that use mathematics in their daily work .....
> 3. The training of mathematics teachers .....
>
> "But there are also other nonvocational aims of mathematics instruction that are of equal importance. I shall talk about two of these:
>
> (1) the development of the kind of literacy that is needed by a citizen in a complex, industrial society in the era of automation and nuclear energy, and
>
> (2) cultivation of the intellect and liberation of the mind." (2, p. 505)
>
> "If the purpose of the new curriculum is exclusively the development of professional competence, it is suitable only for those young people who are preparing to enter the professions.
>
> However, we have seen that there are other purposes, not of a vocational character, that the new curriculum serves as well. These are the purposes of developing mathematical literacy, and cultivating and liberating the mind. These purposes are relevant to all pupils, not merely the gifted or the college-bound. Consequently the new courses of study should not be restricted to the college-bound, but should be extended to all pupils." (2, p. 508)

5.9.3 The possibility of a Mathematics curriculum for all pupils

Adler therefore desires that the proposed reform of curricula should benefit all pupils and not only those who proceed to a university later on. He expects that some teachers will not see eye to eye with him.

> "Some teachers will react to this suggestion by saying, 'You are asking for the impossible .....'
>
> "During World War II, among the first to land on a newly captured island in the Pacific were the Seabees (the Construction Battalions). A magazine

article describing their work, said they were guided by the slogan: 'The difficult we do immediately. The impossible takes a little longer.'

"If a desirable goal is impossible under one set of rules, don't throw up your hands in despair. Just change the rules." (2, p. 509)

5.9.4 Differentiation according to I.Q.

The question is apparently not what the pupils are able to learn, but when they can learn it. Table 5.34 shows clearly that this is where the rub lies. The safest course is apparently to accept that all pupils (except those who reveal definite deviations) can master the mathematical concepts, the only difference being that one will do so sooner than another.

Adler utters a very serious warning against the practice of withholding subject matter from certain pupils merely by reason of their I.Q.

"Those who reply on the I.Q. theory assume that most children are not capable of learning much. A natural consequence is that they do not try to teach them much. They offer these children watered-down courses devoid of significant mathematical content. They take pupil failure for granted, and so neither teacher nor child is required to exert the effort that might prevent failure. The major result is cumulative retardation from year to year. The children become less and less prepared to cope with the work of the higher grades. The process of educational decay is selfreinforcing, because it is controlled by a typical feedback loop: We think the children are not capable of learning, so we teach them less. Then they learn less and become less capable of learning. So again we teach them less, and so on ad infinitum. Teachers and pupils become trapped in a vicious circle.

"We can convert the impossible into the possible by changing the rules of our game. Let us cut the feedback loop by throwing overboard the I.Q. theory and the defeatist attitude that it engenders. Replace the I.Q. theory by a new, yet really odd, hypothesis, that all children except those with serious brain damage are capable of learning and thinking. They are able to master abstractions and generalizations if they are properly taught. So let us not hold back from any children the benefits of the new courses of study. Extend the new program to all classes on all levels." (2, p. 509-510)

5.9.5 Differentiation in South Africa

Differentiation must be approached and carried into effect with very great care. Serious consideration must be given to the question of introducing only one basic Mathematics syllabus for all the pupils in all the provinces.

The courses must then be arranged in such a manner that the pupils of different ability and aptitude will learn new concepts at the stage when they have reached the right level of maturity.

5.10 ARE THE MATHEMATICS SYLLABI MODERN ENOUGH?

5.10.1 Questions to the teachers

In order to ascertain whether the teachers regard the syllabi for Mathematics and Arithmetic as modern, certain questions were put to all teachers of Mathematics and Arithmetic. Those teachers who take mainly Arithmetic were asked to answer the questions in respect of the Arithmetic syllabus. Table 5.35 shows how the teachers reacted to the relevant questions.

TABLE 5.35

VIEWS OF THE TEACHERS IN RESPECT OF THE MODERNITY OF THE SYLLABI

Assertions	Teachers who -								TOTAL
	Confirmed the assertions		Denied the assertions		Replied to the assertions		Did not reply to the assertions		
	Number	%	Number	%	Number	%	Number		
(a) Due account is taken of the increased application of Mathematics in scientific and technical fields	300	34.0	583	66.0	883	51.4	835	48.6	1718
(b) The latest advance of Mathematics itself is adequately reflected in the syllabus	252	27.6	660	72.4	912	53.1	806	46.9	1718
(c) The basic mathematical concepts receive adequate attention in the syllabus	736	64.3	409	35.7	1145	66.7	573	33.3	1718
(d) The syllabus contains material which can be utilized to present a mathematical structure	697	59.5	474	40.5	1171	68.1	547	31.9	1718

5.10.2 Conclusion

The first two questions were answered by approximately half of the teachers and the third and fourth questions by roughly two-thirds. It is a truly grave state of affairs that only about half of the teachers charged with the task of teaching Mathematics and Arithmetic at high schools consider themselves competent to express an opinion on the question whether the Mathematics offered is modern enough.

Of the teachers who submitted a reply, two-thirds (66.0%) considered that the increased application of Mathematics in scientific and technical fields is not taken into proper account in the syllabus.

Almost three-quarters of the teachers (72.4%) are convinced that the latest developments in the mathematical field are not adequately reflected in the curriculum. This is an astonishing pronouncement which shows how right the time is for a radical reform in our teaching of Mathematics.

Approximately two thirds (64.3%) of the teachers (who replied) are under the impression that the basic mathematical concepts receive adequate attention in the syllabi. The attention given to the basic concepts does not in the first place depend upon the syllabus but upon the teachers who are responsible for the subject. The basic concepts are realities which exist irrespective of whether or not they are explicitly mentioned in the syllabus The competent teacher who wishes to teach his subject in such a manner that the thinking of his pupils will be shaped by his tuition will in any case give the necessary attention to the basic concepts.

Three-fifths (59.5%) of the teachers consider that the curriculum includes the necessary material to present a mathematical structure. In this case too it is really the teacher's approach which will be of decisive importance.

5.10.3 Discussion

It is astonishing that the teachers should think that the basic mathematical concepts receive adequate attention. When they are asked what, for example, the central concept is, the majority find themselves unable to give an answer, while it is clear that many of those who did reply, merely guessed. A large number of teachers showed by their replies that they do not even know what a mathematical concept is. In regard to the aims of the syllabi, many teachers are completely in the dark.

5.11 THE IMPROVEMENT OF A CURRICULUM

> "The curriculum is now generally considered to be all the experiences that learners have under the auspices or direction of the school. Curriculum improvement refers not only to improving the structure and the documents of the curriculum but also to stimulating growth, learning, and alteration of perceptions and values on the part of all the persons who are concerned by the curriculum." (31, p. 18)

During the whole course of the history of education changes in syllabi have taken place. Schools with new subjects and new aims were established, and new subjects were taken up in the curricula of existing schools.

The old idea that education is intended only for the children of selected groups and that the children of the rest of the community should merely be trained to perform manual labour, had gradually to make way for the new idea that every child is potentially capable of going a long way in life and that each one should be given the opportunity to develop to the highest level of his ability in one field of study or another. The number of such fields has increased considerably if the present syllabi are compared with those of 200 years, 100 years or even a mere 50 years ago.

The drafters of syllabi require the assistance of psychologists in order to indicate to them where and when they go beyond the limits of the pupils concerned. One thing which should be taken into thorough account is that sustained purposefulness plays an important role. This is revealed very clearly in the study of foreign languages in particular. If a start is made with such a study, perseverance and tenacity of purpose are essential in order to be able to use the languages well.

The attainment of the envisaged goal is considerably expedited when persons put their heads together and share their experiences with one another. Such a course helps to maintain equilibrium. The class teacher must also continually plan in order to reach his goal and he must modify his methods so that they will be adapted to every group of pupils or, where the classes are not too large, to each individual. He must bear in mind that: "Education produces learning not essentially by what a teacher says, thinks or does, but by what a pupil can be encouraged to say, think, do and feel." (31, p. 27)

The importance of this is more readily realised when due account is taken of the fact that "every learner brings with him individualised conditions of learning. These include his personality, his personal experiences, his mental ability and the effects of the social order. The nature of the learner himself and of his background has a marked effect on what he learns and on his style of learning." (31, p. 37)

If it then happens that he displays an interest in the curriculum, he will be a willing learner, but the opposite is also true if the syllabus goes completely beyond his field of interest.

It is essential that the teacher should through careful observation and investigation arrive at a measure of certainty in regard to what each child in his class requires most in order to make good progress and he should then concentrate on assisting the child in the required direction or directions.

Tradition plays a dual role, namely as a means of preventing tested successful methods from being summarily thrown overboard, and also as a brake upon desirable changes. Persons charged with the drafting of syllabi have to keep educational traditions in mind, but they must also not hesitate to eliminate completely what has become outmoded in course of time.

Such persons must moreover study the wishes of the local community. Most members of the community are laymen in the field of education and yet their wishes in regard to lines of study at their schools must be respected. The subject matter must, however, be left in the hands of experienced teachers and those who draw up curricula.

> "The curriculum leader should lead the planning for movement, and should cause the plans to be known and talked about." (31, p. 64)

New ideas can flow from such a discussion and these will help to adapt the curriculum to the community in so far as such adaptation is also desirable for the pupils.

> "Social forces have always had a strong effect in the making of curriculum decisions. Some of these forces have originated, and continue to originate, in the wider society and culture in which man lives. Still other forces develop within communities. Curriculum personnel must reckon with social forces without resenting them or their multiple origins, but the educator has a special responsibility to relate them to elementary and secondary education in ways which will benefit children and youth, who are the precious clientele of the schools." (31, p. 69)

The object of schools is to teach the right subject matter to children and in this way to enable them to play in the community. It is consequently this subject matter which poses a problem to the drafter of syllabi, namely what should be provided for each age group so that the group will be able to understand it. And in the case of the teacher it would be a good thing for him to remember that what is taught is often not so important as the way in which it is taught. The pupil who studies in the correct way makes better progress than one who sets about matters aimlessly.

Two things play an important role in the choice of subject matter for a syllabus. These are, firstly, the learning process which should take due account of the ability of the pupil and, secondly, the demands made upon the school by society and the local community. A third factor has now been added and that is the tenor and practical use of the subject material itself.

It has become clear in recent years that for the purposes of study only such work as is important should be selected and that things which are unimportant should as far as possible be eliminated. The approach must moreover also undergo a drastic change in many respects, especially because the field of human knowledge has expanded tremendously during the past ten to twenty years. According to Robert Oppenheimer a doubling of knowledge occurs every eight to twelve years. Now it has become an art to select what should be included in a syllabus and what should not. This is indeed a very great problem with which those responsible for drawing up a curriculum has to contend. It is fatal for teachers merely to continue presenting subjects such as Physics in the same old mould because developments in this field have been so gigantic that the pupils concerned will be hopelessly behind the times, and if they find themselves in a modern laboratory later on, they will fell like fish on dry land.

Since 1959 extra trouble has been taken in America to have curriculum makers draw up syllabi in co-operation with experienced teachers in order to retain what was useful in the past and to supplement this with what is important and within the reach of the pupils from the store of new knowledge, which has become a necessity for modern man. Provisional curricula are drawn up and tested in a number of schools. After the reports of teachers in regard to the operation of new syllabi have been received, the latter are modified to make them more effective. In due course a final syllabus is then drawn up, but in

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view of the rapid development taking place at present, this can of course never be absolutely final.

In every field of knowledge such as, for example, Chemistry, Physics, History, etc., there are certain fundamental facts of which thorough account must be taken. After that the curriculum maker can make his additions to suit the groups concerned but he must ensure that what is added is directly or indirectly linked with fundamental matters.

> "Fundamental to a resolution of forces in decision-making about the curriculum are the curriculum makers' beliefs about nature, education and destiny of man. In the center of the trilogy nature-education-destiny is education, which expresses what can be done with and for man. What can be done with and for him, is affected by his nature, and what can be done with and for him in turn affects his destiny. Persons who want to improve the quality of education, for themselves and for other people must face an initial question. 'What are the sources of the ideas which will determine the kind of education to be provided?' The answers to this question will imply much about the curriculum improver's view of the nature of man. The answers will also suggest what the curriculum improver thinks man should do in the future. Most of all, however, they will bespeak points of view about the substance and process of education." (31, p. 142)

When a change in a syllabus is contemplated, clarity must first be obtained in regard to what the curriculum wishes to achieve. The latter must be tackled in a strictly scientific manner. Reliable data must be compiled in regard to the desirability of changes.

The school was established by and stands within community. Consequently account must be taken of what the community advocates in respect of the subject matter for the children. It can, however, only indicate broad directions. The final form of the syllabus is a matter for specialists.

The curriculum makers must also give due consideration to eternal truths and the divine will and revelation.

But even these four factors cannot yet ensure completeness. In short, what this amounts to is that:

> "A philosophy of curriculum sources is, then, a function of the interaction of people as they consider ideas." (31, p. 181)

In order to draw up the syllabus in the proper way, due account must be taken of the pupil, the learning process, the social influences and the nature or purport of the subject matter. The following are a few of the aims which must be attained:

> The pupils must be able to fill a useful place in society; they must possess skills in various fields; their interests must cover a wide field; they must be able to think critically or, to put the matter more briefly, they must be able to think; and they must be able to appreciate things and circumstances. Syllabi must therefore be drafted with great wisdom.

The drafter of syllabi must be convinced of the field to be covered in the primary and the high school; he must take careful note of the order of sequence in which the subject matter is presented; there must be the necessary continuity in his syllabi over the years covered so that new knowledge can be built on what has already been acquired, and there must be a definite balance in a complete curriculum for the pupil. The <u>whole</u> individual must be developed.

In the planning of changes the following must serve as criterion:

> "Not too fast, not to slow, not too carelessly planned, not too big, not too insignificant, not too recently considered. This is obviously a rule easier

to state than to live by, but it is extremely relevant to the process of improvement." (31, p. 142)

To this must be added that:

"Curriculum improvement refers not only to improving the structure and the documents of the curriculum but also to stimulating growth, learning and alteration of perceptions and values of the part of all the persons who are concerned with the curriculum." (31, p. 181)

All this requires a new vision on the part of principal, curriculum-maker, teacher and pupil.

Outsiders who have an interest in new syllabi, are business undertakings and individuals with a profit motive in mind. Textbooks are written, programmed material is produced, tests are compiled, etc. These are then commercialised. Unfortunately much of this material may often be fraught with greater disadvantages than advantages for education.

Education departments, groups of laymen who are intensely interested in the education of the child, school administrations, inspectors, teachers and pupils are those who act within the framework of education. The actual compilation and improvement of syllabi falls primarily within the sphere of these bodies and individuals. The pupils are included because the quality of the prescribed work is closely bound up with their capabilities. Furthermore, no matter how large or small, how simple or complicated the educational machinery of an environment may be, one can never get away from the fact that all this revolves round the child and is aimed at preparing it to take his place in the community.

"The six major strategies which are commonly used in American school systems today are:

1. Improving the curriculum through master planning for the entire school system;

2. Improving the curriculum at its heart, i.e. in the classroom and the individual school;

3. Improving the curriculum through in-service education of professional personnel;

4. Improving the curriculum through supervising the work of these personnel;

5. Improving the curriculum by re-organising pupil experiences by re-organizing the school itself;

6. Improving the curriculum through the use of evaluation, research and experimentation." (31, p. 256)

To a greater or lesser extent each of these six strategies plays a role in the planning of improved education. The one or two which enjoy priority at any point of time are dependent upon the circumstances prevailing in a school or school environment at such a stage. It is the duty of those who design the syllabus to act judiciously in order to obtain the best results. As soon as the insight of teachers has broadened and their experience has increased, the fruits of such insight and experience will also be enjoyed by pupils in the classroom.

The teacher must also constantly be in a position to evaluate the progress which his pupils as a group are making in the subject concerned. If this can be extended further to an evaluation of the progress of each individual, not only in the relevant subject but also in his personality and character, the value of the work will be very greatly enhanced. For his own purposes the teacher must set schematic criteria on which he intends to base his judgement so that this can remain as objective and as fair as possible. Without evaluation it is difficult to determine whether any progress has actually taken place and, if so, to what extent. In regard to the revision of Mathematics syllabi, attention should be given to the following:

> "There has been a considerable amount of activity in the field of mathematics education in the past ten years. Much of this activity, particularly in the area of the mathematics curriculum, had been long overdue." (44, p. 211)

At present surprising activity is taking place in this field throughout the world.

> "G. Baley Price has referred to this rapid change in the field of mathematics as 'The Revolution in Mathematics'. Professor Price hinted at what this revolution in mathematics involved when he said, 'The twentieth century has been the golden age of mathematics, since more mathematics, and more profound mathematics, has been created in this period than during all the rest of history'." (44, p. 212)

How should such a reform be tackled:

> "W. Eugene Ferguson has outlined eight points in the implementation of a new mathematics program in the secondary school in the pamphlet, The Revolution in School Mathematics.
>
> 1. Recognition by school authorities of the need for a new mathematics program.
> 2. Adequate preparation of teachers in the mathematics now being taught for the first time in secondary schools.
> 3. Selection of a new program.
> 4. Selection of students for the program.
> 5. Informing parents about the new program.
> 6. Informing other members of the school system about the new program and its implications for the mathematics program K-12.
> 7. Continuation of teacher preparation for carrying the new program to higher and lower grades.
> 8. Provision for adequate time and compensation to carry on the new program year after year." (44, p. 212)

Problems are being experienced in the compilation of new syllabi.

> "The German report points out a problem common to many nations - that the amount of time allocated to mathematics in the curriculum is severely limited. Therefore, the introduction of modern mathematics cannot be thought of as the addition of new topics to an existing curriculum. Rather, one must find topics within the traditional curriculum which, although they may have been worthwhile, are not from a modern point of view indispensable. Modern ideas are introduced by the replacement of such topics with selected ideas from modern mathematics. On the other hand, one often has an opportunity to supplement these topics for the better students in an 'Arbeisgemeinschaft', where students voluntarily go deeper into the subject matter. Apparently such informal courses play an important role in the education of mathematics students in Germany. Not only does Germany propose a new curriculum for high school mathematics, but their report shows evidence of deep thinking on individual topics in this curriculum." (52, p. 195)

"In conclusion, I would like to reiterate a sentiment contained in the German report, namely that it takes at least a generation to complete a major change in the mathematics curriculum. At the rate mathematics is developing, by the time the present reform is completed, we are sure to want a reform of the 'modern curriculum'." (52, p. 197)

5.12 SUMMARY

5.12.1 Introduction

At the beginning of this chapter a number of aims were given for an investigation in connection with the syllabi for mathematical subjects. Direct answers must now be given to the questions which were posed.

5.12.2 Are the syllabi adapted to the latest developments in the specialist field?

The answer is "No". The most recent development in this field is the stressing of basic concepts. The concept of sets in particular has gained prominence in the latest literature and textbooks. Such concepts as ratio, variable, one-to-one correspondence and function should also be given much more prominence.

This adaptation can be effected, provided the prospective and present teachers receive the correct training and retraining beforehand.

5.12.3 Are the syllabi overloaded with obsolete subject matter?

This is indeed the case in respect of some syllabi. The Arithmetic syllabus is so overloaded with commercial subject matter that it provides liitle scope for concept formation. Too much information is given and too little opportunity afforded for justice to be done to such concepts as the number (whole, natural, rational, real), set and ratio.

In the Algebra syllabi the equation plays an exaggerated role and will have to make way for the set and function concepts.

Logarithms and the solution of triangles in Trigonometry can also be regarded as outdated material, in so far as the work in connection with it consists largely of looking up tables. Logarithms and Trigonometry should afford the more gifted pupil in particular greater opportunities for concept through, for example, the treatment of the logarithmic progression and more extended attention to the trigonometrical ratios. Euclidean Geometry is not regarded as obsolete subject matter since it is still one of the best examples of the development of a mathematical structure, its study being of great importance for the development of deductive reasoning in pupils.

5.12.4 Does any uniformity exist in the curricula of the different departments of Education?

University admission

In so far as the Mathematics syllabi for admission to a university are concerned, basic uniformity does indeed exist since the requirements of the "minimum" syllabus of the Joint Matriculation Board must be satisfied.

The most important departure occurs in the Transvaal Department of Education. The university entrance course contains a considerable amount of material which does not occur in the syllabus of the Joint Matriculation Board.

Final examination of the high school

The Transvaal Department of Education and the Natal Department of Education have special courses for pupils who wish to complete Std. 10 but without qualifying for admission to a university. Considerable differences exists between the two syllabi. The Natal syllabus offers the possibility for a course in Mathematical Arithmetic with the development of the number concept of the natural number up to the representation of real numbers by points on a straight line.

This is missing in the Transvaal syllabus. On the other hand, the Transvaal Std. 10 course includes Differential Calculus which is not the case in respect of Mathematics (Ordinary grade) in Natal. In addition there are also a number of other small differences.

Arithmetic

The Arithmetic syllabi of the various bodies have more or less the same scope, but, in so far as the details are concerned, there are considerable differences without its being possible to give reasons for these differences. Each department includes items or omits them according to its own preferences.

Algebra

In Stds. 7 and 8 basically the same instruction is given in the various departments of education. The Transvaal C-stream offers a little less and the Cape Department of Education still less.

Mathematics

In regard to the syllabi for Stds. 7 and 8 differences occur in respect of certain details. Pupils who go from one province to another will experience adjustment problems owing to differences in the syllabus, but these differences are not of radical nature. In one province certain theorems and definitions are taught, and not in another.

Trigonometry

Except in Natal and the Transvaal C-stream a little Trigonometry is offered mainly in Std. 8.

There is therefore only partial uniformity in the syllabi of the different departments of education.

Additional Mathematics

The syllabi of the Natal Department of Education and those of the Joint Matriculation Board correspond only in part, since portions already occur in the Natal syllabus for Advanced Grade Mathematics.

5.12.5 Is it possible to change the syllabus in such a manner that the gulf between the high school and the university can be reduced?

The gulf between the school and the university is not caused primarily by the syllabus but rather by the approach to it. If the same material which occurs in the syllabus, is applied to bring about the right concept formation in the pupils, there will be no question of any gulf between the school and the university.

If the syllabus is changed in such a manner that the mathematical concepts are given prominence and the accompanying training of the pupils leads to concept-forming thinking, the gap between the school and the university will in all probability disappear.

5.12.6 Are the syllabi of the various departments of such a nature that their aim and envisaged educational value can be attained by means of them?

The answer to this question must be "No" because most teachers are not acquainted with the aims and envisaged educational value of the syllabi.

The principal reason for this is the fact that the aims and educational value have been unsatisfactorily formulated. The basic shortcoming is the lack of clarity in regard to the position occupied by the mathematical concept.

CHAPTER 6

THE METHOD OF TEACHING

6.1 INTRODUCTION

This chapter does not offer methods of teaching the mathematical subjects but only discusses the situation as it is revealed in the answers furnished by teachers in reply to the questionnaires.

Use has been made of quotations where study of the literature can throw further light on matters.

Attention is given to the time allocated for the teaching of mathematical subjects, the frequency with which teachers have to repeat a lesson, the number of times a week that homework is given, and the time devoted to revision. The teachers indicate what measures are taken by them to ensure that the pupils work correctly, and also the aids used by them. Finally, the problem of establishing a link between the school and the university is dealt with.

6.2 THE TIME ALLOCATED TO THE TEACHING OF THE MATHEMATICAL SUBJECTS

6.2.1 The number of periods per week allocated to the teaching of General Mathematics

The position in 1962

The number of periods allocated per week to the teaching of General Mathematics are set out in Table 6.1.

The "total" column on the right hand side of the table indicates the number of teachers who furnished data. The "average" column shows the average number of periods allocated per week per class.

TABLE 6.1

THE NUMBER OF PERIODS ALLOCATED PER WEEK PER CLASS TO THE TEACHING OF GENERAL MATHEMATICS

	Standard 6									
Periods Department	1	2	3	4	5	6	7	8	Total: Teachers	Average number of periods
Education, Arts and Science	1	0	1	0	6	11	-	-	19	5.3
Cape of Good Hope	5	7	9	2	37	122	20	4	206	5.5
Natal	0	1	-	-	-	-	-	-	1	2.0
Orange Free State	0	2	4	1	-	-	-	1	8	3.5
Transvaal	3	4	4	4	11	87	84	17	214	6.3
South West Africa	-	-	-	-	8	-	-	-	8	5.0
TOTAL	9	14	18	7	62	220	104	22	456	5.8
Percentage	2.0	3.2	3.9	1.5	13.6	48.2	22.8	4.8		

TABLE 6.1 (continued)

				Standard 7						
Periods Department	1	2	3	4	5	6	7	8	Total: Teachers	Average number of periods
Education, Arts and Science	2	0	1	3	14	2	-	-	22	4.5
Cape of Good Hope	0	3	3	3	29	135	6	-	179	5.7
Natal	-	-	-	-	1	2	-	-	3	5.7
Orange Free State	-	-	-	1	2	-	-	-	3	4.7
Transvaal	-	-	-	4	6	27	16	8	61	6.3
South West Africa	-	-	-	-	-	-	4	2	6	7.3
TOTAL	2	3	4	11	52	166	26	10	274	5.8
Percentage	0.6	1.1	1.5	4.0	19.0	60.6	9.5	3.7		

					Standard 8						
Periods Department	1	2	3	4	5	6	7	8	9	Total: Teachers	Average number of periods
Education, Arts and Science	2	0	0	1	10	-	-	-	-	13	4.3
Cape of Good Hope	2	2	2	1	12	166	14	-	-	199	5.9
Natal	-	-	-	1	3	1	-	-	-	5	5.0
Orange Free State	-	-	-	-	-	2	-	-	-	2	6.0
Transvaal	0	1	0	1	4	8	21	7	2	44	6.7
South West Africa	-	-	-	-	-	7	2	-	-	9	6.2
TOTAL	4	3	2	4	29	184	37	7	2	272	5.9
Percentage	1.5	1.1	0.7	1.5	10.7	67.7	13.6	2.6	0.7		

It is obvious that no uniform policy exists in respect of the number of periods per week for the teaching of General Mathematics. In most cases, however, the number is six periods a week.

6.2.2 The number of periods per week allocated to the teaching of Mathematics

The position in 1962

The numbers of periods allocated per week per class for the teaching of Mathematics are reflected in Table 6.2.

TABLE 6.2

THE NUMBER OF PERIODS ALLOCATED PER WEEK PER CLASS TO THE TEACHING OF MATHEMATICS

Standard 6

Periods / Department	1	2	3	4	5	6	7	Total: Teachers	Average number of periods
Education, Arts and Science	0	1	0	1	6	11	1	20	5.5
Cape of Good Hope	1	6	2	2	10	33	6	60	5.3
Natal	1	21	2	-	-	-	-	24	2.0
Orange Free State	25	21	5	1	-	-	-	52	1.7
Transvaal	1	4	3	2	2	9	2	23	4.1
South West Africa	-	-	-	-	1	-	-	1	5.0
TOTAL	28	53	12	6	19	53	9	180	3.7
Percentage	15.6	29.4	6.7	3.3	10.6	29.4	5.0		

Standard 7

Periods / Department	1	2	3	4	5	6	7	Total: Teachers	Average number of periods
Education, Arts and Science	12	5	1	13	20	4	-	55	3.7
Cape of Good Hope	2	2	2	3	10	51	7	77	5.6
Natal	-	-	-	6	35	8	-	49	5.0
Orange Free State	-	1	-	44	11	-	-	56	4.2
Transvaal	2	2	15	57	76	17	-	169	4.5
South West Africa	-	-	-	-	1	-	-	1	5.0
TOTAL	16	10	18	123	153	80	7	407	4.6
Percentage	3.9	2.5	4.4	30.2	37.6	19.7	1.7		

Standard 8

Periods / Department	1	2	3	4	5	6	7	8	9	Total: Teachers	Average number of periods
Education, Arts and Science	12	7	1	6	39	4	3	1	-	73	4.1
Cape of Good Hope	0	3	0	0	4	42	5	1	-	55	5.8
Natal	1	0	1	1	44	6	-	-	-	53	5.0
Orange Free State	-	-	-	1	53	5	-	-	-	59	5.1
Transvaal	0	2	6	51	98	27	1	1	1	187	4.8
TOTAL	13	12	8	59	238	84	9	3	1	427	4.9
Percentage	3.0	2.8	1.9	13.8	55.7	19.7	2.7	0.7	0.2		

TABLE 6.2 (continued)

Standard 9											
Periods Department	1	2	3	4	5	6	7	8	9	Total: Teachers	Average number of periods
Education, Arts and Science	10	4	5	6	0	16	12	1	-	54	4.5
Cape of Good Hope	2	1	0	1	3	139	36	1	1	184	6.1
Natal	0	1	0	1	1	22	19	2	0	46	6.3
Orange Free State	0	0	2	0	1	16	39	3	3	64	6.7
Transvaal	1	0	1	2	1	53	101	21	1	181	6.7
South West Africa	-	-	-	-	-	5	-	-	-	5	6.0
TOTAL	13	6	8	10	6	251	207	28	5	534	6.3
Percentage	2.4	1.2	1.5	1.8	1.2	47.0	38.7	5.2	1.0		

Standard 10											
Periods Department	1	2	3	4	5	6	7	8	9	Total: Teachers	Average number of periods
Education, Arts and Science	2	4	1	1	4	13	11	3	0	39	5.5
Cape of Good Hope	-	-	-	1	1	132	35	2	0	171	6.2
Natal	0	1	1	0	0	12	25	3	1	43	6.6
Orange Free State	-	-	-	-	-	12	41	7	0	60	6.9
Transvaal	1	0	3	5	0	33	89	25	1	157	6.8
South West Africa	-	-	-	-	-	3	-	-	-	3	6.0
TOTAL	3	5	5	7	5	205	201	40	2	473	6.4
Percentage	0.6	1.1	1.1	1.5	1.1	43.3	42.4	8.4	0.4		

Conclusion

Standard 6: In almost half of the schools in the Orange Free State only one period a week is allocated to the teaching of Mathematics in Std. 6. In Natal the figure is almost without exception two periods a week, while in the other departments of education six periods a week are devoted to the teaching of Mathematics in a fairly large number of cases.

Standard 7: In most schools four or five periods a week are set aside for the teaching of Mathematics.

Standard 8: In most schools five periods a week are allocated to the teaching of Mathematics. In the Cape Province six periods a week occur more generally.

Standard 9: The number of periods a week ranges mainly from six to seven, while in the Transvaal a ninth of the schools have eight periods a week for the teaching of Mathematics.

Standard 10: Six or seven periods a week are encountered fairly generally, while a twelfth of the schools devote eight periods a week to Mathematics.

6.2.3 The number of periods allocated per week to the teaching of Arithmetic

The number of periods allocated per week per class are shown in Table 6.3. The number of teachers who make use of the given number of periods per week per class is also reflected.

TABLE 6.3

THE NUMBER OF PERIODS ALLOCATED PER WEEK PER CLASS TO THE TEACHING OF ARITHMETIC

Standard 6

Number of periods Department	1	2	3	4	5	6	7	8	Total: Teachers	Average number of periods
Education, Arts and Science	0	2	0	8	6	5	1	0	22	4.7
Cape of Good Hope	1	3	9	7	7	9	1	0	37	4.3
Natal	0	0	0	1	3	16	4	1	25	6.0
Orange Free State	0	3	31	17	4	0	2	0	57	3.5
Transvaal	0	1	5	4	9	7	2	1	29	4.9
South West Africa	0	0	4	0	1	-	-	-	5	3.4
TOTAL	1	9	49	37	30	37	10	2	175	4.4
Percentage	0.6	5.1	28.0	21.2	17.1	21.2	5.7	1.1		

Standard 7

Number of periods Department	1	2	3	4	5	6	7	8	9	Total: Teachers	Average number of periods
Education, Arts and Science	0	0	0	2	4	-	-	-	-	6	4.7
Cape of Good Hope	5	31	43	3	1	3	-	-	-	86	2.7
Natal	-	-	-	9	37	2	1	-	-	49	4.9
Orange Free State	-	-	4	41	9	3	0	1	-	58	4.3
Transvaal	1	4	129	34	7	0	0	1	1	177	3.3
TOTAL	6	35	176	89	58	8	1	2	1	376	3.5
Percentage	1.6	9.3	46.8	23.7	15.4	2.1	0.3	0.5	0.3		

Standard 8

Number of periods Department	1	2	3	4	5	6	Total: Teachers	Average number of periods
Education, Arts and Science	0	1	0	7	7	1	16	4.4
Cape of Good Hope	5	14	23	-	-	-	42	2.4
Natal	0	1	0	17	24	6	48	4.7
Orange Free State	0	0	32	14	5	2	53	3.6
Transvaal	1	11	114	30	7	-	163	3.2
TOTAL	6	27	169	68	43	9	322	3.4
Percentage	1.9	8.4	52.5	21.1	13.4	2.8		

It is difficult to deduce a policy in respect of the teaching of Arithmetic from the above data.

Standard 6: There are about just as many schools which allocate seven periods a week for Arithmetic in Std. 6 as there are schools which have to be satisfied with two periods. Approximately 85% of the schools have from three to six periods per week for this subject.

Standard 7: Each province has a different policy. In the Cape Province it is two or three periods a week, in Natal mainly five periods a week, in the Orange Free State mostly four periods a week, and in the Transvaal three.

Standard 8: Three periods a week occur in most cases. Natal and the Department of Education, Arts and Science, however, give preference to four or five periods.

6.2.4 Discussion

Some principals of high schools and teachers of Mathematics would like to have more time for the teaching of this subject. The tables show that there is a fairly large variation from school to school in the amount of time made available for the mathematical subjects. It is obvious why some teachers complain that they are handicapped because of a lack of time. Five periods a week for Stds. 7 and 8 and seven periods for Stds. 9 and 10 are regarded by some as insufficient for the average pupil.

It is noteworthy that those persons who desire more time, wish to use the extra time for tutorial periods in order to enable pupils to manipulate the main processes more effectively.[1)]

Not much has as yet been published on the time which should be allocated to the teaching of Mathematics. In a study on the mathematical relation concept A. J. van Rooy came to the conclusion that five periods a week should be devoted to the teaching of Arithmetic.

> (Translation) Five 35-minute periods a week give 116 hours a year. (107, p. 51) This is really the minimum time which should be devoted to Arithmetic. It agrees with the following American view:
>
> "Hartung estimated that the total time available for arithmetic instruction in the middle grades was roughly 100 to 125 hours per year. He also suggested that to teach meaningfully the arithmetic course now assigned to the middle grades would require more time than is now allotted." (98, p. 77)

He then continues as follows:

> "There seems to be no ready answer to questions about arithmetic time allotment or to the manner in which time should be distributed. Likewise, there are no data regarding the relationship of class size to achievement in arithmetic, nor is it known whether a few, some, or all children could learn meaningfully the arithmetic assigned to the middle grades if given the opportunity. As research findings dealing with objectives and methods of teaching accumulate, such questions will be answered, but classroom teachers generally have very little part in determining how soon definite answers will be available." (98, p. 77)

(Translation) There is a further reason for the backwardness of the teaching of Mathematics at high schools, namely the slow RATE at which pupils perform the work. School inspectors have found that arithmetical and mathematical processes in the primary school and in the high school are done at least fifty per cent too slowly. They observe that pupils make from two to four calculations during a particular lesson hour; and the teacher is highly satisfied with this.

It has frequently been demonstrated in our schools that it is possible to have from 12 to 15 such calculations completed per lesson hour. In Std. 3 for example a multiplication such as 273x189 should take TWO MINUTES; i.e. Std. 3 pupils should finish FIFTEEN such multiplications during a working period of 30 minutes.

So, too, in Std. 7 the simplication of an expression such as $2x(3x^2-5x+2) - 3x^2(3-2x) - 5(2x^3-4x^2-x-6)$ should not take more than THREE

1) Memorandum from a school principal.

MINUTES; i.e. during a working period of 30 minutes Std. 7 pupils should be able to do at least TEN such simplifications. But is this ever the case?

Pupils are made accustomed to the slow rate of working from substandard A, and when they eventually reach the higher classes, this is still the case.[1)]

Serious attention must still be given to the whole question of the amount of time allocated to the mathematical subjects.

6.3 THE NUMBER OF CLASSES TAUGHT IN THE SAME STANDARD BY THE SAME TEACHER OF MATHEMATICS

6.3.1 The new phenomenon

The establishment of large high schools has been accompanied by the new phenomenon that, for example, from six to eight or even more Std. 6 classes are found in the same school. It may consequently happen that a teacher has to repeat the same Mathematics lesson over and over again if he is responsible for more than one class in the same standard. Unless the teacher is ingenious enough to think of some expedient, the matter can eventually become very monotonous.

Table 6.4 indicates how many teachers are expected to take one or more classes in a particular standard. The data have been classified on a provincial basis.

TABLE 6.4

THE NUMBER OF TEACHERS WHO HAVE TO TEACH MATHEMATICS TO THE GIVEN NUMBER OF CLASSES IN THE SAME STANDARD

Standard 6

Province	Number of classes							Total
	1	2	3	4	5	6	7	
Cape Province	40	11	3	4	-	-	-	58
Natal	14	6	2	2	-	-	-	24
Orange Free State	23	18	7	3	1	1	1	54
Transvaal	6	3	5	2	2	1	-	19
South West Africa	-	1	-	-	-	-	-	1
TOTAL	83	39	17	11	3	2	1	156
One or two Std. 6 classes	122 teachers							
Three or more Std. 6 classes	34 teachers							
TOTAL	156 teachers							

1) Letter from an Inspector of Education (Cape).

TABLE 6.4 (continued)

Standard 7								
Province	Number of classes							Total
	1	2	3	4	5	6	7	
Cape Province	58	13	4	-	-	1	1	77
Natal	27	16	6	1	-	-	-	50
Orange Free State	40	10	1	4	1	-	-	56
Transvaal	64	50	24	20	5	3	1	167
South West Africa	-	1	-	-	-	-	-	1
TOTAL	189	90	35	25	6	4	2	351
One or two Std. 7 classes	279 teachers							
Three or more Std. 7 classes	72 teachers							
TOTAL	351							

Standard 8								
Province	Number of classes							Total
	1	2	3	4	5	6	7	
Cape Province	46	9	-	-	-	-	-	55
Natal	34	14	5	-	1	-	-	54
Orange Free State	47	6	3	2	-	-	-	58
Transvaal	80	53	35	9	4	5	2	188
TOTAL	207	82	43	11	5	5	2	355
One or two Std. 8 classes	289 teachers							
Three or more Std. 8 classes	66 teachers							
TOTAL	355							

Standard 9								
Province	Number of classes							Total
	1	2	3	4	5	6	7	
Cape Province	153	24	6	1	-	-	-	184
Natal	28	14	4	-	-	-	1	47
Orange Free State	55	7	1	1	-	-	-	64
Transvaal	85	60	28	9	3	2	-	187
South West Africa	3	2	-	-	-	-	-	5
TOTAL	324	107	39	11	3	2	1	487
One or two Std. 9 classes	431 teachers							
Three or more Std. 9 classes	56 teachers							
TOTAL	487 teachers							

TABLE 6.4 (continued)

Standard 10								
Province	Number of classes							Total
	1	2	3	4	5	6	7	
Cape Province	152	19	3	1	-	-	-	175
Natal	30	11	-	1	-	-	1	43
Orange Free State	54	6	-	1	-	-	-	61
Transvaal	63	63	24	8	2	1	1	162
South West Africa	1	2	-	-	-	-	-	3
TOTAL	300	101	27	11	2	1	2	444
One or two Std. 10 classes	401 teachers							
Three or more Std. 10 classes	43 teachers							
TOTAL	444 teachers							

6.3.2 Conclusion

If we proceed from the assumption that a teacher can give the same lesson twice without finding it monotonous, then the following percentages of teachers, shown in Table 6.5, have no reason for complaint in this respect.

TABLE 6.5

PERCENTAGE OF TEACHERS WHO TAKE ONE OR TWO MATHEMATICS CLASSES IN THE SAME STANDARD

Standard	Percentage
Six	78
Seven	79
Eight	81
Nine	89
Ten	90

Since it is not necessary for a relatively high percentage of the teachers to repeat the same lesson more than once, it is clear that school Principals who do expect their teachers to do so, are out of step with the majority. Cases where multiple repetition is required should be eliminated.

6.4 HOMEWORK

6.4.1 The number of times per week homework is given

Table 6.6 reflects the number of teachers who give Arithmetic homework the given number of times to pupils in the different standards.

TABLE 6.6

THE NUMBER OF TIMES PER WEEK ARITHMETIC HOMEWORK IS GIVEN TO PUPILS

Number of times a week homework is given	Teachers							
	Standard 6		Standard 7		Standard 8		Total	
	Number	%	Number	%	Number	%	Number	%
0	2	0.4	3	0.5	3	0.6	8	0.5
1	19	3.6	48	8.5	48	9.8	115	7.3
2	83	15.9	153	27.2	140	28.5	376	23.9
3	138	26.5	218	38.8	186	37.8	542	34.4
4	150	28.8	95	16.9	59	12.0	304	19.3
5	129	24.8	45	8.1	56	11.3	230	14.6
TOTAL	521	100.0	562	100.0	492	100.0	1575	100.0

Table 6.7 shows the corresponding information in respect of homework in Mathematics.

TABLE 6.7

THE NUMBER OF TIMES PER WEEK MATHEMATICS HOMEWORK IS GIVEN TO PUPILS

Number of times a week homework is given	Teachers											
	Standard 6		Standard 7		Standard 8		Standard 9		Standard 10		Total	
	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
0	10	1.6	6	0.8	4	0.7	4	0.7	5	0.9	29	0.8
1	69	11.3	17	2.4	16	2.7	14	2.3	6	1.1	122	4.0
2	147	24.3	109	15.4	86	14.4	21	3.5	11	2.0	374	12.2
3	122	20.0	167	23.6	142	23.8	62	10.2	52	9.5	545	17.8
4	94	15.4	186	26.2	156	26.2	128	21.0	106	19.4	670	21.8
5	167	27.4	224	31.6	191	32.0	371	61.0	356	65.2	1309	42.7
6	0	-	0	-	1	0.2	2	0.3	2	0.4	5	0.2
7 and more	0	-	0	-	0	-	6	1.0	8	1.5	14	0.5
TOTAL	609	100.0	709	100.0	596	100.0	608	100.0	546	100.0	3068	100.0

6.4.2 The time devoted by teachers to the correction of homework

Arithmetic

Table 6.8 shows how many teachers spent, on an average, the given number of hours weekly at home on the correction of Arithmetic homework.

TABLE 6.8

THE AVERAGE NUMBER OF HOURS SPENT WEEKLY ON THE CORRECTION OF ARITHMETIC HOMEWORK AT HOME

Number of hours per week	Teachers													
	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1	63	30.7	6	6.3	14	15.7	35	11.7	4	40.0	8	17.4	130	17.4
2	77	37.6	25	25.9	30	33.3	85	28.3	3	30.0	13	28.2	233	31.3
3	29	14.1	19	19.8	16	17.8	41	13.7	2	20.0	9	19.6	116	15.5
4	14	6.8	12	12.5	8	8.9	35	11.7	0	-	7	15.2	76	10.2
5	12	5.9	13	13.5	12	13.3	39	13.0	1	10.0	5	10.9	82	11.0
6	4	2.0	9	9.4	3	3.3	19	6.3	0	-	1	2.2	36	4.8
7	3	1.5	4	4.2	0	-	3	1.0	0	-	0	-	10	1.3
8	0	-	2	2.1	6	6.7	14	4.7	0	-	2	4.3	24	3.2
9	0	-	4	4.2	0	-	3	1.0	0	-	0	-	7	0.9
10	2	0.9	2	2.1	1	1.0	11	3.7	0	-	0	-	16	2.1
12	0	-	0	-	0	-	5	1.7	0	-	0	-	5	0.7
14	0	-	0	-	0	-	0	-	0	-	1	2.2	1	0.1
15	0	-	0	-	0	-	4	1.3	0	-	0	-	4	0.5
18	0	-	0	-	0	-	3	1.0	0	-	0	-	3	0.4
20	0	-	0	-	0	-	2	0.7	0	-	0	-	2	0.3
21	1	0.5	0	-	0	-	1	0.3	0	-	0	-	2	0.3
TOTAL	205	100.0	96	100.0	90	100.0	300	100.0	10	100.0	46	100.0	747	100.0
Average number of hours per week	2.5		4.0		3.2		4.3		2.3		3.3		3.5	

The time spent at home on the correction of homework is $3\frac{1}{2}$ hours per week. This can be regarded as quite reasonable. On the other hand, only about half of the teachers devote from 2 to 3 hours per week on the correction of Arithmetic homework. The times range from one hour to twenty-one hours a week. Even if it is accepted that all the teachers do not have the same number of Arithmetic classes, it is clear that the teachers' duties in connection with the correction of homework are construed in various ways.

Mathematics

Table 6.9 reflects the average number of hours spent per week by teachers on the correction of Mathematics homework at home.

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TABLE 6.9

AVERAGE NUMBER OF HOURS PER WEEK SPENT BY TEACHERS ON THE CORRECTION OF MATHEMATICS HOMEWORK AT HOME

Number of hours per week	Teachers													
	Cape Province		Natal		Orange Free State		Transvaal		South West Africa		Education, Arts and Science		Total	
	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
1	57	16.6	2	2.2	11	10.8	26	6.5	4	28.6	12	13.0	112	10.8
2	104	30.4	22	24.2	23	22.5	65	16.3	2	14.3	22	23.9	238	22.9
3	62	18.1	7	7.7	14	13.7	47	11.8	4	28.6	12	13.0	146	14.0
4	32	9.3	22	24.2	15	14.7	29	7.3	3	21.4	17	18.5	118	11.3
5	45	13.1	13	14.2	13	12.7	49	12.3	0	-	11	12.0	131	12.6
6	12	3.5	5	5.5	7	6.9	32	8.0	0	-	7	7.6	63	6.1
7	7	2.0	2	2.2	4	3.9	11	2.8	0	-	1	1.1	25	2.4
8	8	2.3	3	3.3	1	1.0	29	7.3	0	-	5	5.4	46	4.4
9	1	0.3	6	6.6	0	-	7	1.8	0	-	0	-	14	1.3
10	6	1.7	7	7.7	10	9.8	43	10.8	1	7.1	1	1.1	68	6.5
11	0	-	0	-	0	-	2	0.5	0	-	0	-	2	0.2
12	1	0.3	2	2.2	1	1.0	13	3.3	0	-	1	1.1	18	1.7
13	0	-	0	-	0	-	0	-	0	-	1	1.1	1	0.1
14	2	0.6	0	-	0	-	5	1.3	0	-	1	1.1	8	0.8
15	4	1.2	0	-	2	2.0	18	4.5	0	-	0	-	24	2.3
16	0	-	0	-	0	-	5	1.2	0	-	0	-	5	0.5
18	1	0.3	0	-	0	-	3	0.7	0	-	0	-	4	0.4
20	0	-	0	-	1	1.0	13	3.3	0	-	1	1.1	15	1.4
22	1	0.3	0	-	0	-	0	-	0	-	0	-	1	0.1
24	0	-	0	-	0	-	1	0.2	0	-	0	-	1	0.1
25	0	-	0	-	0	-	1	0.2	0	-	0	-	1	0.1
TOTAL	343	100.0	91	100.0	102	100.0	399	100.0	14	100.0	92	100.0	1041	100.0
Average number of hours per week	3.5		4.8		4.5		6.6		3.0		4.1		4.9	

6.4.3 Conclusion

Arithmetic

Approximately a third of the teachers (34.4%) give Arithmetic homework three times a week, a third (33.9%) more than three times a week and slightly less than a third (31.7%) fewer than three times a week.

The tendency exists to give Arithmetic homework more often in Std. 6. More than half (53.6%) of the teachers give homework four or five times a week to the pupils in Std. 6. In the Transvaal and Natal more time is apparently spent on correction work than is the case elsewhere.

Mathematics

The pupils in Stds. 9 and 10 are given homework almost twice as often as the pupils in the lower standards. In Std. 9 Mathematics homework is given five or more times a week by 62.3% of the teachers, as compared with 27.4%, 31.6% and 32.2% in Stds. 6, 7 and 8 respectively. Roughly two thirds of the teachers (67.1%) give homework five or more times a week in Std. 10.

A sixth (17.1%) of the teachers give Mathematics homework fewer than three times a week. This occurs especially in the lower classes. Thirty-six per cent of the teachers, for example, give homework fewer than three times a week to the pupils in Std. 6.

The teachers of the Transvaal Department of Education apparently have to spend a considerable amount of time on the correction of Mathematics homework. There are also exceptions, but probably these will be teachers who also teach other subjects.

6.4.4 The possible extension of the school day

The teachers were asked whether the school day should be lengthened so that the work which the pupils would otherwise have to do at home could be done at school under the supervision of their teachers. The response of the teachers to this idea is reflected in Table 6.10.

TABLE 6.10

THE POSSIBLE LENGTHENING OF THE SCHOOL DAY

		Number	Percentage
(a)	Teachers in favour of lengthening the school day so that "homework" can be done under supervision at school	245	14.2
(b)	Teachers who were against this idea	1288	75.0
(c)	Teachers who did not reply	185	10.8
	TOTAL	1718	100.0

It is clear that three quarters (75.0%) of the teachers who submitted completed questionnaires were opposed to the idea of the school day being lengthened so that work which the pupils would otherwise have to do at home could be done under supervision at school.

6.5 MEASURES TAKEN BY THE TEACHERS TO ENSURE THAT THE PUPILS WORK CORRECTLY

6.5.1 Measures

Table 6.11 lists the measures taken by most of the teachers. The table also shows the number of teachers who indicated the relevant method as the most important. It goes without saying that the teachers will make use of more than one of these methods.

TABLE 6.11

MEASURES TAKEN TO ENSURE THAT THE PUPILS WORK CORRECTLY

Measures	Teachers					
	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
1. The work is corrected regularly	420	56.9	516	53.2	936	54.8
2. The pupils are subjected to tests	49	6.6	45	4.6	94	5.5
3. Corrections are strictly checked	39	5.3	48	5.0	87	5.1
4. Drillwork	32	4.3	37	3.8	69	4.0
5. Individual attention	33	4.5	36	3.7	69	4.0
6. Explanation why the work should be done correctly	19	2.6	12	1.2	31	1.8
7. Pupils made to do the work themselves on the blackboard in front of the class	12	1.6	10	1.0	22	1.3
8. Model answers worked out for the class	3	0.4	10	1.0	13	0.8
9. Impositions	5	0.7	7	0.7	12	0.7
TOTAL	612	82.9	721	74.4	1333	78.1
No replies furnished or other methods mentioned	126	17.1	248	25.6	374	22.9
GRAND TOTAL	738	100.0	969	100.0	1707	100.0

6.5.2 Conclusion

It is clear that there is really only one method of ensuring that pupils work correctly and that is the regular marking of their work. All corrections must be strictly checked and, when necessary, individual attention should be given to the pupils.

It is necessary, moreover, that the pupils should be tested regularly and that drillwork at this stage and for this purpose should be recommended by many teachers. Proper motivation also plays a role.

The working out of model answers for pupils so that they can write them down neatly in their books is suggested as an important procedure by a small minority of the teachers.

For the effective teaching of Mathematics the teachers should be able to give a considerable amount of individual attention to each pupil and to check his written work carefully. For that reason the classes should be small and, where necessary, more teachers should be made available.[1)]

6.6 AIDS

6.6.1 Aids used

The aids most commonly used by the 1718 teachers are set out in Table 6.12. The percentages are furnished in order to indicate the ratio between the number of qualified and unqualified Mathematics teachers who make use of such aids and do not show the percentages in respect of the total number of qualified and unqualified Mathematics teachers.

[1)] Memorandum of a school principal

TABLE 6.12

THE AIDS MOST COMMONLY USED

Aids		Teachers				
		Qualified in Mathematics		Unqualified in Mathematics		Total
		Number	%	Number	%	
1.	Blackboard and blackboard instruments	316	42	442	58	758
2.	Mathematical models and notice board for interesting problems	219	56	171	44	390
3.	Notes, worked out examples of problem solutions	144	46	167	54	311
4.	Instruments for practical Mathematics	139	50	140	50	279
5.	Films on mathematical subjects	58	59	40	41	98

It would appear that relatively more of the qualified teachers of Mathematics make use of mathematical models and films than is the case with unqualified teachers.

6.6.2 Criteria used

The teachers were asked whether they make use of objective measures by means of which the mathematical and arithmetical ability of pupils can be determined. The replies of 1707 teachers were analysed. (Originally 1718 of the returned questionnaires were analysed. It subsequently appeared that the questionnaires of eleven teachers had to be left out of account for various reasons. In those chapters where the replies given to questionnaires are used for statistical purposes, they did not have any effect worth mentioning on the results.)

Table 6.13 reflects the analysis.

TABLE 6.13

THE USE OF OBJECTIVE MEASURES

Teachers	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
who make use of objective measures	178	24	233	24	411	24
who do not make use of objective measures	448	61	442	46	890	52
who gave no reply	112	15	294	30	406	24
TOTAL	738	100	969	100	1707	100

Approximately a quarter (24%) of the teachers make use of objective measures by means of which the mathematical and arithmetical ability of pupils can be determined.

6.6.3 The need for scientifically compiled tests

The replies of the teachers as to whether they experience a need for scientifically compiled tests which will expose the mathematical shortcomings of pupils in various fields, are analysed in Table 6.14.

TABLE 6.14

THE NEED FOR SCIENTIFICALLY COMPILED TESTS

	Field	Qualified in Mathematics		Unqualified in Mathematics		Total	
		Number	%	Number	%	Number	%
(a)	Basic knowledge						
	Teachers who require tests	463	63	582	60	1045	61
	Teachers who do not require tests	202	27	146	15	348	20
	Teachers who did not reply	73	10	241	25	314	19
	TOTAL	738	100	969	100	1707	100
(b)	Concept formation						
	Teachers who require tests	475	64	550	57	1025	60
	Teachers who do not require tests	179	24	128	13	307	18
	Teachers who did not reply	84	12	291	30	375	22
	TOTAL	738	100	969	100	1707	100
(c)	Ability						
	Teachers who desire scientific tests	462	63	565	58	1027	60
	Teachers who do not desire scientific tests	197	27	136	14	333	20
	Teachers who did not reply	79	10	268	28	347	20
	TOTAL	738	100	969	100	1707	100
(d)	Attitude towards subject						
	Teachers who require tests	434	59	509	53	943	55
	Teachers who do not require tests	221	30	167	17	388	23
	Teachers who did not reply	83	11	293	30	376	22
	TOTAL	738	100	969	100	1707	100

Of the unqualified teachers only about 15% declare that they feel a need for scientifically compiled tests. Apparently there is a smaller percentage of unqualified Mathematics teachers who feel the necessity for the tests in comparison with those who are qualified in Mathematics. This must be ascribed to the fact, however, that from 25% to 30% of the unqualified teachers did not reply to the relevant questions.

More than 60% of the teachers feel a need for objective measures but according to Table 6. 13 only 24% of the teachers make use of such aids. There is therefore still a very great need which must be met.

6.6.4 The Mathematics classroom

At present just any room in the school is assigned to the Mathematics teacher. The question is, if greater justice is to be done to Mathematics, whether a special room together with a workroom should also not be built for this subject and in any case at least for the Mathematics teachers of Stds. 8, 9 and 10. The apparatus received by the teacher should also consist of more than just a pair of compasses, a protractor, 60^{o} and 45^{o} setsquares, and a T-square.

The room should also be equipped in such a manner that films on mathematical projects can be shown. A considerable number of these are obtainable from the Division of Audio-Visual Education of the Department of Education, Arts and Science.

A slide rule should also be available for teaching pupils how to make calculations with it.

It is furthermore necessary that there should be a bookshelf for other books besides textbooks so that pupils can learn how to consult other books and especially books which will give them an insight into the history of the development of Mathematics. They should also be afforded the opportunity through the medium of books to come into close touch with the importance of Mathematics in many spheres of life. (9)

6.7 THE DRILL METHOD

6.7.1 The teachers

In the questionnaire the teachers were asked whether they often employ the drill method in the teaching of Mathematics and whether they consider that the type of question which generally occurs in the final school examination encourages the use of the drill method.

The opinions of the teachers are reflected in Table 6.15.

TABLE 6.15

THE DRILL METHOD IN THE TEACHING OF MATHEMATICS

	The use of the drill method	Teachers who						Total
		answered in the affirmative		answered in the negative		gave a reply	gave no reply	
		Number	%	Number	%	Number	Number	
(a)	The drill method is frequently used	859	76.8	259	23.2	1118	600	1718
(b)	The type of question in the final school examination encourages the drill method	703	100.0	0	-	703	1015	1718

6.7.2 Conclusion

Approximately three quarters (76.8%) of the teachers who replied to the question often made use of the drill method.

Those who replied to the following question, were unanimous in their opinion that the type of question which generally occurs in the final school examination encourages the use of the drill method in the teaching of Mathematics.

The university lecturers were in full agreement with the teachers. (Translation) The preparation of candidates for an external examination paper unfortunately only too often has the result that the teacher has to coach his pupils by drillwork in order for them to be able to answer types of questions.[1)]

6.8 THE LINK BETWEEN THE SCHOOL AND THE UNIVERSITY

6.8.1 The gap between Mathematics in the high school and that in the first year at a university

In Questionnaire N.B.377 a number of questions were put to the teachers in order to obtain their opinions in connection with the extent to which the work in the high school links up with that at a university. The replies of the qualified and the unqualified teachers of Mathematics were not analysed separately. It can

[1)] Memorandum of a university lecturer.

be accepted that it is for the most part the unqualified teachers of Mathematics who failed to give replies since these Std. 10 classes are preferably entrusted to the qualified Mathematics teachers.

The first question was whether the gap between Mathematics in the high school and that at the beginning of the first year at a university is too great.

The replies of 1718 teachers of the mathematical subjects are analysed in Table 6.16. Three teachers did not complete this part of the questionnaire and their returned questionnaires were excluded from the previous total of 1721.

TABLE 6.16

THE GAP BETWEEN HIGH SCHOOL MATHEMATICS AND THAT AT THE BEGINNING OF THE FIRST YEAR AT A UNIVERSITY

Opinion	Teachers	
	Number	%
The gap between the university and high school is too great	758	44.1
The gap is not too great	203	11.8
Cannot answer the question	467	27.2
Did not reply to the question	290	16.9
TOTAL	1718	100.0

The teachers who consider the gulf too great outnumber by approximately four times those who think that there is not really any gap between high school Mathematics and that of the first-year at university (44.1% as against 11.8%). The fact that 44.1% of the teachers did not reply to the question must be ascribed to the possibility that they do not teach Mathematics in Std. 10 and/or did not take the first year course in Mathematics at university.

6.8.2 Reasons why the gap is too great

Table 6.17 sets out the reasons given by some teachers why the gap between the teaching of Mathematics at school and the first year at university is too great. In addition an indication is given of the number of teachers who are of this opinion.

TABLE 6.17

THE REASONS WHY THE GAP BETWEEN THE TEACHING OF MATHEMATICS AT SCHOOL AND AT UNIVERSITY IS TOO GREAT

Possible reasons	Teachers who replied in the affirmative		Teachers who replied in the negative		Teachers who gave a reply	Teachers who did not give any reply	Total
	Number	%	Number	%	Number	Number	
(a) The standard of high school Mathematics is too low	342	45.6	408	54.4	750	968	1718
(b) University Mathematics starts off at too rapid a pace	669	84.7	121	15.3	790	928	1718
(c) University Mathematics starts off at too high a standard	188	32.6	388	67.4	576	1142	1718
(d) Variations in the use of mathematical language	163	28.7	404	71.3	566	1152	1718
(e) Too many variations in respect of subject matter in Mathematics	251	44.9	308	55.1	559	1159	1718
(f) There is too much difference in the method of studying	552	82.0	121	18.0	673	1045	1718

6.8.3 Discussion

According to the opinion of the teachers and certain university lecturers in mathematical subjects, the following are the reasons why the link between the teaching of Mathematics at school and at university is not what it should be (reasons as furnished in memoranda sent in by them):

Rate

The problem lies primarily in the universities. A large majority (83.7%) of the teachers consider that new material is offered to young first year students at too rapid a rate at the beginning of the first year. They cannot adjust themselves to this rapid presentation of the material and the course has advanced fairly far before they begin to realise what is going on. Both the teachers and university lecturers are in complete agreement on this important matter. The latter regard the university course as the logical continuation of the high school course. There is a discontinuity however in the rate at which the material is taught when students proceed from the one course to the other. The lecturers therefore recommend the following:

The rate at which new concepts are presented should be accelerated during the school years so that this rate will be almost the same in the second half of the last school year as that in the first university year.

Progressive teachers should be allowed more freedom in the treatment of subject matter which does not occur in the syllabi so that more can be done to develop good pupils to the utmost of their ability.

The teachers are too strongly bound to the existing examination system which thwarts efforts at reform. Some teachers also express the opinion that the universities should think of lengthening their academic year by beginning a little earlier in the year and consequently going a little slower with the first-years in the initial lectures.

Method of teaching

The second problem experienced by first-year students is that the difference between the method of teaching at school and that at the university is too great. This is the opinion of 82.0% of the teachers who replied to this question.

It is common knowledge that the last six months of the high school pupil's Std. 10 year is spent on revision and preparation for the final examination. The assimilation of new concepts is inevitably relegated to the background while the whole learning process is focussed on rounding off the high school work. After a relatively long summer vacation the young students are not prepared to make rapid acquaintance with new concepts.

Many prospective students have to undergo nine months' military service at the end of their high school career. If no precautionary measure in one form or another is taken, much of the schoolwork is forgotten and the academic atmosphere is largely lost.

It is worthy of serious consideration that at least the three months during which the ballotees do not undergo military training in that particular year should be spent on post-school and pre-university instruction.

In addition the pupil is accustomed to the care of a teacher who is eager to show the parents and authorities a collection of favourable symbols and who consequently directs his teaching methods towards that end. When a student subsequently has to become accustomed to the method of teaching of a lecturer who perhaps does not even have a teacher's certificate, and such a student has himself to accept responsibility for the execution of assignments, substantial problems are created for the former Std. 10 pupil.

Some university lecturers are also of the opinion that the differences in teaching methods at the school and the university demand serious attention. They mention a few matters to which attention should be given in the high school. This naturally does not apply to all schools, but they nevertheless observe that disquieting features occur fairly generally.

The impression exists that pupils are taught certain methods in order to obtain particular results. The lecturers recommend that the general and basic principles should first be thoroughly driven home before the rule and the formula are presented. Intuitive concepts and axioms should be clearly outlined, but after that the logical conclusions should receive the necessary attention. This work of the university lecturers is in agreement with the aim of the teachers for the teaching of Mathematics.

Since it is in fact logical reasoning which should be stressed, the boundaries between the various fields of Mathematics should not be strictly maintained. The use of Algebra and Trigonometry should also not be eliminated in geometrical reasoning, provided that the whole is logical.

Great value should be attached to the pupils' own efforts so that their initiative can be encouraged in this way. A pupil should not be penalised unnecessarily if he makes use of his own method, and the so-called standard methods should be avoided as far as possible.

Some university lecturers advocated a more modern approach to the syllabi. It is recommended that a teacher should first discuss sets before he defines a function. The function concept occurs in the syllabus.

The standard of high school Mathematics

The other possible reasons given in explanation of the gap between high school Mathematics and that of the first year student at university are supported by only a minority of the teachers.

The most important of these reasons is that the standard of high school Mathematics is too low. That 45.6% of the teachers were of this opinion, must probably be ascribed to the fact that at the time of this investigation some provinces did not as yet meet the minimum requirements prescribed by the Joint

Matriculation Board. In the meanwhile the position has changed and it can now be accepted that the new syllabi will give many teachers more satisfaction. The university lecturers are not greatly concerned about the prescribed standard as long as the method of teaching cultivates the right habits of thinking in the pupils.

The standard of university Mathematics

Approximately a third (32.6%) of the teachers who replied to this question consider that university Mathematics maintains too high a standard at the beginning. It is clear that the rate of presentation is a much greater obstacle than the standard itself.

The mathematical language

Differences in mathematical language are mentioned as an obstacle by only 28.7% of the teachers. If high school teachers lay the necessary stress on concepts such as function and sets, and if they remain conversant with the latest literature, this stumbling-block will disappear. University lecturers who ensure that they are fully acquainted with the terminology in the high school syllabi will also find the pupils will understand them more readily. These matters are really self-evident.

6.8.4 Preparation of pupils for the university

Not all high school pupils proceed to a university. For that reason it would also not be fair or desirable that the whole high school should be geared to meet the requirements of future university students.

On the other hand it must also be admitted that future students form a very important group and that an investment in their future careers as mathematicians can yield rich dividends.

Possibly the solution lies in co-operation between the school and the university to offer selected pupils something extra, perhaps by means of vacation courses. The following paragraph tells what is being done in the State of New York in the U.S.A. in this connection:

> "During the full semester of 1962-63, Rosary Hill College of Buffalo, N.Y., offered twenty-one entering freshmen the opportunity to enroll in a mathematics seminar. They were all students who had demonstrated above-average ability during four years of high school mathematics. The major purpose of the seminar was to increase this mathematical maturing so that they will be equipped to work independently, and perhaps creatively at an early stage in their college careers." (59, p. 998)

> "As a result of this seminar, the students gained experience in independent research methods. They learned to read and interpret mathematics books and periodicals. They found that becoming discouraged and finding ways to overcome that discouragement, is a normal part of the process of learning mathematics. They also learned a great deal about many topics in mathematics and they learned to share their techniques and skills, as well as their factual knowledge, with each other." (59, p. 999)

Some people may still have misgivings when they see some of the courses offered to high school pupils on the threshold of the university. Doubts are felt particularly about the axiomatic method.

> "Still we find concepts and theorems being introduced without the pupils' participation, and therefore the sources and paths of mathematical creation remain unknown to them and the axiomatic method, the ideal of every science, is not understood."

> "It is not the axiomatic method which is the source of failure in mathematics teaching but its epistemological interpretation. Hilbert's idea, that the empirical source of a mathematical theory should not form a part of that

theory, developed through radical formatism to become completely independent of mathematics. This is the cause of the failure and of all peculiarities of mathematics teaching."

"If an axiomatic system is a free and spontaneous creation of man's mind, mathematics teaching must be carried out dogmatically, a cleavage must exist between axiomatic theories and other sciences, and it is impossible for an understanding of the axiomatic method to be awakened. Efforts at reform, based on psychological principles are bound to fail as long as any psychological interference with mathematics teaching is considered as a 'desecration of the temple of pure mathematics'."

"Thus, in geometry axiomatics is treated in such a way that the pupil fails to understand it; in other fields it is not treated at all, as secondary education lacks the theory of sets and mathematical logic." (97, p. 39)

6.8.5 The cultivation of independent study habits

The good textbook

One of the best ways in which pupils can be taught to study independently is to provide them with Mathematics textbooks in which they will find not only problems to do, but which will really enable them to read Mathematics.

"South African pupils simply do not learn to read the text in a mathematical textbook, which is used as a source of examples and nothing else. Only the inadequate teacher finds himself reading the text. The sad thing is that this has been going on for so long that it is seldom recognized as a problem." (62, p. 31)

McGee discusses a Mathematics textbook which appeared recently in the U.S.A. and the writers of which he regards as a particularly fortunate choice:

"Note first who the authors are. Dolciani is a university mathematician, Berman a high-school mathematics teacher, Freilich a high-school principal and former mathematics teacher, and Meder a dean of a university and sometime director of the College Entrance Examination Board. This healthy co-operation between university and school teachers is sadly lacking in the South African system. These authors have a number of aims, including the encouragement of reasoning and discovery, progression from the concrete to the abstract, motivation by means of historical notes and practical applications, the use of set theory to unify the approach, the use of graded examples to promote achievement, and the presentation of all materials in a book of pleasing format. In these aims they seem to have been very successful." (62, p. 32)

In the U.S.A. in particular this Mathematics textbook has received special attention. Perhaps people in that country were too textbook-conscious and initially devoted too little attention to the training and retraining of the Mathematics teacher.

Various study groups concentrated their attention particularly on the textbook. A very well known group is the School Mathematics Study Group. Of great importance too is the University of Illinois Committee on School Mathematics (UICSM).

"Originally the UICSM staff was inspired by three major theses. They believed that a consistent exposition of high-school mathematics was possible; they believed that high-school students were most interested in ideas; and they believed that acquiring manipulative skill and understanding basic concepts were complementary activities. They were concerned with helping pupils to discover mathematics but were careful not to require the student to verbalize his discovery too soon. Their criticism of traditional mathematical curricula was that they resulted in isolated skills and

> techniques for solving standardized problems and did not help to familiarize students with formal algebraic notions. 'Also we don't beat the drums for mathematics by telling the student the subject will be important to him when he grows up. If we cannot make it important to him while he is studying it, we cannot justify making him study it'." (62, p. 32)

It is important moreover that the definitions and explanations in the textbooks should be very accurate. The use of words must also be checked very carefully.

> "........ precision in exposition is something we expect of the textbook and the teacher, rather than of the learner. Precise communication is a characteristic of a good textbook and a good teacher; correct action is a characteristic of the good learner." (62, p. 33)

A tendency to follow textbooks slavishly exists on the part of some teachers. What makes matters worse is that schools in the different provinces and also within the same province make use of a great variety of textbooks. A pupil who proceeds from one school to another often finds it difficult to adapt himself to new textbooks especially where the textbook is followed too slavishly.

The reasons why textbooks are followed too slavishly, are the following:

(1) Schemes of work are modelled wholly on a particular textbook.

(2) Schemes of work are too concise to be of any use to the teacher.

(3) Lack of interest in his work on the part of the teacher. [1)]

The attitude of the teacher in the classroom

Great value must be attached to independent efforts on the part of pupils. For that reason standard methods of solving problems must be approached by the teacher with the greatest circumspection. A teacher can recommend a good method, but he should never penalize a child if the latter uses his own method with some degree of success. Through incorrect action a teacher can destroy a pupil's initiative.

The impression is nevertheless created that the method of teaching often consists in providing pupils with certain methods in order to obtain certain results - the rule of thumb and formula, instead of the general and basic principles which should be thoroughly understood before the rule and formula can be used intelligently. [2)]

Differentiation

If a better link between the school and university is to be obtained, it is essential that the standard of the Mathematics taught to the best pupils should be raised. To bring this about, pupils should be placed in separate streams in all Mathematics classes according to their mathematical ability. The gifted pupils should follow a more difficult course and should also write a special examination.

Edwin J. Swineford drew up a list of ninety hints on the teaching of Mathematics at a junior high school. Hints 85 to 90 are the following:

"85. Try three levels of work in your class: an enriched program for the accelerated group; a core program for the average or normal group; a minimum program for the slow-moving group.

"86. Determine the level at which each student is functioning in each of the seven different areas of arithmetic.

[1)] Contribution of a committee member.

[2)] Contribution of a committee member.

87. Provide for individual differences, since good teaching separates individuals.

88. Practice student grouping in mathematics.

89. Accept the student on the levels which he can perform.

90. Remember that you teach and that they learn - in their own way and their own rate." (95, p. 148)

CHAPTER 7

THE TEXTBOOKS FOR THE MATHEMATICAL SUBJECTS AT THE HIGH SCHOOL

7.1 TEXTBOOKS FOR GENERAL MATHEMATICS

7.1.1 Textbooks used in the Cape Province

The three textbooks used by most teachers at the time of the survey (June, 1962) are given in Table 7.1

TABLE 7.1

TEXTBOOKS FOR GENERAL MATHEMATICS MOST WIDELY USED IN THE CAPE PROVINCE

Code	Authors	Title	Number of teachers who use it
B	Boshoff & le Roux	Mathematics for Junior Certificate Stds. 6 to 8	102
D	Dreyer	General Mathematics Stds. 6 to 8	70
G	Gonin, Archer and Slabber	Graded Mathematics Stds. 7 and 8	33

These three books are discussed together in Table 7.2. In the columns an indication is given of the percentage of the replies which were in the affirmative. As in the table above, the three books are indicated by the letters B, D and G. The answers are not given at the back of the book in all three cases, and all three are classified in such a manner that Algebra, Geometry and Arithmetic occur in separate sections.

TABLE 7.2

COMPARISON OF THE THREE BOOKS MAINLY USED FOR GENERAL MATHEMATICS IN THE CAPE PROVINCE

Characteristic		B %	D %	G %
1. The contents of the book cover the syllabus of the department		95	99	100
2. The facts, theorems and definitions are accurate		97	99	94
3. The questions are arranged in order of difficulty		91	97	88
4. The language is clear and mathematically correct		99	100	100
5. The development of the fundamental mathematical concepts is sufficiently emphasised		87	86	88
6. The book is more modern in its approach than earlier publications		85	76	73
7. The personal opinions on the book:	Very good	36	13	9
	Good	54	64	64
	Fair	5	17	15
	Poor	-	-	-
	Very poor	-	-	-
	No opinion	5	3	12

According to this table the most modern book has the best chance of enjoying a good sale. It should be borne in mind in this connection, however, that Boshoff and le Roux's book also makes provision for Std. 6 and for this reason should therefore enjoy larger sales than the other two books which only make provision for Stds. 7 and 8.

7.1.2 Textbooks used in the Orange Free State

The three textbooks used by most teachers are given in Table 7.3.

TABLE 7.3

THE THREE MOST POPULAR TEXTBOOKS FOR GENERAL MATHEMATICS IN THE ORANGE FREE STATE

Code	Authors	Title	Number of teachers who use it
M	Marquard & Faure	Mathematics for Stds. 7 and 8	10
H	Hugo, Barnard and De Wet	Mathematics for Stds. 7 and 8	10
R	Hugo, Barnard and De Wet	Arithmetic and Mathematics for Std. 6	6

The three books are analysed in Table 7.4. In this table, percentages are not used.

TABLE 7.4

BOOKS FOR GENERAL MATHEMATICS IN THE ORANGE FREE STATE

Characteristic		M	H	R
1. The contents of the book cover the syllabus of the department		10	10	5
2. The facts, theorems and definitions are accurate		10	10	6
3. The questions are arranged in order of difficulty		8	10	5
4. The language is clear and mathematically correct		10	9	5
5. The development of the fundamental mathematical concepts is sufficiently emphasised		8	7	5
6. The book is more modern in its approach than earlier publications		6	6	3
7. The personal opinions on the book:	Very good	1	1	-
	Good	7	6	5
	Fair	2	2	1
	Poor	-	-	-
	Very poor	-	-	-
	No opinion	-	1	-
Maximum		10	10	6

The contents of all three books are classified in such a manner that there are separate sections for Arithmetic, Mathematics and Algebra. The answers are not given at the back of these books.

Since only a few teachers expressed their opinions on these books, no further discussion of the works is given here.

7.1.3 Textbooks used in the Transvaal

For General Mathematics use is made mainly of the textbooks of one group of authors: Behr, Steyn, Grobler, Lange: General Mathematics for Std. 6: Behr, Steyn, Grobler: General Mathematics for Std. 7.

Two hundred and eighteen teachers expressed their opinion on these books as shown in Table 7.5.

TABLE 7.5

BEHR, LANGE, STEYN AND GROBLER: GENERAL MATHEMATICS

Characteristic		Number	Percentage
1. The contents of the books cover the syllabus of the Department		212	97
2. The facts, theorems and definitions are accurate		189	87
3. The questions are arranged in order of difficulty		190	87
4. The language is clear and mathematically correct		195	89
5. The development of the fundamental mathematical concepts is sufficiently emphasised		155	71
6. The book is more modern in its approach than earlier publications		172	79
7. The personal opinions on the books:	Very good	27	12
	Good	122	56
	Fair	36	17
	Poor	4	2
	Very poor	2	1
	No opinion	27	12

The books are not divided into separate sections for Arithmetic, Algebra and Geometry. The answers are not given at the back of these books, but use is made of separate answer books for teachers.

7.1.4 Natal

Apparently no textbooks for General Mathematics were used in Natal in 1962, the reason apparently being that the subject had not then been generally introduced.

7.2 TEXTBOOKS FOR ALGEBRA

7.2.1 Cape Province

The three textbooks most commonly used in this Province are given in Table 7.6.

TABLE 7.6

THE TEXTBOOKS MOST COMMONLY USED FOR ALGEBRA IN THE CAPE PROVINCE

Code	Authors	Title	Stds.	Number of teachers who use it
G.J.	Gonin, Archer and Slabber	Graded Mathematics for Junior Certificate	7 & 8	44
G.S.	Gonin, Archer and Slabber	Graded Mathematics for Senior Certificate	9 & 10	30
B	Boshoff en le Roux	Mathematics for Senior Certificate	9 & 10	20

The three books are discussed together in Table 7.7. In the columns an indication is given of the percentage of the replies which were in the affirmative.

No answers are given at the back of the books by Gonin, Archer and Slabber, but answers are given in those of Boshoff and le Roux.

TABLE 7.7

THE TEXTBOOKS USED FOR ALGEBRA IN THE CAPE PROVINCE

Characteristic		G.J. %	G.S. %	B %
1. The contents cover the syllabus of the department		91	97	100
2. The book contains separate sections for Algebra and Graphs		Yes	Yes	Yes
Graphs are treated as an inherent part of Algebra		No	No	No
3. The central concept of the book is:	Sets	2	3	15
	Equations	23	23	25
	Functionality	16	7	15
	Variation	-	3	-
	Ratio	-	-	-
	Relation	5	-	-
	Other	5	7	15
	No opinion	49	57	30
4. The book is written in clear language		81	87	75
Grammatically correct		25	37	50
Mathematically correct		32	40	50
Mistakes in the language and in the presentation of mathematical facts occur		7	2	5
5. The book develops a clear mathematical structure		93	83	80
6. The book is more modern in its approach than earlier publications		77	67	35
7. Personal opinions:	Very good	41	27	-
	Good	45	63	65
	Fair	5	3	20
	Poor	-	-	5
	Very poor	-	-	-
	No opinion	9	7	10

Some doubt apparently exists on the part of the readers in regard to what can be regarded as the central concept of the Algebra section of these books.

7.2.2 Natal

The three textbooks most widely used are given in Table 7.8.

TABLE 7.8

THE ALGEBRA TEXTBOOKS MOST COMMONLY USED IN NATAL

Code	Authors	Title	Stds.	Number of teachers who use the textbook
K	De Kock & de Waal	Algebra for Junior Certificate	7 & 8	34
D	Dreyer & Schmidt	Algebra for Senior Certificate	9 & 10	14
H	Hall	School Algebra, Part I and II	9 & 10	6

The three books are discussed in Table 7.9. In the columns an indication is given of the percentages of the replies which were in the affirmative.

TABLE 7.9

THREE ALGEBRA TEXTBOOKS USED IN NATAL

	Characteristic		K %	D %	H %
1.	The contents cover the syllabus of the department		94	71	83
2.	The book contains separate sections for Algebra and Graphs		Yes	Yes 50%	No
	Graphs are treated as an inherent part of Algebra		No	Yes 43%	Yes
3.	The central concept of the book is:	Sets	3	-	17
		Equations	9	36	17
		Functionality	12	21	-
		Variation	3	-	-
		Ratio	-	-	-
		Relation	9	-	-
		Other	6	7	-
		No reply	58	36	66
4.	The book is written in clear language		79	57	50
	Grammatically correct		3	7	17
	Mathematically correct		6	14	17
	Mistakes in the language and in the presentation of mathematical facts occur		6	14	-
5.	The book develops a clear mathematical structure		74	43	67
6.	The book is more modern in its approach than earlier publications		50	14	33
7.	Personal opinion on the book:	Very good	6	14	33
		Good	53	29	50
		Fair	29	43	17
		Poor	3	-	-
		Very poor	-	-	-
		No opinion	9	14	-
8.	Answers given at the back of the book		No	Yes	Yes

Since very few teachers replied to these questionnaires it is dangerous to draw any conclusions. It is clear that the newer textbooks which appear, do not really give more satisfaction than those of Hall which were written as long ago as the nineteenth century.

7.2.3 Orange Free State

The three textbooks most commonly used for the teaching of Algebra are given in Table 7.10.

TABLE 7.10

TEXTBOOKS MOST COMMONLY USED FOR THE TEACHING OF ALGEBRA IN THE ORANGE FREE STATE

Code	Authors	Title	Stds.	Number of teachers who use the textbook
M	Marquard and Faure	Mathematics	7 & 8	14
H.J.	Hugo, Barnard & de Wet	Mathematics	7 & 8	10
H.S.	Hugo, Barnard & de Wet	Mathematics	9 & 10	19

The three books are discussed in Table 7.11. In the columns an indication is given of the percentages of teachers who replied in the affirmative.

TABLE 7.11

THE TEXTBOOKS USED AT HIGH SCHOOLS IN THE ORANGE FREE STATE FOR ALGEBRA

Characteristic		M	H.J.	H.S.
1. The contents cover the syllabus of the department		100	100	100
2. The book contains separate sections for Algebra and Graphs		Yes	Yes	Yes
Graphs are treated as an inherent part of Algebra		No	No	No
3. The answers appear at the back of the book		No	No	Yes
4. The central concept of the book is:	Sets	7	10	11
	Equations	14	20	5
	Functionality	7	10	16
	Variation	-	20	-
	Ratio	7	10	-
	Relation	14	10	-
	Other	-	-	-
	No reply	50	20	63
5. The book is written in clear language		71	80	68
Grammatically correct		-	-	5
Mathematically correct		21	20	16
Mistakes in the language and in the presentation of mathematical facts occur		7	-	5
6. The book develops a clear mathematical structure		86	80	53
7. The book is more modern in its approach than earlier publications		57	40	58
8. Personal opinion:	Very good	14	10	-
	Good	57	70	53
	Fair	-	-	42
	Poor	-	-	5
	Very poor	-	-	-
	No opinion	19	20	-

7.2.4 Transvaal

The three most popular textbooks used for the teaching of Algebra are given in Table 7.12.

TABLE 7.12

THE TEXTBOOKS MOST COMMONLY USED FOR THE TEACHING OF ALGEBRA IN THE TRANSVAAL

Code	Authors	Title	Stds.	Number of teachers using the textbook
H	Hofmeyr, Ogden and Marais	Algebra & Graphs Part II	9 & 10	156
B	Behr, Lange, Steyn and Grobler	General Mathematics	6 & 7	11
C	Channon and Smith	A School Algebra	9 & 10	4

These books are discussed in Table 7.13.

The "book" which is coded B is actually two different books, namely:

A.L. Behr, G.H.A. Steyn, J.S. Grobler and J.W. Lange:

General Mathematics for Standard VI and

A.L. Behr, G.H.A. Steyn and J.S. Grobler:

General Mathematics for Standard VII

The third book in the series, i.e.

G.H.A. Steyn and T.S. Welman:

Mathematics for Standard VIII is not discussed here.

TABLE 7.13

THE ALGEBRA BOOKS USED IN THE TRANSVAAL

Characteristic		H	B	C
1. The contents cover the syllabus of the department		89	82	50
2. The book contains separate sections for Algebra and Graphs		Yes	Yes	No
Graphs are treated as an inherent part of Algebra		No	No	Yes
3. The answers appear at the back of the book		Yes	Yes	No
4. The central concept of the book is:	Sets	2	-	-
	Equations	26	27	50
	Functionality	17	-	-
	Variation	3	9	-
	Ratio	1	-	-
	Relation	2	-	-
	Other	5	-	25
	No reply	44	64	25
5. The book is written in clear language		86	73	100
Grammatically correct		2	-	-
Mathematically correct		8	9	-
Mistakes in the language and in the presentation of mathematical facts occur		2	9	-
No reply given		3	9	-
6. The book develops a clear mathematical structure		81	73	100
7. The book is more modern in its approach than earlier publications		64	73	75
8. Personal opinion:	Very good	19	9	25
	Good	60	45	75
	Fair	13	27	-
	Poor	2	9	-
	Very poor	-	-	-
	No opinion	6	9	-

7.2.5 Condusion

The replies are vague here. Not all of the teachers have a clear idea of what the central concept in these books is, and no unanimity exists in regard to the quality of the books of Hofmeyr and others or of Behr and others; some regard the books as very good and others as poor, while the majority consider that the quality of the books lies somewhere between these two extremes.

7.2.6 Discussion

A. J. Dekker discussed the Algebra textbooks used in schools in the Transvaal. (119) He devotes attention to the following works:

Marais, Ogden and Hofmeyr	:	Algebra, Part I, Stds. 7 and 8
Van Dalsen	:	Mathematics for Std. 7
Steyn and Welman	:	Mathematics for Std. 7
Van Dalsen	:	Mathematics for Std. 8
Steyn and Welman	:	Mathematics for Std. 8
Hofmeyr, Ogden en Marais	:	Algebra and Graphs, Part II, Stds. 9 and 10
Steyn and Welman	:	Algebra and Graphs, Stds. 9 and 10

Since the books of Hofmeyr, Ogden and Marais and also those of Steyn and Welman are very widely used, brief summaries of Dekker's views in connection with these books are given here.

7.2.7 P. J. Marais, R. Ogden and W. H. Hofmeyr: Algebra, Part I

This book appeared for the first time as long ago as 1946. According to Dekker the overwhelming majority of the schools used the book in Stds. 7 and 8 during the period 1958 to 1962.

In the course of this investigation it became clear that the services of many unqualified Mathematics teachers were used to teach Mathematics in the lower secondary classes. For that reason it is gratifying that the authors give a number of examples of mathematical operations at the beginning of each subject. It is clearly stated, however, that the examples should not be followed slavishly but that pupils should constantly be encouraged to show personal initiative.

The book contains sufficient subject matter for Stds. 7 and 8 but there are too few illustrations. Although there are numerous little problems for pupils to work out, there are few which can give the idea of the development of a mathematical structure. Oral exercises, graphs, factors and the solutions of three linear equations with three unknowns should receive more attention.

According to Dekker the preliminary steps are tackled in an incorrect manner. After a few definitions letters are substituted for figures and immediately afterwards directed numbers are tackled. A Std. 7 pupil cannot follow the high degree of abstraction and proceeds to manipulation without having mastered the concepts.

The first one hundred and ten pages require mainly manipulative processes without making it clear to the pupils what their purpose is. Only then are expressions in symbolic form and the solution of problems dealt with.

Opportunities to make use of algebraic processes in a practical manner are neglected. The difference between squares in the form of factors can be written down as follows:

$$72 \times 68 = (70 + 2)(70 - 2) = (70)^2 - (2)^2 = 4900 - 4 = 4896$$

Use is seldom made of geometrical figures in order to explain algebraic relations.

Subject matter which should form an entity, is dealt with separately. So, for example, the solution of a linear equation with two unknowns should be linked with the graph of the straight line and with the solution of simple problems.

According to Dekker's finding, this textbook is no more than fair in so far as its quality is concerned.

7.2.8 <u>W.H. Hofmeyr, R. Ogden and P.J. Marais: Algebra and Graphs Part II, Stds. 9 and 10</u>

Dekker passes a more favourable judgment on this book than on the first part. The work covers the Transvaal syllabus for Stds. 9 and 10 very fully.

Provision is made for the revision of work which has already been dealt with. This revision is effected at a higher level than that at which the subject matter was originally presented. In the exercises attention is given to skill, accurate thinking and accurate work.

A lack felt in Part I also occurs in Part II. Equations with two unknowns, problems, graphs, and the solution of quadratic equations are treated as completely separate subjects without one being related to another.

Following the recent extension of the syllabus with a view to making provision for pupils intending to proceed to a university,the additional subject matter has been added at the back of the book in the form of an appendix. It is strongly recommended that this subject matter should be inserted in the right places in later editions.

In this book justice is not done to the concept of functionality.

There is considerable scope for improvement in the book. A teacher with the necessary training and initiative will know how to use this book to advantage.

7.2.9 <u>G.H.A. Steyn and T.S. Welman: Mathematics for Std. 7 and Mathematics for Std. 8</u>

These two books were first used in 1961 and have rapidly become popular in schools in the Transvaal. Dekker foresees that they will soon take the place of Marais' book and also the publications of other authors in most schools. In 1961 nineteen per cent of the schools were using them and by 1962 the figure had risen to fifty-five per cent.

These books contain a good supply of exercises which can be done orally before the teacher proceeds to written work. They also cover the syllabus very fully and have sufficient exercises on every phase of the syllabus. Dekker regards the books as the best on the market as school textbooks for Mathematics.

They have the added advantage that the Algebra and Geometry appear in a single book, and the subject can therefore be seen as an entity by the pupils. The subdivisions of Algebra which go together are treated together and the close connection between Algebra and Geometry is indicated where feasible. The approach to the work is particularly modern, especially in regard to the illustrations.

Both educationally and psychologically the presentation in these books is better than in other publications which deal with the same syllabus.

In both books it is clearly indicated which exercises are intended for pupils who wish to enter a university, which exercises should be done by school-leaving pupils, and which exercises are intended for those following the Std. 8 stream.

The books contain sufficient exercises for the revision of old work. The examples become increasingly difficult and stimulate pupils to do things themselves.

Problems and the function concept play an important role in the two books. Graphs also occupy a prominent position.

7.2.10 G.H.A. Steyn and T.S. Welman: Algebra and Graphs for Stds. 9 and 10

The book is modern because it contains numerous and good illustrations. It covers the whole of the prescribed syllabus and treats the work in such a manner that things which belong together follow closely upon one another, as for example certain equations, graphs and problems. The way in which the work is set out is satisfactory and pupils are afforded the opportunity to fathom and master questions themselves.

In this book, as in those for Stds. 7 and 8 by the same authors, a clear indication is given of what work is intended for the Std. 10 course and what for the university entrance course; in addition, it deals very fully with the syllabus as prescribed for Transvaal secondary schools, and makes provision for sufficient exercises to enable the teacher to revise completed work from time to time.

The function concept plays an important role in this textbook.

The presentation of the work is logical and scientific although not always psychologically adapted to the pupils, especially because more emphasis is laid on the subject than on the child. Sufficient knowledge to be able to pass the examination still plays a large role - in fact, too large a role. The child's ability to apply the acquired knowledge in practice remains under-developed. It will remain so until the examination system undergoes a spectacular change. In 34 of the 41 subdivisions made by Dekker he considers that the book serves its purpose "Well or excellently", and only in one aspect, namely "Problems which lead to quadratic equations", does he think that the book is poor.

7.3 TEXTBOOKS FOR GEOMETRY

7.3.1 Cape Province

Since the same books as those for Algebra are used, further discussion is unnecessary.

7.3.2 Natal

The three textbooks about which most comments were received are given in Table 7.14.

TABLE 7.14

GEOMETRY TEXTBOOKS USED IN NATAL

Code	Authors	Title	Stds.	Number of teachers who commented
D	Durell	Geometry for Schools	7 - 10	27
K	De Kock & de Waal	Geometry for Junior Certificate	7 & 8	25
W	De Kock & de Waal	Geometry for Senior Certificate	9 & 10	15

The three books are analysed in Table 7.15. In the columns an indication is given of the percentage of replies which were in the affirmative.

TABLE 7.15

TEXTBOOKS USED IN NATAL FOR GEOMETRY

Characteristic		D %	K %	W %
1. The contents cover the syllabus of the department		100	92	87
2. The wording of the axioms, definitions and theorems is mathematically correct		100	83	67
3. The drawings are clear and accurate		100	92	100
4. The book is more modern in its approach than earlier publications		81	63	33
5. Personal opinion on the book:	Very good	67	13	-
	Good	22	46	33
	Fair	11	38	67
	Poor	-	-	-
	Very poor	-	3	-

It is clear that the teachers do not regard the two books of the South African authors as highly as the book of the overseas author, Durell. The teachers apparently look forward to a book which will at least match overseas standards.

7.3.3 Orange Free State

The textbooks on which most teachers submitted comments are given in Table 7.16.

TABLE 7.16

THE GEOMETRY TEXTBOOKS ON WHICH MOST TEACHERS IN THE ORANGE FREE STATE SUBMITTED COMMENTS

Code	Authors	Title	Stds.	Number of teachers
H	Hugo, Barnard and De Wet	Mathematics	9 & 10	20
B	Hugo, Barnard and De Wet	Mathematics	7 & 8	9
M	Marquard and Faure	Mathematics	7 & 8	12

The three books are analysed in Table 7.17. In the columns an indication is given of the percentage of replies which were in the affirmative.

TABLE 7.17

TEXTBOOKS USED IN THE ORANGE FREE STATE FOR GEOMETRY

	Characteristic		H %	B %	M %
1.	The contents of the book cover the syllabus of the department		90	100	92
2.	The wording of the axioms, definitions and theorems is				
	mathematically correct		75	78	67
	grammatically correct		5	-	17
	Mistakes in the language and in the presentation of mathematical facts occur		5	-	8
	No reply given		15	-	8
3.	The drawings are clear and accurate		95	89	83
4.	The book is more modern in its approach than previous publications		60	44	58
5.	Personal opinion on the book:	Very good	-	-	8
		Good	50	67	50
		Fair	40	33	33
		Poor	5	-	-
		Very poor	-	-	-
		No opinion	5	-	9

7.3.4 Transvaal

The textbooks on which most teachers submitted comments are given in Table 7.18.

TABLE 7.18

THE GEOMETRY TEXTBOOKS ON WHICH MOST TEACHERS IN THE TRANSVAAL SUBMITTED COMMENTS

Code	Authors	Title	Stds.	Number of teachers
G	Gray and Smith	A New Sequence Geometry	7 & 8	11
M	Garson and Malherbe	Geometry in Transvaal schools	9 & 10	74
W	Welman and Steyn	Geometry and Analytical Geometry	9 & 10	102

The three books are analysed in Table 7.19. In the columns an indication is given of the percentage of replies which were in the affirmative.

TABLE 7.19

TEXTBOOKS USED FOR GEOMETRY IN THE TRANSVAAL

Characteristic		G %	M %	W %
1. The contents of the book cover the syllabus of the Department		64	96	98
2. The wording of the axioms, definitions and theorems is mathematically correct		82	93	85
3. The drawings are clear and accurate		91	96	100
4. The book is more modern in its approach than earlier publications		27	78	87
5. Personal opinion on the book:	Very good	18	30	25
	Good	36	61	62
	Fair	46	8	7
	Very poor	-	-	-
	Poor	-	-	-
	No opinion expressed	-	1	6

7.4 TEXTBOOKS FOR TRIGONOMETRY

7.4.1 Cape Province

Here the same books are used as those discussed under the heading "Algebra".

7.4.2 Natal

The book discussed by most teachers is that of B. F. Schmidt : Trigonometry for Senior Certificate and Matriculation. The opinions of twenty-seven teachers are set outin Table 7.20.

TABLE 7.20

DISCUSSION OF THE TRIGONOMETRY TEXTBOOK USED IN NATAL

Characteristic		Number	Percentage
1. The contents cover the syllabus of the department		5	19
2. The wording is	mathematically correct	25	93
	grammatically correct	1	4
3. The drawings are clear and accurate		27	100
4. The book is more modern in its approach than previous publications		9	33

The most important objection against this book, which is intended for Stds. 9 and 10, is the fact that it does not completely cover the syllabus.

7.4.3 Orange Free State

The books on which most teachers submitted comments are given in Table 7.21.

TABLE 7.21

TEXTBOOKS USED FOR TRIGONOMETRY IN THE ORANGE FREE STATE

Code	Authors	Title	Stds.	Number of teachers
H	Hugo, Barnard and De Wet	Mathematics	9 & 10	22
W	Wilkinson, Van Staden and Strauss	Mathematics according to the Orange Free State syllabus	9 & 10	30

The two books are analysed in Table 7.22. In the columns an indication is given of the percentage of replies which were in the affirmative.

TABLE 7.22

TRIGONOMETRY TEXTBOOKS USED IN THE ORANGE FREE STATE

Characteristic	H %	W %
1. The contents cover the syllabus of the department	91	90
2. The wording is mathematically correct	77	70
grammatically correct	9	-
Mistakes in the language and in the presentation of mathematical facts occur	5	-
3. The drawings are clear and accurate	91	93
4. The book is more modern in its approach than previous publications	55	80

7.4.4 Transvaal

The textbooks on which most teachers submitted comments, are given in Table 7.23.

TABLE 7.23

TEXTBOOKS USED IN THE TRANSVAAL FOR TRIGONOMETRY

Code	Authors	Title	Stds.	Number of teachers
B	Brink, Ogden and Garson	Trigonometry for the high schools and secondary schools	9 & 10	114
W	Welman and Steyn	Trigonometry	9 & 10	77
G	Garson and Malherbe	Trigonometry	9 & 10	15

The three books are analysed in Table 7.24. In the columns the percentages of replies which were in the affirmative are indicated.

TABLE 7.24

ANALYSIS OF THE TEXTBOOKS USED FOR TRIGONOMETRY IN THE TRANSVAAL

Characteristic	B %	W %	G %
1. The contents cover the syllabus of the department	96	97	87
2. The wording is mathematically correct	85	94	87
grammatically correct	3	3	-
Mistakes in the language and in the presentation of mathematical facts occur	7	1	7
3. The drawings are clear and accurate	94	99	73
4. The book is more modern in its approach than previous publications	68	79	47

Remarks

Use is made of a great variety of textbooks. It would appear that, as the central province, the Orange Free State is flooded with textbooks from all quarters. The largest number in that province using one textbook, that of Wilkinson, van Staden and Strauss on Trigonometry, is thirty.

7.5 A GENERAL REVIEW OF THE TEXTBOOKS USED

7.5.1 Introduction

The following paragraphs will deal with the authors of textbooks, the books used, and the teachers' opinions on certain aspects of the textbooks used by them.

7.5.2 The authors of textbooks on mathematical subjects

The teachers were asked whether they are the authors or co-authors of textbooks for Mathematics and/or Arithmetic. Of the teachers who replied in the affirmative, thirty (30) can be regarded as qualified in Mathematics, while fifteen (15) of the authors of textbooks successfully completed fewer than two degree courses in a mathematical subject.

Two-thirds of the authors are therefore mathematically qualified, while one-third of them are unqualified. These figures relate only to the authors who submitted completed questionnaires.

7.5.3 The textbooks used

The names of the textbooks used in the high schools in the various provinces are given in Appendix 9. Table 7.25 shows how many different textbooks are used in Stds. 6, 7 and 8.

The totals have intentionally not been given, since the same textbooks can be used in more than one province. If these figures were to be added, a distorted picture would be obtained.

Table 7.26 indicates how many different textbooks are used in Stds. 9 and 10.

TABLE 7.25

THE VARIETY OF TEXTBOOKS USED IN STANDARDS 6, 7 AND 8

Subject	Provinces			
	Cape Province	Natal	Orange Free State	Transvaal
General Mathematics	15	3	9	18
Algebra	5	5	5	9
Geometry	9	9	7	6
Algebra and Geometry	9	3	6	5
Arithmetic	20	23	12	16

TABLE 7.26

THE VARIETY OF TEXTBOOKS USED IN STDS. 9 AND 10

Subject	Provinces			
	Cape Province	Natal	Orange Free State	Transvaal
Algebra	4	13	7	7
Algebra and Graphs	1	2	0	3
Algebra, Geometry and Trigonometry	13	11	9	9
Algebra and Geometry	0	1	0	0
Geometry	6	10	7	5
Analytical Geometry	1	5	0	2
Trigonometry	7	6	7	6
Commercial Mathematics	5	2	4	5

7.5.4 Condusion

Use is made of a relatively large variety of textbooks. This practice has its advantages and its disadvantages. On the credit side it should be mentioned that there is no deadly uniformity. On the other hand, there are no books of such a standard that they dominate the scene. With some exceptions, the general impression created by this picture is that of mediocrity. For that reason the most popular books have been discussed in the foregoing paragraph.

7.5.5 The textbooks and the syllabi

Table 7.27 reflects the opinions of 1712 teachers in regard to the manner in which the contents of the textbooks cover the subject matter prescribed in the syllabi. Only those teachers who replied to the relevant questions have been taken into account.

TABLE 7.27

THE MANNER IN WHICH TEXTBOOKS MEET THE REQUIREMENTS OF THE SYLLABI

Subject to be covered by the textbook	Teachers: Qualified in Mathematics: Satisfied: Number	%	Not satisfied: Number	%	Total	Unqualified in Mathematics: Satisfied: Number	%	Not satisfied: Number	%	Total
General Mathematics	389	94.2	24	5.8	413	508	96.6	18	3.4	526
Algebra	463	91.1	45	8.9	508	312	96.0	13	4.0	325
Geometry	438	96.1	18	3.9	456	280	95.9	12	4.1	292
Trigonometry	370	92.0	32	8.0	402	140	94.0	9	6.0	149

The overwhelming majority of the teachers are wholly satisfied with the manner in which the syllabi are covered by the textbooks used by them.

7.5.6 Algebra and graphs together or separately

Table 7.28 shows how many teachers make use of textbooks which deal with Algebra and Graphs separately, and how many teachers use textbooks in which Algebra and Graphs are treated as an entity.

TABLE 7.28

ALGEBRA AND GRAPHS

Classification of Algebra textbooks used	Teachers: Qualified in Mathematics: Number	%	Unqualified in Mathematics: Number	%	Total: Number	%
1. Algebra and Graphs separate	337	66.8	213	71.2	550	68.5
2. Graphs treated as an inherent part of Algebra	155	30.8	78	26.1	233	29.0
3. Another classification	12	2.4	8	2.7	20	2.5
TOTAL	504	100.0	299	100.0	803	100.0

7.5.7 Conclusion

Approximately half of the 1712 teachers who completed the questionnaires, replied to the relevant questions. More than two-thirds of the teachers (66.8% and 71.2%) still use textbooks in which Algebra and Graphs are treated as two separate subjects. It would appear however that the qualified Mathematics teachers rather than the unqualified ones tend to concentrate on books which deal with the two subjects as one.

If the concepts of sets and functionality are to be given prominence, then it is essential that graphs and the other algebraic material should be dealt with as an entity. Graphs are visual material and as such belong rather to the beginning of a course than the end. In view of the foregoing it is clear that the position in connection with Algebra textbooks at the time of the survey was still far from satisfactory.

7.5.8 Fundamental mathematical concepts and structure

As has already been mentioned the questionnaires completed by 1712 teachers were analysed. The questions on the textbooks for the various subjects were not answered by all the teachers. This is, after all, only to be expected since some teachers teach only Arithmetic, others only Mathematics, and a group only General Mathematics. In the analysis of the replies only those teachers who did reply to the relevant questions, will be taken into account.

General Mathematics

The teachers who make use of textbooks for General Mathematics had to answer the following question:

"Is the development of the fundamental mathematical concepts sufficiently emphasised by the book?" An analysis of the replies is given in Table 7.29.

TABLE 7.29

STRESSING OF THE FUNDAMENTAL MATHEMATICAL CONCEPTS IN THE TEXTBOOKS FOR GENERAL MATHEMATICS

Teachers' opinions on whether the fundamental concepts are adequately stressed	Teachers					
	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
Yes	311	78.7	486	91.7	797	86.2
No	84	21.3	44	8.3	128	13.8
TOTAL	395	100.0	530	100.0	925	100.0

Those teachers who do not possess the minimum mathematical qualifications (two degree courses in a mathematical subject) are almost unanimous (91.7%) in their opinion that justice is being done to the fundamental mathematical concepts. Almost a quarter (21.3%) of their better qualified colleagues think differently about the matter.

Algebra

The teachers were asked whether the Algebra textbook which they use develops a clear mathematical structure. The number of teachers who replied in the affirmative and those who replied in the negative are reflected in Table 7.30.

TABLE 7.30

THE DEVELOPMENT OF A CLEAR MATHEMATICAL STRUCTURE IN THE ALGEBRA TEXTBOOKS

Teachers' opinions on whether or not a clear mathematical structure is developed	Teachers					
	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
Yes	404	83.0	281	92.4	685	86.6
No	83	17.0	23	7.6	106	13.4
TOTAL	487	100.0	304	100.0	791	100.0

The percentage of unqualified mathematics teachers who are of the opinion that a clear mathematical structure is developed in the Algebra textbook which they use is greater than in the case of the qualified teachers.

It is possible that complete clarity does not exist in regard to what is meant by the development of a clear mathematical structure.

7.5.9 The central concept

The teachers were expected to indicate the central concept in the textbooks used by them. Table 7.31 reflects the replies out of a total of 734 mathematically qualified and 978 mathematically unqualified teachers.

TABLE 7.31

THE CENTRAL CONCEPT IN THE ALGEBRA TEXTBOOK

Mathematical concept	Teachers					
	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
Equations	156	47.0	77	42.1	233	45.2
Functionality	93	28.0	44	24.0	137	26.6
Other concept	38	11.4	13	7.1	51	9.9
Sets	24	7.2	21	11.5	45	8.7
Relation	10	3.0	11	6.0	21	4.1
Variation	8	2.4	7	3.8	15	2.9
Ratio	3	1.0	10	5.5	13	2.6
TOTAL	332	100.0	183	100.0	515	100.0

In 1962 equations were still the central concept in almost half of the Algebra books used in the schools. The concept of functionality is given prominence in only slightly more than a quarter (26.6%) of the textbooks used. Furthermore, considerable confusion exists in regard to what is actually the central concept in the various textbooks.

In view of the numerous complaints that pupils do not learn to think, it is essential that complete clarity should be obtained in regard to the place of the mathematical concept in teaching.

7.5.10 Accuracy

A question posed in regard to the textbook used, namely whether the language is clear and grammatically and mathematically correct, was not answered satisfactorily. Unfortunately many teachers were under the impression that they either had to say that a textbook was grammatically correct or that it was mathematically correct. It should be borne in mind in this connection that these qualities vary greatly from book to book and that a discussion of individual textbooks would be more meaningful.

General Mathematics

In regard to the textbooks used for General Mathematics the question was asked whether the facts, definitions and theorems are accurate. The replies are reflected in the figures given in Table 7.32.

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TABLE 7.32

THE ACCURACY OF THE FACTS, DEFINITIONS AND THEOREMS IN THE TEXTBOOKS FOR GENERAL MATHEMATICS

Opinions of the teachers on textbooks used for General Mathematics	Teachers					
	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
The facts, theorems and definitions are accurate	396	95.9	515	96.6	911	96.3
The facts, theorems and definitions are not accurate	17	4.1	18	3.4	35	3.7
TOTAL	413	100.0	533	100.0	946	100.0

Almost all the teachers are satisfied with the accuracy of the facts, theorems and definitions in the textbooks for General Mathematics.

Geometry and Trigonometry

In regard to the textbooks used for Geometry and Trigonometry the question was asked whether the diagrams are well drawn and accurate. The replies are analysed in Table 7.33.

TABLE 7.33

THE CLARITY AND ACCURACY OF THE DIAGRAMS IN THE TEXTBOOKS

Opinions of the teachers	Teachers					
	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
Geometry textbooks						
Diagrams well drawn and accurate	462	98.1	284	96.6	746	97.5
Not well drawn and accurate	9	1.9	10	3.4	19	2.5
TOTAL	471	100.0	294	100.0	765	100.0
Trigonometry textbooks						
Diagrams well drawn and accurate	407	98.3	145	96.7	552	97.9
Not well drawn and accurate	7	1.7	5	3.3	12	2.1
TOTAL	414	100.0	150	100.0	564	100.0

Practically all the teachers are satisfied with the clarity and accuracy of the diagrams in the textbooks used.

7.5.11 Are the textbooks modern?

Table 7.34 reflects the replies of the 1712 teachers in answer to the question whether the textbooks used by them are more modern than earlier publications.

TABLE 7.34

THE NUMBER OF TEACHERS USING TEXT-BOOKS WHICH CAN BE REGARDED AS MORE MODERN THAN EARLIER PUBLICATIONS

Opinions of teachers	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
Textbooks for General Mathematics						
More modern	290	78.0	414	90.0	704	84.6
Not more modern	82	22.0	46	10.0	128	15.4
TOTAL	372	100.0	460	100.0	832	100.0
Textbooks for Algebra						
More modern	329	74.6	199	79.3	528	76.3
Not more modern	112	25.4	52	20.7	164	23.7
TOTAL	441	100.0	251	100.0	692	100.0
Textbooks for Geometry						
More modern	330	78.8	213	82.9	543	80.3
Not more modern	89	21.2	44	17.1	133	19.7
TOTAL	419	100.0	257	100.0	676	100.0
Textbooks for Trigonometry						
More modern	269	72.3	114	81.4	383	74.8
Not more modern	103	27.7	26	18.6	129	25.2
TOTAL	372	100.0	140	100.0	512	100.0

If account is taken of the fact that no enquiries were made whether the textbooks used are modern, but only whether they are more modern than earlier publications, it is significant that approximately a quarter of the qualified Mathematics teachers replied in the negative. We can accept that they look forward to textbooks which will be "different" and which will create the impression of being completely new and modern.

7.5.12 The teachers' assessment of the textbooks used by them

The teachers were expected to determine the merits of the textbook used by them, according to a scale of five points.

The results of the merit assessment are given in Table 7.35.

TABLE 7.35

THE ASSESSMENT BY THE TEACHERS OF THE TEXTBOOKS USED BY THEM

Assessment	Teachers											
	Qualified in Mathematics						Unqualified in Mathematics					
	General Mathematics		Algebra		Geometry		General Mathematics		Algebra		Geometry	
	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
Very good	68	16.5	96	33.2	117	25.4	91	18.6	64	21.8	78	26.2
Good	253	61.7	95	33.2	260	58.9	309	63.5	176	59.4	160	53.2
Fair	80	19.4	87	30.1	64	13.9	82	16.7	52	17.8	56	19.3
Poor	9	2.2	10	3.5	7	1.6	4	0.8	3	1.0	3	1.0
Very poor	1	0.2	0	-	1	0.2	2	0.4	0	-	1	0.3
TOTAL	411	100.0	288	100.0	449	100.0	488	100.0	295	100.0	298	100.0
Replied	411	56.0	288	39.2	449	61.2	488	49.9	295	30.2	298	30.5
Did not reply	323	44.0	446	60.8	285	38.8	490	50.1	683	69.8	680	69.5
GRAND TOTAL	734	100.0	734	100.0	734	100.0	978	100.0	978	100.0	978	100.0

Most of the teachers regard the textbook as "good".

It is noteworthy that approximately a third (33.2%) of the qualified Mathematics teachers regard their Algebra textbooks as "Very good", a third (33.2%) as "Good", and a third (33.6%) as "Fair" or "Poor".

Although there are not many complaints, the opinions of the teachers in regard to their textbooks show a fair amount of variation.

7.6 DISCUSSION

Dr. Victor E. McGee, who was born in South Africa, but who is at present Assistent Professor at Dartmouth College, New Hampshire, U.S.A., writes as follows on the South African Mathematics textbook:

> "Without doubt the textbook in use is the last thing in the world to inspire interest, and because of its uninspiring format it is not read. South African pupils simply do not learn to read the text in a mathematical textbook, which is used as a source of examples and nothing else. Only the inadequate teacher finds himself reading the text. The sad thing is that this has been going on so long that it is seldom recognized as a problem." (62, p. 31)

He then makes the following definite recommendation:

> "It is recommended that the place of the textbook be radically revised. From the earliest years in school it should be so organized that the books be used as reading material. In their present form they are unable to evoke the slightest enthusiasm in the pupil, so that new texts are sorely needed. A glance at a modern American textbook will reveal just what can be done. And, parenthetically, let me say a word about the recent wave of interest in programmed instruction. For many in America now, it is felt that the value of programmed instruction lies mainly in its usefulness for checking the logical readability of textbook material. While the teacher helps propagate the idea that the textbook is a source of examples and no more, there is no place for programmed instruction. All teachers could be urged forthwith to make the material of the text important to the learner. Instead of making all arithmetical and mathematical homework into a mere doing of

find many of the constructions interesting and a few of them astonishing so that they will want to know why these things happen. If the teacher can use such opportunities to make the pupils realize the need for proving things in geometry, the geometry done in Standard 6 will be very valuable."

The foregoing is not in all respects a suitable foreword to the textbook made available to the pupils, but it is quoted in order to indicate the idea behind these textbooks.

The Std. 6 Algebra Manual for Teachers is divided into two parts.

"The first 20 pages of this book consist of examples and the last 11 pages of notes or worked examples."

Two questions which now involuntarily arise are whether only this **one** English boys' school in the Cape Province should enjoy the benefit of this reform in respect of textbooks and whether it cannot be extended to all schools throughout the country.

7.7 POSSIBLE EXPERIMENTAL TEXTBOOKS

7.7.1 Experimental textbooks

Experimental textbooks are described as follows in Questionnaire N.B.377:

"Experimental textbooks" are textbooks made available to schools in preliminary form to be tried out. The remarks of the teachers are welcomed and a new set of books is compiled incorporating suggestions that seem feasible. This process is continued until a satisfactory result is obtained.

When one considers the vast number of textbooks used today for the various mathematical subjects, the question arises whether the need does not perhaps exist for a few standard works - books which are the best products of the best authors and which can serve as a model for future writers of textbooks. Even the best textbook will in course of time become obsolete, but then it may become necessary to set yet another standard. It is impossible to predict whether any of the existing textbooks, for example, will survive for as long as the well-known Algebra textbook of Hall and Knight which was written as long ago as the nineteenth century and which is used in some schools even to this day. The publication of another book of such enduring value will demand the concerted efforts of exceptionally able persons drawn, quite possibly, from our schools and universities.

To have such experimental textbooks written and to try them out in actual practice is a very ambitious project requiring considerable funds. That is why the teachers who teach mathematical subjects at high schools were asked for their opinions with regard to the matter of this investigation.

7.7.2 The teachers

In the analysis of the replies, a distinction is drawn between two groups of teachers, namely those who completed at least two degree courses in one or more of the following subjects, namely Mathematics, Applied Mathematics, and Mathematical Statistics, and those who do not possess any such qualification. For the purposes of this investigation, the first group is called the qualified Mathematics teachers and the second group the unqualified Mathematics teachers.

7.7.3 The replies to the questions in the questionnaire: The necessity for a revision of the situation

The first question was whether there is a necessity for a complete revision of the teaching of Mathematics. By this is meant the teaching of Mathematics at South African high schools.

examples, it is surely possible to require a little research into a new topic by asking that the pupils read the text as reading matter." (62, p. 34)

A teacher who has already written a considerable number of articles in periodicals on the teaching of Mathematics expressed himself as follows in a memorandum which accompanied his completed questionnaire:

"Most textbooks make one of two mistakes:

(a) They contain only examples, or at any rate far too little explanatory matter so that the pupil can't learn from them even if he wants to.

(b) They try to 'teach' the subject (even explaining how to look up logs). There is so much explanatory matter that the pupil does not know what to learn.

"I believe that examples and notes should be kept separate. The notes should be written so that the pupil can follow them and learn what work to learn (that is, all his notes). Worked examples of every type of example must be included.

"At our school we have been working out of this kind of textbook since 1957. The system is successful and both the teachers and the boys very much prefer it."

In the school concerned textbooks are compiled by the teachers themselves and made available to the pupils in roneoed form. Some of their complaints against certain existing textbooks are the following:

Theorems on similarity are called axioms because the syllabus does not call for any formal proofs. Postulates are not even mentioned.

The teacher in question submitted copies of these textbooks which are provided by the school itself. Their format is folio size. The covers are neatly printed, while the inside pages are roneoed.

A separate booklet is compiled in this manner for each standard. The title of the copies submitted reads as follows:

"Algebra and Geometry for Standard Six
Cape Province Syllabus, 1961"

This copy is intended for the pupils.

The teacher uses a book with the same format. It contains instructions in connection with the work to be done, as well as the answers to the problems which appear in the copy used by the pupils.

The following introductory remarks are given in the copy intended for the pupils:

"In the Algebra in this book, the examples and the notes have been kept separate. In many cases the examples are so designed that the pupils will learn the new ideas from doing the examples. In most cases the teacher will introduce the new ideas himself, but it is not the intention of the notes that the pupils should get new ideas from them - they are there to be studied after that part of the work has been done in class. The notes contain all the Algebra that the pupils should be expected to learn.

"The 'teaching examples' are intended to introduce new ideas. They should all be done, preferably in class. In the Geometry the notes have not been kept separate, but all bookwork is enclosed in a full border. The idea behind the Geometry syllabus is to give the pupils experience in the use of instruments and to let them become familiar with such things as triangles and parallel lines before they start with formal geometry. It is hoped that they will

The replies are analysed in Table 7.36.

TABLE 7.36

THE NECESSITY FOR A COMPLETE REVISION OF THE TEACHING OF MATHEMATICS AND ARITHMETIC

Subject	Teachers							
	Consider a complete revision necessary		Do not consider a revision necessary		No reply given		Total	
	Number	%	Number	%	Number	%	Number	%
Qualified Mathematics teachers:								
Mathematics	340	46.2	240	32.7	155	21.1	735	100.0
Arithmetic	260	35.4	157	21.4	318	43.2	735	100.0
Unqualified Mathematics teachers:								
Mathematics	243	24.7	199	20.1	544	52.2	986	100.0
Arithmetic	243	24.7	199	20.1	544	52.2	986	100.0
Teachers who teach mathematical subjects:								
Mathematics	583	33.9	439	25.5	699	40.6	1721	100.0
Arithmetic	503	29.2	356	20.7	862	50.1	1721	100.0

Qualified Mathematics teachers:

Unanimity does not exist with regard to the necessity for a completely new approach to the teaching of both Mathematics and Arithmetic. It is noteworthy that

(i) of the 340 teachers who said "yes" in the case of Mathematics, 197 said "yes" in respect of Arithmetic while 26 said "no" and 117 gave no reply;

(ii) of the 240 who said "no" in the case of Mathematics, 48 said "yes" in respect of Arithmetic, while 123 said "no" and 69 gave no reply;

(iii) of the 155 who gave no answer in the case of Mathematics, 15 said "yes" in respect of Arithmetic, while 8 said "no" and 132 did not reply.

Unqualified Mathematics teachers:

Less agreement exists on the part of unqualified Mathematics teachers in regard to the necessity for a revision of the teaching of both Mathematics and Arithmetic than would appear from the figures given in the table.

(i) of the 243 who said "yes" in the case of Mathematics, 160 also said "yes" in respect of Arithmetic, while 35 said "no" and 48 gave no reply;

(ii) of the 199 who said "no" in the case of Mathematics, 13 said "yes" in respect of Arithmetic, while 120 said "no" and 66 gave no reply;

(iii) of the 544 who did not give an answer in the case of Mathematics, 70 said "yes" in respect of Arithmetic, while 44 said "no" and 430 failed to give a reply.

Conclusion

Of the qualified Mathematics teachers more than three quarters answered the questions in connection with the teaching of Mathematics, and the majority were in favour of a complete reform in the teaching of this subject.

That 43.2% of the teachers gave no reply to the questions in connection with the teaching of Arithmetic must be ascribed to the fact that few qualified Mathematics teachers also teach Arithmetic. Of those who did reply to the questions, the majority are in favour of a radical revision of the teaching of Arithmetic. In the case of "Mathematics" the majority was 100 out of a total of 680, while in the case of "Arithmetic" the figure was 103 out of 417.

Approximately a quarter of the unqualified Mathematics teachers are interested in a completely new approach to the teaching of Mathematics and Arithmetic. They form a small majority of the teachers who replied to these questions.

7.7.4 The role which experimental textbooks can play in the overhaul of the teaching of Mathematics and Arithmetic

Tables 7.37 and 7.38 show how many teachers are of the opinion that experimental textbooks can play an important role in the revision of the teaching of Mathematics and Arithmetic.

TABLE 7.37

THE IMPORTANCE OF EXPERIMENTAL TEXTBOOKS IN THE OPINION OF QUALIFIED MATHEMATICS TEACHERS

Teachers	Teachers who consider that experimental textbooks can play an important role		Teachers who consider that experimental textbooks cannot play an important role		Teachers who did not reply to the question		Total
	Number	%	Number	%	Number	%	
(1) In favour of a revision	312	91.8	19	5.6	9	2.6	340
(2) Not in favour of a revision	116	48.3	105	43.8	19	7.9	240
(3) No opinion expressed for or against	39	25.2	9	5.8	107	69.0	155
TOTAL	467	63.5	133	18.1	135	18.4	735

TABLE 7.38

THE IMPORTANCE OF EXPERIMENTAL TEXTBOOKS IN THE OPINION OF UNQUALIFIED MATHEMATICS TEACHERS

Teachers	Teachers who consider that experimental textbooks can play an important role		Teachers who consider that experimental textbooks cannot play an important role		Teachers who did not reply to the question		Total
	Number	%	Number	%	Number	%	
(1) In favour of a revision	227	93.4	8	3.3	8	3.3	243
(2) Not in favour of a revision	112	56.3	72	36.2	15	7.5	199
(3) No opinion expressed for or against	175	32.1	42	7.7	327	60.1	544
TOTAL	514	52.1	122	12.4	350	35.5	986
GRAND TOTAL	981	57.0	255	14.8	485	28.2	1721

Conclusions

(1) Almost all the teachers (91.8% and 93.4%) respectively) who are of the opinion that there is room for a completely new approach to the teaching of Mathematics in our schools, consider that experimental textbooks can play an important role in the reform of the teaching of Mathematics and Arithmetic.

(2) The majority of the teachers (48.3% and 56.3% respectively) who do not desire to effect any radical reform of the teaching of Mathematics, are however in favour of experimental textbooks. There is of course no absolute majority in the case of the qualified Mathematics teachers, but more (48.3%) are in favour of such a revision than against it (43.8%).

(3) The teachers who expressed no opinion on the possible reform of the teaching of Mathematics, for the most part (69.0% and 60.1%) also remained silent on this point. Of those who did give an answer, more were in favour (25.2% and 32.1% respectively) of such a revision than against it (5.8% and 7.7% respectively).

In regard to the total group of teachers who completed the questionnaires, the majority (57.0%) were of the opinion that experimental textbooks can play an important role in the new approach to the teaching of Arithmetic and Mathematics. Their number is almost four times greater than the figure for the teachers (14.8%) who hold the opposite opinion. The teachers who expressed no opinion on this matter are mainly those who have not completed at least two degree courses in their mathematical subject (35.5% as compared with 18.4% qualified Mathematics teachers).

7.7.5 By whom the proposed experimental textbooks should be written

Table 7.39 reflects the opinions of those teachers who teach mathematical subjects in regard to possible authors of such experimental textbooks.

TABLE 7.39

POSSIBLE AUTHORS OF EXPERIMENTAL TEXTBOOKS

Possible Authors	Number of teachers in favour of type of possible author			
	Teachers qualified in Mathematics		Teachers unqualified in Mathematics	
	Number	%	Number	%
(1) A group of teachers	191	26.0	276	28.0
(2) A group of university lecturers	2	0.3	2	0.2
(3) An independent author	14	1.9	11	1.1
(4) A committee of teachers and university lecturers	390	53.1	338	34.3
(5) Other	8	1.1	6	0.6
(6) No opinion	130	17.6	353	35.8
TOTAL	735	100.0	986	100.0

It is interesting to observe that more unqualified Mathematics teachers than qualified Mathematics teachers are of the opinion that teachers are the proper persons to write such textbooks.

The majority of the latter group of teachers would rather see that the task be entrusted to a committee consisting of teachers and university professors and lecturers.

7.7.6 The aims of experimental textbooks

Table 7.40 reflects the views of teachers in connection with the aims of such textbooks.

TABLE 7.40

THE AIMS OF EXPERIMENTAL TEXTBOOKS

Aims	Qualified Mathematics teachers who						Unqualified Mathematics teachers who					
	replied in the affirmative		replied in the negative		gave no reply		replied in the affirmative		replied in the negative		gave no reply	
	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
(a) A model for future authors of school textbooks	381	51.8	128	17.4	226	30.8	396	40.2	117	11.9	473	48.0
(b) An aid in the training of teachers	448	61.0	56	7.6	231	31.4	485	49.2	52	5.3	449	45.5
(c) An example of how the mathematical concepts should be developed	499	47.9	33	4.5	203	27.6	465	47.2	23	2.3	498	50.5
(d) An example of the development of a mathematical structure	429	58.4	36	4.9	270	36.7	393	39.8	28	2.7	565	57.5
(e) Preliminary school textbooks until such books are obtainable commercially	326	44.4	137	18.6	272	37.0	348	35.3	112	11.4	526	53.3

Approximately a third of the qualified Mathematics teachers and half of the unqualified teachers failed to reply to the questions. Of those who did give an answer, the overwhelming majority were of the opinion that the given aims are correct. According to the "number of votes" the order or importance of the aims is as follows:

(1) An example of how the mathematical concept ought to be developed in high schools;

(2) An aid in the training of teachers;

(3) An example of the development of a mathematical structure;

(4) A model for future authors of school textbooks;

(5) Preliminary school textbooks until such books are obtainable commercially.

7.7.7 Where the answers should appear

Some teachers consider that the answers given at the back of textbooks are merely a source of annoyance, while other like them there. For that reason teachers were asked where such answers should appear. The way in which they responded is reflected in Table 7.41.

TABLE 7.41

WHERE THE ANSWERS TO PROBLEMS SHOULD APPEAR IN TEXTBOOKS

Where the answers should appear	Qualified Mathematics teachers		Unqualified Mathematics teachers		Total	
	Number	%	Number	%	Number	%
(a) At the back of the same book	238	32.4	136	13.8	374	21.7
(b) In a separate booklet intended for teachers only	188	25.6	219	22.2	407	23.6
(c) In a teachers' manual also containing the teaching methods	187	25.5	302	30.6	489	28.5
(d) Other ideas	10	1.4	2	0.2	12	0.7
(e) No reply	112	15.1	327	33.2	439	25.5
TOTAL	735	100.0	986	100.0	1721	100.0

It is doubtful where the answers to problems should appear in textbooks. Some teachers who are regarded as qualified in Mathematics consider the answers at the back of the textbook to be a definite impediment. They apparently prefer a manual for teachers in which the method of teaching the subject is explained in the light of the textbook concerned and in which the answers are given.

Such supplementary series are obtainable overseas today, an example in this connection being the Kingsway series. (116)

The fact that it is the unqualified Mathematics teachers who feel the strongest need for answers at the back of textbooks is really self-evident. In some provinces such teachers are in the majority, and since their services will undoubtedly still be used in this manner, for a long time serious consideration should be given to the question of ensuring that these particular needs are met.

7.7.8 Who should be responsible for writing experimental textbooks

Teachers were asked: "Under whose aegis should these experimental textbooks be written?" Table 7.42 shows how many teachers are in favour of each of the different bodies mentioned.

TABLE 7.42

BODIES WHICH SHOULD BE RESPONSIBLE FOR THE PUBLICATION OF EXPERIMENTAL TEXTBOOKS

Possible bodies	Qualified Mathematics teachers		Unqualified Mathematics teachers		Total	
	Number	%	Number	%	Number	%
(a) The National Council for Social Research	35	4.8	42	4.3	77	4.5
(b) The South African Mathematical Association	156	21.1	146	14.8	302	17.5
(c) A committee representing different bodies interested in the teaching of Mathematics	397	54.0	406	41.2	803	46.7
(d) No opinion	147	20.0	392	39.7	539	31.3
TOTAL	735	100.0	986	100.0	1721	100.0

The majority of the teachers are in favour of the proposed experimental textbooks being published by a committee representing various bodies which are interested in the teaching of Mathematics. Examples of such bodies are the Departments of Education of the four provinces and South West Africa, the Department of Education, Arts and Science, the National Bureau of Educational and Social Research, the Atomic Energy Board, the Council for Scientific and Industrial Research, the Joint Council of Professional Engineers, the South African Statistical Association, the South African Mathematics Association, the Committee of University Principals, the Federal Council of Teachers Associations in South Africa and the Association of Science and Mathematics Teachers.

CHAPTER 8

THE EXAMINATION

8.1 INTRODUCTION

8.1.1 Examinations and co-ordination

In South Africa part of the education system is controlled by the provincial authorities and another part by the central government. One of the most important co-ordinating bodies is the Joint Matriculation Board which has met regularly since 1918. The functions of this Board have not been confined to conducting examinations, but have also included the important work of drawing up syllabi for high schools. As a result there is usually a very close connection between the examinations controlled by the Board and the subject matter presented in the classroom.

On 1 January, 1963, the National Advisory Education Council began its activities. This body also acts in a co-ordinating capacity and operates over a wide educational field.

Meanwhile the examinations written at the end of Std. 10 continue to be of great importance and a considerable part of the discussion will therefore be devoted to them.

8.1.2 Classification

In this chapter the aim and value of the Mathematics examination are first discussed and then the examination is viewed in its relation to the pupil, the subject matter, the syllabi, the method of teaching, the textbook and the teacher. Attention is also given to the form of the examination.

Since not much of the new material which was brought to light as a result of the research work conducted will be included in this chapter, the discussion will be confined mainly to generalities. There is room for considerable basic research in this connection.

8.2 THE AIM AND VALUE OF TESTING

8.2.1 The testing and evaluation of the teaching and knowledge of Mathematics

The evaluation of imparted and acquired knowledge in Mathematics is a means which can contribute to the constant improvement of the teaching of mathematics. As soon as the teacher has determined what progress his pupils have made, he can modify the scheme of work in order to adapt it to new circumstances.

Evaluation is an integral aspect of teaching. It is admittedly time-consuming, laborious and sometimes also discouraging when it is found how little of what has been imparted has been absorbed, and yet it is an essential procedure which demands thorough attention in order to obtain the best from every pupil.

> "An effective evaluation program emphasizes three important aspects. First, there are the theoretical elements, involving the determination of objectives, content, method and organisation; the statistical procedure accepted for the treatment of test scores; and the technical qualities of the measuring instrument itself such as reliability, validity, and discriminatory power.
>
> "A second aspect of evaluation consists of such procedures as the writing of test items, recording and reporting of scores and interpreting of specific scores;
>
> The third aspect we emphasize is the establishment of a comprehensive evaluation program. Such a program should apply the theoretical and

practical aspects of evaluation to a specific school situation." (47, p. 1-2)

Where sufficient material and data are available, evaluation in the Mathematics classroom is a highly satisfactory experience for both the pupil and the teacher, provided matters are tackled in a conscientious and precise manner. The teacher must moreover be constantly on the look-out for improvements which can be effected in his evaluation programme, and he must continually endeavour to draw up newer and better tests. He must also make liberal use of Mathematics literature so that his own insight with regard to Mathematics can be deepened.

Evaluation is an essential part of the teaching programme and must take due account of the level already attained by the pupil. In the course of instruction attention should not be given mainly to the mechanical operations. The teacher should much rather determine the extent to which growth is taking place in the pupil's logical mathematical thinking. He should ask himself such questions as: Is there an improvement in the insight of the pupil into the subject and to what extent he is succeeding in solving problems by himself and, if needs be, in his own correct manner?

> "Evaluation, an essential of the mathematics program at every level, should be the handmaiden of instruction and learning. It is not a separate entity in a good school program. It may serve to improve the instructional program in the school, to enhance the effectiveness of the teacher, to aid the student in learning mathematics and to furnish valid data for research." (93, p. 7)

In the classroom evaluation contributes to an improvement in the methods of teaching. It can also be used to stimulate a competitive spirit amongst pupils. This rivalry can assume various forms. One of the most important of these is to allow the pupil to compete against himself. With each new endeavour he must try to improve on any previous efforts, to beat the school record, or to fare better than is indicated by the national norms. Competition of this nature also evokes greater diligence on the part of the individual pupil, stimulates his zest for work, and in general stands him in good stead. If he is then also given the opportunity to assist in the marking of tests and the correction of mistakes which have been made, his knowledge will be greatly increased and he will be put on his guard against making the same kind of mistake again. Incorrect methods of study are replaced by correct or more efficient methods.

The considerable expansion which is at present taking place in the field of Mathematics is compelling experts in the subject to determine accurately what work will be understood by the majority at every level. It is here that evaluation of the subject matter comes to the forefront. This work can be done thoroughly only by persons who have a sound theoretical knowledge of the principles of Mathematics and at the same time have also had thorough experience in the teaching of the subject. The teacher must have a deep insight into the conceptual ability of his pupils and their capacity to build upon acquired knowledge on their own initiative.

> "The mathematics curriculum is currently in a ferment. Conflicting claims are made for traditional and newer mathematics at the elementary, the secondary and the college levels. Decisions must be made about the topics included, the sequence of topics, and the immediate and remote goals of mathematical instruction. If these are to be wise decisions they must be based on experimentation and research. This calls for careful evaluation to furnish the information upon which final decisions may be based." (93, p. 13-14)

If these goals are to be realised, schools must keep proper records and carefully preserve the data for future use. In this way schools will be able to make a careful comparison between the work of new classes and that of classes of the previous year or even earlier years.

Evaluation brings important facts to light which can be laid before parents and used to examine the curriculum critically and, if need be, to improve it. It furnishes a survey of the progress made by the pupil in each subject at school, and the data are useful for finding out just where he fits in when he has to proceed to another school. Employers also frequently ask for particulars with regard to prospective employees, and if proper records have been kept, the desired information can be provided with little trouble.

Assessment also contributes greatly to the teacher's knowledge of a pupil's potentialities in Mathematics, his likes and dislikes, his social development, his emotional maturity, his determination to reach the top of the ladder, his health, and the knowledge which he has already acquired at a certain stage at school. All these things have an influence on the measure of success which he can attain in Mathematics. The observer even obtains an insight into the qualities which go to make up the character of the pupil and is enabled "..... to guide the student into study for which he is ready in terms of mental maturity, aptitude and background of experience and learning." (93, p. 15)

It is desirable that he should take the subject as far as his mathematical ability allows and to the stage where his interest can be actively maintained.

Where courses in Mathematics are conducted in different streams, an attempt should be made to place each pupil in the most suitable group. For this purpose use should be made of:

> "..... achievement test scores, aptitude test scores, achievement records, and teachers' judgement. Student attitudes are also important with regard to potential success in a given mathematics course." (93, p. 15)

The teacher must be constantly on the alert and remain fully informed of the level which the pupil has reached at every stage of his instruction and particularly of the extent to which the pupil has learned to think for himself. Once the pupil has learned to think independently, a very great victory will have been achieved, because he will then not readily shrink from the subject again.

It is of the utmost importance that pupils should have a good grounding in the numerous directions in which a knowledge of Mathematics is important. This will eliminate all the aimlessness in learning the subject and help the pupil to decide on his future course. If he sees his guiding star before him - even if this might be at some distance - he will steer his course towards that goal in a spirit of enthusiasm which would otherwise never have been possible. Fixity of purpose is inevitably a great incentive for the attainment of success.

A teacher should constantly evaluate his own work, his methods and the work of his pupils.

> "Evaluation can help answer such questions as the following: How much mathematics can various types of students learn? How much should they learn? What areas of study are most valuable to various types and levels of pupils? At what age can the gifted be identified and what might be done with them? How can a teacher best instruct for transfer at various levels?" (93, p. 16)

Evaluation has the further advantage that it is conducive to remedial teaching and also that it keeps the teacher informed of those parts of the subject which the pupil does not readily grasp. He can then concentrate on such parts in greater earnest in order to enable his pupils to understand them.

Careful observation can ensure that "the modes of work of one student will give a clue for helping those of another. Evaluation of the thinking and procedures employed by students usually is better done by careful observation and interview than by objective testing." (93, p. 17)

Objective tests can be used to evaluate the following: the quality of the syllabus, the achievements of the pupils, the methods of the teacher, and the advice and guidance given to the pupils.

"Evaluation is one of the most difficult tasks a teacher faces. No one will ever invent a magical way to arrive at a mark on a piece of paper which satisfactorily describes the achievement or ability of a student. Certainly in a course of this type evaluation is even more difficult than usual. There is no simple way to measure a student's power of understanding and his capacity to think. Clearly we must not base our judgments entirely upon computational skills and the ability to remember facts." (17, p. iv)

8.3 EXAMINATION AND THE PUPIL

8.3.1 The attitude towards Mathematics

In the investigation of the pupil's attitude towards Mathematics as a subject, both confidence and feeling play a role. Confidence is engendered in the pupil that he can master the subject, and, as he gradually masters this, he derives enjoyment from such realisation. By degress that which is mastered becomes more difficult, and the progressive victories which he achieves increase his love for the subject, instilling in him an intensified desire to reach still higher peaks. Corcoran and Gibb put the matter as follows:

"The student who feels that his attitude is well expressed by the statement: 'Mathematics thrills me and I like it better than any other subject' shows a stronger interest than one who says that his attitude is better described by the statement: 'Sometimes I enjoy the challenge presented by a mathematics problem'." (25, p. 106)

While the teacher must know for certain what the pupil's attitude towards Mathematics is, he must also consider the latter's attitude towards the various subdivisions of the subject and the intensity of his interest in it. Such interest will depend not only upon whether he finds the subject easy or difficult, but particularly on whether he has a sound idea of what the subject means to him at school and also what it can and will mean to him in later life.

It is important in this connection to realise that the student's attitude towards life as a whole also exercises an important influence on his attitude towards aspects which fall within this sphere.

It will greatly benefit the student if he can receive guidance which will enable him to analyse his attitude towards Mathematics. In this way he will obtain better insight into the general value of the subject.

8.3.2 Differentiation

The idea that a pupil should have a special aptitude in order to master Mathematics has often been denied in the past. On the other hand it is obvious that some pupils fare better in the subject than others. Some pupils express a desire to continue their studies of the subject after Std. 10, while others have strong feelings in the opposite direction. It is essential that there should be differentiation between these two groups of pupils.

A teacher who is the convener of the subject committee in his school, writes as follows:

"At present our students (throughout the country) leave school with a certain ability in factorising, solving equations, proving the truth of certain geometrical facts, etc. They do not recognise problems occurring in everyday life as soluble by these processes (where these are relevant) because the problems are not couched in mathematical language.

"The majority of school leavers (that is those who do not proceed to more advanced study) consequently never use their mathematical ability and in a short time they lose it, wasting all their previous efforts in this direction and those of their teachers.

"Had they been taught to solve problems, merely using mathematics as a tool, the picture would be very different.

"In this connection if we can develop separate classes - particularly for our potential university pupils - and let them follow different courses e.g. A, B and C as suggested - we should be able to accomplish very much more."

A woman teacher at a high school for girls in the Cape Province writes as follows:

"Most teachers of senior Mathematics classes would welcome any attempt to raise the standard. In the Cape Province, a revision of the present syllabuses, particularly from the point of view of the children who are gifted with mathematical ability, is certainly needed. At present, and even considering the improved syllabus which is now being introduced, they are in many schools being offered too little, too late.

"Experiments in the United States with a more stimulating and more rigorous course for the best pupils seem to be producing good results, both in interesting the children and achieving a much wider knowledge of Mathematics and ability to use mathematical concepts. These courses seem to make much use of the set notation, and include, for instance, discussion of the assumptions underlying the fundamental processes of Arithmetic. It seems unlikely however that these methods will appeal to or benefit those children who are not capable of going beyond school Mathematics. At any rate, until the new approaches and concepts have been thoroughly tried out with the better pupils, and until teachers are familiar with them, and convinced that they are better than the traditional methods, any attempt to use them with weaker classes would be disastrous.

"It follows that an improvement for our Mathematics depends on recognition of the necessity of separate streams in all Mathematical classes. The best pupils must be taught differently, at a different pace, and must be expected to take a different examination. Experimental textbooks might well be tried out with these classes."

A Mathematics teacher in the Cape Province writes:

"I am convinced that there should be at least two mathematics syllabuses for 8, 9 and 10. I will not repeat here the usual argument in favour on this, but tell you what we have been doing here since 1956.

"Although the best boys are put in one class in Std. 6 (on the basis of Primary Mental Abilities tests and their primary school record), all the Standard sixes do the same mathematics.

"In Std. 7, the top 35 or so mathematicians are put together. They do about the same work as the others, but their approach is more theoretical (e.g. they do far more and more difficult problems in geometry).

"In Std. 8 about the 25 best mathematicians are put together (some of these then come from a lower Std. 7 mathematics group). This group starts with Std. 9 mathematics and keeps up with the lower groups of Std. 9 even though they only do 5 periods per week of mathematics (as opposed to arithmetic) instead of 6.

"In Std. 9 the top class is reduced to about 20 if necessary, (our present Std. 10 class has been in three levels for mathematics since Std. 9. This has worked well, but uses more staff). With the top set we finish almost the whole of the Senior Certificate syllabus by September of Std. 9. For the next 6 months we then do university work.

"(Trigonometry: Sin (A+B), ratios of multiple angles, ratios of the general angle, radian measure, the squares formulae, the maximum value of a Sin x + b Sin y, much practice in manipulation; Calculus: up to the function of a function, much practice in curve tracing; Algebra: Proof by induction

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undetermined coefficients, arithmetic and geometric progressions).

"This system would work better if these boys had to write a different examination at the end of 9 and some tests in 10. I have been trying to get permission to let them write their final Senior Certificate mathematics examination in December of Std. 9 or March of Std. 10 so that they can then really get down to first-year university mathematics. It would then be possible to get the U.C.T.[1] to put them and boys who had done extra mathematics in Rhodesia and at Bishops and St. Andrews etc. in a special class. I have not succeeded so far, but the Transvaal experiment of dropping Matric in a few schools will give me some new ammunition.

"In the top set, there is a tremendous ability range .... The top boys do really outstanding work and at the end of the course it is easy to predict which boys will do well at mathematics at the university."

From the foregoing it is clear that some form of differentiation is desirable.

8.3.3 Differentiation and university admission

One form of differentiation applied in the past was that of the selection of pupils. Pupils who do well in Mathematics are encouraged to take this subject, while the other pupils are advised of all the advantages of the optional subjects. This form of differentiation sometimes has the disadvantage that at the end of his high school career a pupil makes the disillusioning discovery that he does not satisfy the requirements for admission to a university.

Unconditional admission to a university is granted by the Joint Matriculation Board to a candidate who, at one and the same examination, offers six subjects and passes in at least five, three of which must be the two official languages and a third language or Mathematics. If Mathematics is not chosen, a Natural Science must be taken and since 1960 this includes one of the following: Physical Science (Physics and Chemistry) or pure Physics, pure Chemistry, Biology, Zoology, Geology or Botany.

Table 8.1 gives an indication of the subject options necessary for admission to a university

TABLE 8.1

SUBJECT OPTIONS NECESSARY FOR ADMISSION TO A UNIVERSITY ACCORDING TO THE REQUIREMENTS OF THE JOINT MATRICULATION BOARD

Subject	1	2	3	4	5	6	7
1	Afrikaans	Afrikaans	Afrikaans	Afrikaans	Afrikaans	Afrikaans	Afrikaans
2	English	English	English	English	English	English	English
3	Third language	Third language	Mathematics	Mathematics	Mathematics	Mathematics	A Natural Science
4	A Natural Science	Mathematics	A Natural Science	History	Geography	History or Geography	History or Geography
5) 6) 7)	Two further subjects						Mathematics or a third Language

The fifth and sixth subjects can furthermore be chosen from a large range of subjects such as, for example, History and/or Geography (if not already taken as one of the first four subjects), another Natural Science, Additional Mathematics, a Bantu language, Portuguese, Hebrew, Agriculture, Domestic

[1] University of Cape Town.

Science, Bookkeeping and Commercial Arithmetic, Hygiene, Music, Art, Shorthand and Typing, etc.

The entrance requirements as set by universities agree in the main with those of the Joint Matriculation Board. Since each university offers its own variety of courses, however, the university itself also sets certain requirements for admission to study in particular faculties. (90, p. 13-14)

The requirements set by the University of Pretoria are the following (Translation):

> Mathematics is a compulsory matriculation subject for all first-year students in the Natural Sciences and Commercial Sciences. Examples of professions which are included in these two categories, are Architects, Quantity Surveyors, Biologists, Chemists, Dieticians, Physicists, Geologists, Home Economists, Militarists, Agricultural Scientists, Land Surveyors, Engineers (all branches), Doctors (physicians), Metallurgists, Radiographers, Statisticians, Dentists, Pharmacists, Veterinarians, Nurses (B.Sc.), Accountants, Executives, Economists, Meteorologists and Teachers of the Natural Sciences at High Schools. Mathematics is not necessary for the Teacher's Diploma in Home Economics and the Nursing courses (B.A.). (90, p. 14)
>
> From the foregoing it would therefore appear that in South Africa stress is laid not only on the two compulsory official languages but also on a third language, Mathematics and a Natural Science. (90, p. 15)
>
> We would like to associate ourselves with the policy of the Joint Matriculation Board and the South African universities because the basic subjects such as Mathematics, Latin, German, the mother tongue, the Natural Sciences and History are responsible for the general basic moulding of the child. As a result of their nature, contents, form and the method in which they are presented the basic subjects have a more formative value. General moulding of the pupil is essential for future academic study, and in the case of the matriculant who wishes to proceed to a university, it is disastrous to start with specialisation at school at too early a stage.
>
> For this reason de Waal (De Waal, H. L., Skakelblad No. 2, July 1956, U.P., 5) consequently utters a warning against a choice of subjects which include such subjects as Agriculture and Domestic Science at the expense of Mathematics or Chemistry. The basic subjects provide the matriculant with a general foundation for further study in any direction at the university. (90, p. 15)

A form of differentiation which causes Mathematics to be replaced by some other subject is fraught with risks in connection with admission to a university. For that reason it is essential to investigate other forms of differentiation.

8.3.4 Differentiation within the subject

As is known, two different public examinations are written in the Transvaal, namely the Std. 10 examination and the University Entrance Examination.

In 1962 a total of 10,006 pupils wrote these two examinations. Of this number 8,161 passed. Of the candidates for admission to a university, 1,429 passed in the first class, while the figure for the Std. 10 candidates was 384[1]. Pupils who succeed in obtaining a first-class pass in the Std. 10 examination, must surely have the ability to be admitted to a university. These 384 candidates were either too modest to take the university entrance examination or their choice of subjects was such that they could not qualify for admission to a university. Differentiation by means of two separate examinations apparently does not solve all the problems, and apparently there are gifted pupils who still suffer as a result of this differentiation.

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[1] Data obtained from the Transvaal Department of Education.

8.3.5 <u>Differentiation by means of an extra examination paper</u>

The possibility exists that differentiation could be applied in a modified form in which the pupils write the same Standard X examination. For those pupils who wish to proceed to a university an additional paper is set. In this paper the necessary stress can be laid on the mathematical concept and the other matters which university lecturers regard as important. The paper may also be subjected to investigation from year to year in order to determine its discriminatory capacity and its validity, in this way ensuring that the right prospective students are selected for further study of Mathematics.

In England it is also advocated that Mathematics should be taught as a <u>single subject</u> at school.

> "The best thing about the policy of the School Mathematics Project is that it is so firmly in favour of Mathematics as a single subject. In the decisions and recommendations made after the university inquiry last December, the Project has ratified its earlier proposals for a single subject A level, with even the special paper merely a more searching test of the same syllabus." (22, p. 1473)

It is interesting to note that here too mention is made only of an extra paper for admission to a university. The most important point is that the unity of the pupils as a body is preserved as far as possible. "One could criticise the actual syllabus chosen; but the principle is absolutely right."

This striving for unity even entails the abolition of a subject such as Additional Mathematics.

> "Can one not appeal to those universities which still require double mathematics as a condition of entry to think again?"

In South Africa, Additional Mathematics is not set as a requirement for admission by universities, and it has therefore hitherto not been a bone of contention. In England, however, strong feelings have been roused by this matter.

> "A requirement for double mathematics buttressed by physics may be a very good way of getting the first year work of a university mathematics department done at school, and of keeping down the supply of future mathematics teachers, but it is an appallingly narrow and abstract training for the mind of a schoolboy, and another science is educationally a far better bet." (22, p. 1473)

8.4 THE EXAMINATION AND THE SUBJECT MATTER

8.4.1 <u>A new approach to the subject matter</u>

The Joint Matriculation Board was established 45 years before the National Advisory Council for Education. The latter body is concerned with all aspects of education while the former, which is an examining body, was compelled from the outset to draw up syllabi as well. In this way the subject matter is traditionally determined by the examination.

Reports of an overhaul of education are being received from various countries: (Translation)

> In so far as the rate and quality of revisions are concerned, France stands in the forefront. This is made possible by the high scientific level of the French teaching of Mathematics.
>
> In the Scandinavian countries and in the Netherlands, government commissions have been appointed to study the modernisation of the teaching of Mathematics. This attention which is being given to the subject, relates particularly to the subject matter.

> It is a striking fact that in the various countries such great uniformity exists in regard to the proposals for the treatment of new subjects. Elementary set theory, an introduction to logic, certain subjects from modern Algebra, and introduction to the theory of probability and statistics are being widely encountered.
>
> It is being repeatedly pointed out that it is essential to adapt the language of Mathematics and the structure of school Mathematics to modern views.

Not only is the matter being studied and written about, but experimentation is also taking place.

> Germany occupies a leading position as a result of the numerous publications which have appeared on the strength of completed experiments. Specific attention is drawn to the relation which exists between the modernisation of teaching and the training of teachers. Shortcomings in the training of teachers unnecessarily increase the distance between school mathematics and modern mathematics. (45, p. 104)

In South Africa experiments in connection with examinations can to all intents and purposes be launched only by departments of education and only after the approval of the Joint Matriculation Board has been obtained.

8.4.2 A new approach to the subject matter in the primary school

No public final examination is written in the primary school. There is therefore nothing to prevent subject matter from being tackled in a determined manner at that level. In some countries an acquaintance is made with Algebra and Geometry as early as Std. 5.

> "Having studied arithmetic and geometry, mostly informally, in the elementary school, the student will be prepared for a sound treatment of geometry and the algebra of polynomials, beginning in the seventh grade. The mathematics curriculum for the secondary school can therefore go much further than it commonly does at present. The program of a student who elected mathematics each year will, at the end of the twelfth year, have contained a closely knit presentation of calculus, linear algebra, and probability, involving a brief introduction to other mathematical topics." (20, p. 197-8)

The high school can therefore build upon the work of the primary school with great advantage if provision is made to ensure that the work of the latter links up with that of the former. "It was felt ..... that is was desirable to adopt the 'spiral' approach, in which every new topic is introduced under low pressure and is then reconsidered repeatedly, each time with more sophistication, and each time showing more of its interconnections with the rest of the subject." (20, p. 198)

The teaching of Mathematics must be seen as an entity from the primary school to the end of the pupil's school career, and even as far as the university.

> "The key of the modern mathematical curriculum is that it develops consistently from the introduction to numbers in the junior school, through high-school mathematics, and on through modern university mathematical courses. Topics are not taught in isolation. Independent or isolated skills are not encouraged, and a by-product of the continuity of the programme is a maximizing of transfer." (62, p. 33)

8.5 EXAMINATION AND THE SYLLABI

8.5.1 Mention of examinations in the official syllabi

When a teacher examines official syllabi of the departments of education, he is immediately confronted with the examination.

One of the latest editions (71) reads as follows:

ARITHMETIC SYLLABUS

ORDINARY STREAM

JUNIOR CERTIFICATE 300 marks

GENERAL INSTRUCTIONS

(Here follow six examination instructions)

FIRST PAPER (30 marks) (20 minutes) etc.

Of the ten inches of print on the first page, approximately three and a half inches are devoted to subject matter. The rest of the page is concerned with the examination.

On the following page a considerable amount of subject matter appears, but also instructions such as the following are given:

"(N.B. Questions relating to profit and loss must clearly show whether costs have to be taken as part of the buying price or whether they are to be subtracted from the selling price.)

(N.B. Unelapsed period only in full months; no specific dates will be given.)

(A knowledge of income-tax questions will not be set.)

(Questions on National Loan Certificates and questions requiring a knowledge of current rates of interest, e.g. those of the Post Office Savings Bank, will not be set.)"

In this "syllabus" not a word is said about the object of teaching the subject or about any concept or concept formation. The examination is so strongly stressed that one could much rather speak of an examination plan than a syllabus.

Some of the other syllabi contain not only the subject matter but also clear instructions addressed to the examiner in regard to what he may do and what he may not do. So, for example, "An examiner may not require a candidate to give the proof of a theorem marked with an (x) or the converse of any of the following theorems." (101, p. 34)

8.5.2 Discussion

Something to be guarded against is the possible domination of the syllabus by the examination. When the syllabi are published, this should preferably be done with less reference to the examinations concerned. The space occupied can be used to much better advantage by setting out the aims of the work, together with reasons for their importance, and also by indicating the concepts which should be stressed. One or two months before an examination is written, instructions and other directions in connection with the examination for that particular year can be sent to schools.

There are departments of education which mention the aims in the published syllabi. In some cases a more consistent separation of the syllabus from examination instructions could be adopted in order to divest testing of its present primary position.

To expect a teacher to live under the cloud and strain of an examination from the first day of school to the last cannot be conducive to satisfactory teaching since his freedom is restricted unnecessarily.

> "And secondary teachers had less freedom still, hamstrung as they were by encroaching examination pressures." (92, p. 852)
>
> "Without knowing that there is such a thing as a 'modern' mathematics curriculum for high schools, it is possible to be dissatisfied with the state of affairs in the South African school system.
>
> "Fortunately there is a germ of discontent among the more enlightened mathematics teachers and, given time, there is hope that some necessary changes will be made." (62, p. 31)

This stressing of examinations does not pass unobserved by the public. At a public meeting held on 27th May, 1964, Dr. T. van der Walt dealt with the features of our present times. One of these, said Dr. van der Walt, is that we are ensnared in an almost insane pursuit of achievement on the field of sport, in student life, in our studies, and in the community. (Translation) Life is really one unending examination fever. As a result we become 'jittery' and discouraged. And all this is varnished with a thin coating of pleasure as motive. (105, p. 7)

8.5.3 A possible first step in the revision of syllabi

It is clear that the revision of syllabi must be tackled with circumspection. Dr. V. E. McGee, who is a zealous protagonist of educational reform and who knows South African conditions well, suggests the following:

> "In view of the present situation, it would be impractical to suggest a completely new syllabus for all high schools. Instead, it is suggested that a new topic, set theory, be introduced into the mathematical syllabus, starting with the matriculation class and working downwards, rather than starting in the lowest high-school class. As a separate topic for matriculation, the student would be able to modify what he has previously learnt (for instance, in solving simultaneous equations) in terms of what is learned in set theory. It would introduce the teacher to the new topic gradually and would not require a complete change of approach at one time. Included in the new topic would be the elements of logic and the notion of truth tables." (62, p. 34)

8.5.4 Evaluation of the development of syllabi

Evaluation is the keystone of research. At the present time considerable research is being done to discover what should appear in the mathematics syllabi for the various classes. Serious efforts are being made to find at least a small place in the syllabi for the enormous development which is at present taking place at a tremendous rate in the field of Mathematics. A great amount of work is necessary in order to determine what part of the new subject matter falls within the conceptual ability of pupils, but the subject is so important that no trouble should be considered too great in order to reach the envisaged goal. Much of the old material which is not conducive to reasoning because the work connected with it is purely mechanical can readily be omitted and be replaced with work which stimulates the pupil's zest for work and his desire for progress. Such work will really be of greater benefit and much more enduring value to the pupil.

In an analysis which is being made in 14 states and in 18 schools and school systems in the U.S.A., Robert Kalin points out the following things to which evaluation techniques can be applied with advantage:

"1. Selection of students for accelerated classes
2. Guidance for ninth grade mathematics
3. Test selection
4. Test construction
5. Item construction
6. Diagnostic tests

7. Diagnostic check lists
8. Records and reports in guidance
9. Administration of an evaluation program
10. Prediction of success in high school mathematics
11. Evaluation of curriculum development." (60, p. 146)

8.6 THE INFLUENCE OF EXAMINATIONS ON THE METHOD OF TEACHING

8.6.1 The drill method

According to Table 6.13 the teachers are of the opinion that the kind of examination set for the school-leaving certificate encourages the drill method. Certain drill work is good and beneficial, especially where it is used to drive home the basic concepts and principal processes. Excessive drill work aimed at the answering of certain types of examination questions does not bring about much of a positive or permanent nature and must be condemned for that reason.

One method of combating these abuses is not to allow teachers to have access to the examination questions. To effect this, all examination questions should be handed in again after the examination and teachers should not be allowed to see them. This will mean that the examination must be conducted by an examining body which can manage without assistance from teachers. The feasibility of such a system in South Africa will not be considered now. In some countries this system is already applied with success and the teacher is consequently much less examination-conscious and is compelled to concentrate his attention on concepts and basic processes.

> (Translation) Our teachers will, as was formerly the case, again have to be prepared to make wall charts. We shall have to pause longer when we come to a new concept; we shall have to make use of the hints and experience of others; we shall have to read more on the method of teaching the subject; we shall have to give much more and have to rest much more often along the road, and I am sure that after many days we shall find ourselves again and also suffer less from nerves. (106, p. 353)

8.6.2 The advantages of testing

Daily evaluation of the progress of a pupil's work in Mathematics can be recorded in a subject record book. All other information relating to this can also be entered under various headings. In due course the school will possess a cumulative record of the work and capacities of the pupil concerned. If the pupil's achievements are followed up in this manner for a number of years, the record will provide a good clear indication of the pupil's ability. Whether figures or letters or single words are used to describe the phase in the pupil's life, is a matter of minor importance. The main thing is that the persons who have to undertake the task of interpreting the symbols, will have a very good idea of what each symbol is intended to convey to the interpreter. The interpreter must be strictly objective and must under no circumstances attempt to read into the symbol what is not so intended.

Increasing use is being made of electronic machines when records are compiled. The punch cards are then used for the compilation of data in regard to individuals, no matter what the object may be.

In practice the evaluation should serve to provide information about pupils in a broader and a narrower sense. Where information is necessary in a particular direction, the testing should ensure that the tests will throw light on that particular field. The nature of the tests must be adapted to the situation in regard to which information is required. Use can be made of routine tests, of compositions dealing with specific matters, of the interrogation method, of protracted observation, of the execution of technical instructions, etc., according to what information is desired.

A careful record of the information collected must be kept, and this must be filed in such a manner that it is readily accessible. Persons who will need it are the guidance teacher, the school principal and research workers in the field of tests.

The information obtained is used to keep parents acquainted with the progress of their children, to obtain admission to higher educational institutions, to enlighten employers on the potentialities of prospective workers, and also to furnish data to government departments which wish to make use of the information for research or other purposes. It is an art to sort out the data which must be given to various bodies, things which are important to a college are often of little or no value to an employer.

When evaluation is practised, one must make sure that the item is capable of evaluation. Without evaluation not much can be learned about performance, or at least about its quality. Evaluation makes it possible for considerable light to be thrown on the attitudes and behaviour of a pupil. In this connection Robert S. Fouch says: "In evaluation we gather information about student behavior; therefore, in order to evaluate achievement of objectives we need to know which kinds of behavior we should observe. An especially important example, in this time of new trends and emphases in mathematics education, is any objective that refers to understanding. Understanding may be interpreted as a kind of behavior, but for any sort of evaluation we probably need to break this down into directly and easily observable kinds of behavior. We need to ask, for example, how we can recognize the presence or absence of understanding the associativity of operations. How can we decide that one student has more or less understanding of this concept than another? We cannot simply ask him to state the associative property of multiplication, because we would probably be testing only rote memory. Then what can we do? There are, of course, levels of understanding of a concept to be considered." (37, p. 169-170)

The better a matter is understood, the higher the level at which the matter can be entered into, and the more complicated the variety of applications which can be made.

Testing of a concept is a very difficult matter and for that very reason it is a challenge to teachers to devise plans for its evaluation. Thorough records of these findings must also be kept, even if they are perhaps to a very large extent subjective.

8.7 EXAMINATIONS AND THE TEXTBOOK

8.7.1 The value of textbooks

In certain countries, especially the U.S.A. and Canada, the writing and publication of new textbooks became the spearhead of the movement towards a reform in the teaching of Mathematics. The textbooks were introduced into the schools and the results are eagerly awaited.

> "When the 1960's began the new mathematics programs had evolved to the point where textbooks were being produced.
>
> "As the persons charged with determining the secondary Mathematics curricula and purchasing the instructional materials for Mathematics in the many schools of the country surveyed the situation, they found an extreme lack of experimental evidence upon which to base their decisions. Should they accept the expressed opinion of authorities or follow the easy road of continuing in the patterns of the past?" (115, p. 495)

Testing of the pupils concerned is the proper way to determine whether the textbooks have brought about a change for the better. Use will also have to be made of control groups. What must be taken into account, however, is the fact that testing and examining cannot be separated from textbooks. The writers of textbooks will always have to think of the examination. This also applies to the authors of the most modern textbooks.

> "The authors of this book have been noticeably influenced by the Commission

on Mathematics of the College Entrance Examination Board and by the School Mathematics Study Group. Both of these groups have done an outstanding service in focussing the attention of the country on the need for changes in the high school mathematics curriculum." (48, p. iv)

It should be strongly recommended that the examining body (in this case the Joint Matriculation Board) should issue a publication on the policy in connection with a reform of the teaching of Mathematics so that prospective writers and publishers of textbooks can obtain some guidance.

"The order of topics in a great number of textbooks is Substitutions, Addition of Like terms, Brackets etc., and it is not until Chapter 10 or thereabouts that Problems were reached. Thus when problems are reached they are often found to be difficult, but the way examinations are set it is quite possible for a pupil to omit them and still pass on the manipulative work which means little, if anything to him.

"I feel that a more direct approach to the essential stuff of the subject is desirable ....."

There is therefore a need of textbooks which will offer much more than sets of old examination questions!

8.8 THE FORM OF EXAMINATIONS

8.8.1 The public examination

The procedures in connection with the present Matriculation and Matriculation Exemption Examination will now be examined more closely. Since most of the facts relating to the manner in which this examination is applied are well known, the searchlight will be thrown on the moderator for Mathematics who is appointed by the Joint Matriculation Board. Seen in proper perspective, the problems which have arisen in connection with this mode of examination may stand out more clearly. A member of the committee who was himself for many years a moderator for Mathematics furnished information which made a considerable contribution to this part of the report.

8.8.2 The present state of affairs

Each department of education has its own syllabi, its own examiners and its own examination papers. The only exception is South West Africa, but the Joint Matriculation Board also has its own syllabi, examiners and examination papers. There are therefore six different examinations.

8.8.3 Matriculation Exemption

In order to confer exemption from Matriculation, the departmental examination papers must meet with the approval of the moderator of the Joint Matriculation Board.

Since two examination papers per examination must be written for Mathematics, this means that the above-mentioned moderator must express his opinion on twelve examination papers for the examination at the end of the year and then a further twelve for the supplementary examination in March. Although it is not altogether necessary, the examination paper for the Transvaal Std. 10 course is also submitted to the Moderator.

8.8.4 The task of the Moderator

The moderator is now expected to determine whether the various bodies are maintaining the same relative level and whether the standard is the same from year to year.

To draw up equivalent examinations and tests is of course not impossible. This however demands a very high degree of professional schooling, considerable time and patience, and the opportunity to test hundreds of items (examination questions). The drawing up of three equivalent forms of the test is in itself a very great achievement and it seldom happens that a person succeeds in drafting more than five such forms. A careful item analysis must first be carried out through the application of preliminary tests before the composition of such equivalent tests can be tackled.

In addition differences in language must also be taken into account. The Afrikaans text of the Geometry syllabus of Natal will for example not be easily understood by a child in the Transvaal. Expressions such as "concurrent lines" may therefore not occur in the examination papers in other provinces.

In reality therefore the moderator has to assess not twelve examination papers per examination but twenty-four, since every paper has to be drawn up in English and in Afrikaans, and even in Mathematics differences in language can play an important role.

It must also be borne in mind that the syllabi of the various departments are not identical in all respects. Material in use in one province may not be offered in the neighbouring provinces.

To draw up two sets of twelve equivalent examination papers in this manner is virtually an impossible task. Even a bureau of full-time professional persons would find it difficult to do so, even though they had every opportunity to conduct preliminary tests and also had the most modern apparatus at their disposal.

The task of the moderator is therefore even more difficult. He does not draw up the papers himself. The examination papers are drawn by different examiners in far corners of the country and the moderator must ensure that in one way or another equivalence is reached. There is no opportunity for pre-testing and consequently also no item analysis. Nobody knows what the relative degree of difficulty of the different items (examination questions) is. And still less is anybody able to give an indication of how the relevant items distinguish between good and poor candidates. For these all-important examinations the moderator must rely upon his knowledge, experience and innate intuition.

As far as is known, the error of measurement of such examination papers has not yet been determined but this has been done for standardised tests and an error of measurement of four points in a hundred frequently occurs. In the past however the moderators have inspired so much confidence that the Joint Matriculation Board was able to decide to raise the minimum for university admission from 40% to 45%. Despite the fact that there must in the nature of things be new failure rates, the moderator should necessarily be attuned to change and yet ensure that the standard of the examination papers remains unaffected both severally and jointly.

8.8.5 The modus operandi of the moderator

Although each examination paper is accompanied by a memorandum, the moderator must himself answer each question in all the papers fully and carefully and do all the calculations required in the questions in order to -

(i) eliminate the mistakes of the examiners, and
(ii) be able to express a sound opinion (while at the same time confining himself to the horizon of experience of the Std. 10 pupil).

With an average of 10 questions per examination paper, the figure amounts to 240 questions. In addition, correspondence on a fairly considerable scale with the 24 examiners then also sometimes follows. In such correspondence other proposed amendments are dealt with, difficulties in connection with the terminology are ironed out, and misunderstandings in connection with the correct use of language are eliminated.

Thereupon the examination papers must once again be scrutinised in their final form and the English and Afrikaans versions must be compared very carefully and meticulously. The final step is the proof-reading which must be done once or twice by both the moderator and the examiner.

8.8.6 Reviewing of the scripts

(a) As soon as the examiners have completed the marking, 20 scripts in respect of each examination paper are sent to the moderator (who must furnish a list of his holiday addresses in good time) so that he can determine whether the allocation of marks by the examiner (and numerous sub-examiners) is correct, uniform and of the right standard. The moderator must therefore study the examination papers very intensively and ensure that his judgment is maintained. This work embraces the scrutiny of

12 test packets of 20 each in December and
12 test packets of 20 each in March.

(b) Then the moderator's assistance is called in where he has to decide upon borderline cases.

(c) Ultimately the moderator is an appeal court because in all cases of appeal he must carefully judge the scripts and determine the final marks.

8.8.7 The checking done by a moderator during a particular calendar year

The data furnished by a person who has himself acted as moderator for some considerable time, are shown in Table 8.2. One examination script may of course consist of several examination booklets.

TABLE 8.2

THE NUMBER OF EXAMINATION SCRIPTS CHECKED BY A MODERATOR DURING A CALENDAR YEAR

Examination	November/December				March			
	Test	Border-line	Appeal	TOTAL	Test	Border-line	Appeal	TOTAL
Joint Matriculation Board	40	84	80	204	40	14	10	64
Cape Province	42	0	58	100	40	0	2	42
Natal	40	4	136	180	40	0	4	44
Orange Free State	40	40	28	108	40	0	6	46
Transvaal	60	0	342	402	54	0	60	114
National Senior Certificate	40	0	114	154	40	0	6	46
TOTAL	262	128	758	1148	254	14	88	356

The grand total is therefore

November/December	1,148	
March	356	
	1,504	examination scripts

The moderator had therefore to judge 1,504 examinaton scripts according to 24 examination papers (each in both official languages) amongst which they were divided.

8.8.8 Reports and complaints

When an examination programme has been completed, the examiners draw up their reports on the work submitted, and it is the moderator's duty to check these reports.

After the examination has been written, the teachers concerned generally go over the examination papers and they have the right to lodge complaints. Any complaints received are referred to the examiner and moderator, each of whom then draws up and submits his reply. In this manner pressure is involuntarily exerted on the examiner and moderator to tread the old proven paths.

8.8.9 Occasional assistance

The moderator (for any subject) is generally a senior member of the staff of a university, the head of a department, and in addition possibly also Dean of a Faculty. He is charged with the work of examining his own students and perhaps also, as external examiner, with those of some other university. Time must moreover be found to undertake the work of moderating for the Joint Matriculation Board. It may rightly be asked whether the time of a person of this calibre could not be spent in some better way. There are much greater tasks which await Mathematics pedagogues with wide experience of teaching not only in secondary schools but also at universities.

8.8.10 The marks awarded by examiners

Examining and the way in which this should take place has been receiving attention for many years. In 1931 a conference was held in Eastbourne at which England, France, Germany, Scotland, Switzerland and the United States of America were represented. An investigation was instituted into the manner in which marking is done in certain examinations. The conclusion at which the commission arrived is that the reliability of marking left much to be desired. The type of examinations set at that time made it difficult for the examiners to maintain the same standard individually and in relation to one another.

After a careful study the committee came to the following conclusion: "It is only by careful and systematic experiment that methods of examination can be devised not liable to the distressing uncertainties of the present system." (41, p. vii)

8.8.11 "Modern" examinations

Since that time various bodies have succeeded in drawing up tests the reliability and validity of which can be accurately determined and which have in course of time been greatly improved. The tests are objective in this respect that the personal judgment of the examiner cannot play an important role. The questions and answers are shorter and the memoranda and answer keys are consequently simpler. This does not connote that the answering of the tests has necessarily become easier.

In the United States of America the "modern examination" has been applied with so much success that other countries which initially were very critical towards it, are now displaying increasing interest as is apparent from the following quotation:

(Translation) Every year a test is drawn up by the Mathematical Association of America and The Society of Actuaries and this is then submitted to pupils of American high schools. The object of this test is inter alia to distinguish the exceptionally gifted pupil. This object is not achieved if the test is drawn up in such a manner that a not inconsiderable group of pupils all obtain approximately the maximum score. The examination questions have consequently been chosen in such a manner that one obtains a proper distribution and that it is almost impossible to obtain the maximum number of points.

The twelfth annual test set on the 9th March, 1961, ... consists ..... of 20 questions for each of which a total of 3 marks can be obtained, 10 questions for each of which a total of 4 marks can be obtained, and 10 questions for each of which a total of 5 marks can be obtained. Next to each question 5 answers are given, and of these only one is correct. The pupil receives a separate sheet of paper on which he tries to give the correct answer to each question by writing down A, B, C, D or E. In this way it is possible to obtain marks by guessing. To compensate for the effect of random guesswork penalty marks are given for every incorrect answer, the penalty marks being $\frac{1}{4}$x3, $\frac{1}{4}$x4, $\frac{1}{4}$x5 respectively. The final score is obtained by reducing the number of marks gained by the number of penalty marks. (110, p. 136)

This particular test is being applied to an increasing extent in England and the Netherlands, while this method of testing has already been adopted fairly generally in the United States of America.

The principal advantages of this form of testing is the fact that the answers can be marked very easily, guesswork is discouraged, and the reliability and validity indices can be calculated with relative ease. It is also possible to calculate the difficulty value, the discriminatory value and even the validity of every item (examination question).

8.8.12 The procedure in drawing up tests

The drafting of a test or examination satisfying the foregoing requirements is a considerable and comprehensive task.

M. L. Hartung defines evaluation as follows:

"Evaluation is the process of finding the extent to which the actual experiences conform to the objectives." (42, p. 22)

It is necessary to notice the difference between evaluation and measurement. Measurement is a means which helps to make it possible to evaluate. Measurements at intervals provide an opportunity to determine whether general progress has taken place. The progress is evaluated in order to ascertain whether it is satisfactory and whether it can be improved upon.

If evaluation is to be made possible, there must be an aim of which both the teacher and the pupil are aware and which they must strive to realise in a purposeful manner. In the second place examples must be set which can serve as guides, and in the third place proper records must be kept of what has been achieved in the examples, stress being laid on the results of personal initiative. In the fourth place it is necessary that the records should be studied scientifically in order to draw conclusions from them. All this leads to the application of the findings in order to modify the curriculum when necessary and to adapt the method of working to the experience gained.

It is unlikely that the situation will after this be one hundred per cent perfect. The process must be continued to eliminate any remaining shortcomings.

When measurement is applied, steps must be taken to ensure that the tests are as objective as possible and that the standard of reliability is high. The drafting of such a test is a difficult task, especially if, in addition, it has to be relatively short. After such measurements have been completed, the question of interpretation arises. Interpretation must be undertaken only by persons with a thorough idea of psychological measurement. The results must then serve to give the pupils guidance individually and in groups in regard to what is expected of them and the extent to which they are able to meet the demands made upon them. The information obtained is also used to effect desirable amendments of the curriculum.

To be of real value, the valuation programme must (i) be comprehensive. While it really tests knowledge and skill, there should also be an opportunity to determine the pupil's interest, value appreciation and ability to apply his knowledge in new situations. (ii) The programme must be balanced, i.e. the same emphasis must be laid on each subdivision. (iii) The programme must moreover be subjected to systematic evaluation. By this is meant that the tests should be given at regular intervals. A comparison of the results will make it possible to determine whether and to what extent real progress has been made.

J.C. Merwin distinguishes between the following three steps in the drafting and use of a test:

> "1. Planning the test; making decisions about the course objectives to be covered, the type of items to be constructed, and the number of items to be used for sampling each of the objectives.
>
> 2. Preparing the situations to be presented to the students, or item writing.
>
> 3. Organizing, administering and scoring the test.
>
> These three steps should be designed to yield reliable, relevant and objective information about the achievement of students." (63, p. 44)

For Mathematics the co-operation of all the teachers charged with mathematical work must be secured. They must ensure that no test question contains the slightest suggestion of what the answer actually is. If the slightest clue is given, the test result cannot be accepted as reliable. This also applies to cases where questions have a tinge of ambiguity.

A well constructed test provides information in regard to the standard attained by the class and the level attained by each individual in it, this being the criterion of the quality of the test.

The tests are as a rule of such a nature that they throw light on the things which can most easily be measured, as, for example, basic skill and a knowledge of mathematical relations. It is even more important however to ascertain whether there is growth in the logical reasoning capacity of the pupil, his creativity, his constructive attitudes and his good habits. Tests must be drawn up in such a manner that light is also thrown on the development of the characteristics mentioned.

Here follows a list of the things on which Sobel and Johnson would like light to be thrown. They say -

> "The student should :
>
> have a knowledge and understanding of mathematical processes, facts and concepts;
>
> have skill in computing with understanding, accuracy and efficiency;
>
> have the ability to use a general problem-solving technique;
>
> understand the logical structure of mathematics and the nature of proof;
>
> use mathematical concepts and processes to discover new generalizations and applications;

recognize and appreciate the role of mathematics in society;

develop study habits essential for independent progress in mathematics;

develop reading skill and vocabulary essential for progress in mathematics;

demonstrate such mental traits as creativity, imagination, curiosity and visualization;

develop attitudes that lead to appreciation, confidence, respect, initiative and independence." (89(a), p. 72)

This is a weighty list but worthy of being set as the ideal in the teaching of Mathematics and the evaluation of what pupils have attained. Unfortunately most tests cover only a meagre part of the whole field and yet a greatly exaggerated value is attached by inexperienced persons to results yielded by such tests. There are however factors such as attitude, appreciation, and habits which cannot be measured objectively. Progress in this sphere must be determined by careful observation.

It is of the utmost importance that teachers who stress the great value of the teaching of Mathematics will ensure that adequate time is spent on doing justice to the important process of evaluation.

8.8.13 The procedure followed in the drafting of a particular set of tests

S.S. Myers compiled and published the following report in connection with the construction of certain Mathematics tests by the Educational Testing Service, Princeton, N.J.:

"By the early part of 1959, an advisory committee of ten members had been established to help in the development of test specifications. Some members of the committee had expressed apprehension at developing new achievement tests in mathematics at a time when so much development of new mathematics curricula was taking place. It was felt, however, that since the entire test development process required about four years, work on the new tests should proceed, even though compromises would be necessary now and revisions of the new tests needed several years later.

"By the summer of 1959, the specifications for 11 new tests (Arithmetic, Structure of the Number System, Algebra I, Algebra II, Algebra III, Geometry II, Trigonometry, Analytic Geometry, Calculus I and Calculus II) were developed with the indispensable assistance of the advisory committee. Consideration of three other tests - Probability and Statistical Inference, Elementary Analysis, and Finite Mathematics was postponed until a later date.

"Since the Arithmetic Tests involved three parallel forms, and the others, two parallel forms each, a total of 23 new forms was planned. Two pretests for each of these forms were scheduled, making a total of 46 new pretests to be written by 46 item writers, for a total of over 1850 questions. These writers began work during the summer of 1959 after an elaborate kit of instructions, sample items, and test specifications had been sent to each. From November, 1959, to March, 1960, the Mathematics Section of Test Development Division at ETS selected and reviewed 1850 items for 46 new pretests which were tried in a selected national population of schools in May, 1960. Each of the pretests was reviewed by four competent mathematics teachers.

"In our zealous efforts to modernize and up-date the content of these end-of-course tests, we learned to our dismay that the pretests were too difficult for our sample of schools, with mean scores averaging about 35 per cent of the total possible scores. After much careful discussion and planning, it was decided to prepare, as a first step, final forms only in Arithmetic, Algebra I, Algebra II, Geometry I, Geometry II. The purpose of this was to see whether it was possible to select appropriate items from the pretests and revise them so that forms appropriate in content and difficulty could be constructed for pretesting and norming in May, 1961.

"After may revisions, the Algebra and Geometry Tests were given a quick, experimental try-out in two large high schools in metropolitan Philadelphia, in January, 1961. This try-out revealed that the forms still required drastic revision to make them appropriate in difficulty. After another thorough overhauling which involved the writing of many new, more appropriate questions, Arithmetic, Algebra, and Geometry were pretested during May, 1961. These tests were administered to students in three selected school populations - suburban, urban, and rural. The results justified our labors and the Cooperative Test Division is now proceeding with the publication of the following tests: Arithmetic, Forms A, B and C; Algebra I, Forms A and B; Algebra II, Forms A and B; and Geometry, Forms A and B (the original two levels for Geometry are represented by Parts I and II in the final forms)." (66, p. 224-5)

8.8.14 Comments

The existing South African examination system is cumbersome and consequently results in high costs.

The procedure followed to ensure a uniform standard does not satisfy the requirements set.

The examiners and moderators must do the examining work in their free time on a part-time basis, while some departments of education already have facilities for test services.

"The system of testing in South Africa is not flexible and does not allow individual interests to develop. It should also be criticized for not allowing any research to be done on the way in which particular questions are answered by different matriculation classes over the years. For instance, there is no hope of ever reaching the conclusion that the matriculation candidates in 1960 were better students than those of 1955." (62, p. 33)

8.8.15 Possible solutions

(a) A controlled internal examination system: The Transvaal Department of Education is at present planning an experiment which, if it succeeds, may eliminate most of the evils of the present system. Sooner or later internal examination systems give rise to a need for an external test. The need for outside criteria becomes increasingly apparent with the passing of time, this being the reason why such a need exists.

(b) Objective Mathematics examinations

"The recommendation here is that a start be made in developing objective examinations for mathematics for matriculation. This is a more flexible system of examining than the present one and would enable research to be conducted into the content of the mathematics examination. The development of questions for such objective tests would require the active co-operation of university professors and school teachers, something which cannot be recommended too strongly." (62, p. 34)

8.8.16 The necessity of a solution being found

The reason why the organisation of the present examination system is absolutely imperative is that (translation) impossible demands are made upon the moderator under the existing system. This alone is sufficient reason for characterising the existing practice as impractical, inefficient and time and energy wasting.[1)]

[1)] Observations of a former moderator.

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8.9 EXAMINATIONS AND THE TEACHER

8.9.1 Enthusiasm for testing

T. Sampson, a teacher at Oakville, Ontario, Canada, writes as follows:

> "I like tests and I feel that my classes are beginning to not dislike them. They have so many that they could not keep up any strong dislike all the time. Less than half of the tests are for term marks and I give no warning, so there is no cause for stagefright. Some of the tests are unusual enough to be interesting. And, most important, many students are becoming aware that the tests are for their benefit and information more than for mine. (In some cases, I have the students trade papers to mark, and I don't collect the papers afterwards).
>
> "My enthusiasm for tests started as a lack of enthusiasm when I was tired of making them up each year and when I bogged down with reading through piles of answers. I decided to make a set that could be duplicated and used over again each year. The tests were planned systematically for the whole year. Each test had three or four questions on the topic just recently covered and two or three questions on other topics for review. Since I give no warning and do not use the tests in exactly the same order with all classes, I do not worry about anyone memorizing the solutions ahead of time. In preparing the tests, I tried wherever possible to set them up in such a way that a mask could be used to mark the papers easily and quickly. Only one or two questions were used to test good form or the ability to work through a detailed solution.
>
> "In the course of preparing these tests, my lack of enthusiasm disappeared and tests were no longer a means of getting a mark to pass into the next grade or a signal to the student that he had learned enough about that topic and could now forget it. Tests became introductions to a new lesson." (84, p. 117)

Every teacher can make testing a source of pleasure if a new approach is possible.

> (Translation) Rigid examination requirements as well as rigid interpretation of flexible examination requirements have a very strong stifling effect on the reform of teaching. (102, p. 19)

8.9.2 What should be tested

When tests are conducted, four factors inter alia must be borne in mind, namely (1) the manner in which the pupil's knowledge has expanded, (2) the extent to which insight has been inculcated, (3) the increase in the pupil's skill in approaching and dealing with the questions set, and (4) how the pupil succeeds in marshalling his reasoning.

In his lesson the teacher must constantly bear these four factors in mind so that he can stimulate the pupil's development in this field.

> (Translation) Every effort towards a reform in education and instruction is doomed to failure if we proceed from the assumption that education and examination or test programmes cannot be amended. (118, p. 223)

CHAPTER 9

THE TRAINING OF MATHEMATICS TEACHERS

9.1 INTRODUCTION

After a long period of stagnation mathematics teachers are now faced with an important task, namely the radical revision of the teaching of Mathematics in high schools. "The present program of mathematics education, despite calls for reform, is for all practical purposes more than a hundred years old. Very few new ideas have been added since 1900 A.D., and there has been no real shift in direction." (35, p. 15)

In the following chapter an indication is given of the great shortage of Mathematics teachers in 1962. In so far as the future is concerned ..... "there will be an increasing need for qualified teachers of mathematics at all levels of instruction." (76, p. 141)

The question which now arises is the manner in which these new teachers should be recruited. Is it possible to expect that their qualifications should be as high as in the past or even higher, or must we be satisfied with a lower qualification for Mathematics teachers?

The success of Mathematics instruction at school rests particularly on the competent teacher. For that reason the key to the teaching of Mathematics is the training of teachers who are properly qualified in their subject.

> "Anyone who believes as I do, that the complaints are just indeed, is inevitably led to consider the education of teachers; for it is obvious that if their preparation is faulty changes must be made in the education of teachers before any substantive ones can be hoped for in public schools." (54, p. 3)

The question, however, is: When can such a teacher be regarded as properly qualified?

> "To obtain answers to these queries, it is necessary to know
>
> (a) what conditions exist in the training of mathematics teachers, and
> (b) what needs will have to be supplied." (76, p. 141)

9.2 PRINCIPLES FOR THE TRAINING OF THE MATHEMATICS TEACHER

An American, Turner, has drawn up an important list of principles for the training of Mathematics teachers.

(1) Future Mathematics teachers must take a thorough course in higher Mathematics.

(2) This training must be given at a university or a college by persons who are themselves competent mathematicians, but who, in addition, also understand and appreciate the inherent difficulties of Mathematics in the teaching and learning process (Klein in Germany, Perry in England, Young in America).

(3) Mathematics teachers must make a study of the important branches of pure Mathematics, Mechanics, the History of Mathematics, the applications of Mathematics in other fields, its logical foundations, and especially the essential connection between the various branches of advanced Mathematics.

(4) During his period of training the Mathematics teacher must also make a somewhat less intensive study of an additional subject, preferably a subject related to Mathematics.

(5) During his period of service as a teacher he must strive not only to become acquainted with other parts of Mathematics, but also to master them.

(6) A period of professional training as a teacher of his subject after his academic course is absolutely essential.

(7) The contents of such a professional course of training must be concentrated mainly on showing teachers how to teach the subject Mathematics in the best possible manner.

(8) The same applies to the second subject.

(9) The period of professional preparation of Mathematics teachers should include ancillary courses in the theory and practice of Education and Psychology.[1)]

The question is whether the teachers who teach mathematical subjects in South African high schools meet these requirements. In Europe this is not the case. "Most countries today face the serious problem of how to replace or upgrade the 20 per cent who are not fully qualified." (76, p. 150)

9.3 THE TRAINING OF MATHEMATICS TEACHERS IN OTHER COUNTRIES

9.3.1 Europe

"The mathematical study required to teach in the upper cycle of the secondary school in Europe may be stated as follows:

"The teachers must hold a four-year diploma from the department of pure science or mathematics of a university. The acquisition of this diploma usually takes five or six years; rarely does a candidate complete the programme in four years. The mathematics course studied include higher algebra, advanced geometry, mathematical analysis, differential equations, theory of real and complex functions, numerical analysis, probability, theory of numbers and courses in applied mathematics. In many cases, no distinction is made between those students who intend to teach and those who intend to go into mathematical research."

"Some European countries permit teachers to achieve the highest teaching licence through successive steps. Thus, after completing three years of university study, one can apply for examination to qualify as a mathematics teacher in the first year of the upper cycle. Then by continued study and passing of examinations, one can qualify to teach in the next two years ..." (76, p. 147)

9.3.2 The Netherlands

L.N.H. Bunt gives an outline of the training of the Mathematics teacher for academic schools which accord admission to a university in the Netherlands. (18, p. 660-664)

A Mathematics teacher can obtain the required qualifications in two different ways, namely:

(a) by attending a university or a technological institute and by passing the necessary examinations, and

(b) by taking extramural courses and by passing in an examination organised by the Ministry of Education.

For admission to a university or a technological institute a student must pass the final examination of the high school and more particularly that in the stream for the natural sciences.

This study lasts from six to seven years. The first part occupies three years and ends in the candidate examination, and the second part in the

1) Contribution of a Committee Member.

doctoral examination. In his article Bunt then sets out the requirements for the candidate examination and mentions the textbooks which indicate the level of study. The subject matter is Linear Algebra with applications in Analytical Geometry, and an elementary introduction to the Theory of Probability and Statistics. Further subjects are Traditional and Advanced Analysis, while students who wish to continue their mathematical studies later on must also give attention to Modern Analysis.

Much greater freedom is allowed in regard to the subject matter for the Doctor's degree and the student can for the most part arrange his course to suit himself.

To obtain the teacher's diploma, the student must choose Mathematics either as a major or as an ancillary subject. If Mathematics is taken as a major subject, the course consists of subjects such as the following: Analysis, including Differential Equations; Legendre Polynomials; spherical Bessel's functions; Fourier analysis; the Theory of Complex Functions; Riemann planes; the general theory of Differential Equations with real and complex coefficients;

Integral calculus; Banach and Hilbert spaces;
the Theory of Groups; the Fundamentals of various Geometries;
and in addition a few other optional subjects.

At the universities in the Netherlands there is also another type of training for a Mathematics teacher, namely that intended for students who do not undergo the full training for the candidate examination. They attend the ordinary Mathematics lectures at the university and write the preliminary examinations but not the candidate or doctoral examinations. There is an A examinations parallel to the candidate examination and a B examination parallel to the doctoral examination. In this way the Secondary School Teacher's Certificate can be obtained. In order to qualify fully as teachers, however, students must also attend lectures in the Theory of Education, Psychology of the Adolescent and General Didactics, the Didactics of Mathematics and write an examination in this subject, and attend Mathematics lessons as a student teacher at a high school. Historical and Theoretical Education also have special sections of interest to the Mathematics teacher. A matter to which attention is being given is the difference between the mode of thinking in Mathematics and that of the natural sciences, and the difference between the mathematical mode of thought and that in use in everyday life.

> "And if we are aware of the fact that in a near future the community will need a much larger supply of mathematics teachers than ever before, we may well ask if we are not throwing away good potential teachers who, by being subjected to some different kind of introduction to abstract notions and methods and to the study of some more concrete material, might be saved for the teaching profession." (18, p. 664)

9.3.3 Great Britain

In almost every country a relatively intensive professional training is required. In a country such as Britain where academic training was formerly regarded as sufficient, professional training is today being demanded to an increasing extent.

In England and Wales professional training is not a requirement for appointment, but the percentage of prospective teachers who do receive it at universities and training colleges is increasing day by day. In the United States of America an intensive professional course is offered and, as in Britain, considerable practical teaching is required. In the United States and Britain an increasing amount of in-service training is provided while in Germany it is compulsory. Vacation courses are arranged in all the above-mentioned countries. In Britain The Mathematical Association displays great interest in the teaching of elementary Mathematics; by means of thorough reports of expert committees and discussions at annual conferences the association is making a weighty contribution to the professional in-service training of Mathematics teachers. The Association's official journal, "The Mathematical Gazette", assists Mathematics teachers in all problems relating to the teaching of Mathematics not only in their own country but also abroad. The corresponding associations in America are "The Mathematical Association of America", "The National Council of Teachers of Mathematics", and "The Central Association of Science and Mathematics Teachers"; their respective journals being "The Mathematics Teacher", "School Science and Mathematics", and "The Yearbooks of the National Council of Teachers of Mathematics". These British and American periodicals and research reports should be on the bookshelves of every inexperienced and experienced Mathematics teacher.

9.3.4 Germany, Denmark, France and Sweden

Germany, Denmark, France and Sweden undoubtedly maintain the highest standard in the world for the selection and preparation of teachers for secondary schools. Prospective teachers receive a considerable part of their remarkable training from the best teachers in the schools themselves. Germany lays much less stress than France on brilliant and comprehensive mathematical achievements but (like Denmark and Sweden) concentrates more specifically on theoretical and practical professional training. Names that come to mind in this connection are Klein, Schimmack, Lietzmann and Wolff. In-service training forms an important part of the training of the prospective teacher in Germany.

In Germany the name of the prospective teacher is placed on a list of candidates for appointment in secondary schools, and after he has completed his academic and professional training and satisfactory reports on his period of probation have been received he is given a permanent appointment. Teachers make great use of vacation courses and conferences. Through the medium of the committee of "Die Internationale Mathematische Unterrichtskommission" (IMUK) and "Deutscher Ausschuss für Mathematischen und Naturwissenschaftlichen Unterright" (DAMNU) the best thinkers and the most experiences teachers in the country place their ideas and methods at the disposal of every progressive teacher. Periodicals such as the "Zeitschrift für Mathematischen und Naturwissenchaftlichen Unterricht" and "Unterrichtsblätter für Mathematik und Naturwissenschaft" give excellent advice which promotes the in-service training of the active teacher.[1)]

9.3.5 Canada and the United States

> "In Canada and the United States of America, the requirement to begin teaching is graduation from a four-year college or university with at least a minor or a major (a minimum of 12 up to 30 or more semester hours) in mathematics. (A semester hour is one hour per week for sixteen weeks of class lectures. Each semester hour demands a minimum of two additional hours of outside study)."

When this minimum is increased by further study to 18, or in some areas 24, semester hours, the teacher's licence is made permanent. In these countries, a high school teacher can begin his professional work at 22 and has his permanent licence at 24 years of age. (76, p. 148)

In the United States the adequacy or otherwise of the traditional academic undergraduate curriculum for Mathematics at the colleges and universities is discussed. The question is whether this curriculum is suitable for social scientists including Mathematics teachers.

> "The old curriculum no longer meets our needs in the training of scientists, engineers and mathematicians.
>
> "Today there is hardly a branch of mathematics that does not find application in economics and psychology.
>
> "There seems to be fairly general agreement that social scientists need a mathematical curriculum that brings them quickly into contact with modern mathematics with the omission of traditional topics that no longer contribute

1) Information received from a member of the Committee.

either to applications or understanding.

"The proposals for curricular change made by social scientists are very similar to those proposed by committees of mathematicians who have recently considered the high-school and elementary college programme for all students." (61, p. ix, x)

"In the 20th century mathematics has been characterized by an explosive creativity, rapid expansion of its fields of application, an increasing use of abstract theories, and a tendency toward unification in terms of the concepts of set theory. 'Modern' has come into use as a catchword to describe mathematical teaching that reflects these features .....

"It does not signify that everything of old vintage has been rejected, but rather the belief that everything, of whatever age, should be reconsidered in the light of our current knowledge of mathematics and its applications." (61, p. 11)

It is very difficult to obtain an over-all picture of the training of teachers in the United States of America. James D. Koerner investigated the matter, and was so strongly impressed by the things which he considered unsatisfactory in this field that he published a book entitled "The miseducation of American teachers". In the introduction he states that he has attempted to write a fair and objective report.

What is responsible for the shortcomings? "Most of the ailments of teacher education are the natural result of narrowly concentrated power protecting accumulated professional orthodoxies." (54, p. 204)

9.3.6 Russia

Since considerable interest is being displayed in Russian education, a fairly lengthy quotation is given here from an article by Vogeli and Lindquist.

"Two official curricula are available to the Soviet pedagogical student preparing to teach mathematics - the Mathematics - Technical Drawing curriculum and the Mathematics - Physics curriculum. Both programs are of five year duration.

Mathematics	Total class hours	Professional	Total class hours
Mathematical Analysis	408	Elementary Mathematics	400
Higher Algebra	192	Method of Teaching Mathematics	194
Analytic Geometry	172	Pedagogy	120
Projective and Descriptive Geometry	110	Special practical training in Mathematics	92
Seminar in Mathematics	84	Psychology	84
Foundations of Geometry	64	History of Pedagogy	72
Complex Variable	54	Special Preparation	48
Elective in Mathematics	54	School Hygiene	36
Real Variable	50	Visual Aids	36
Theory of Numbers	48	Practice Teaching	16 weeks
Foundations of Arithmetic	36		

The official government syllabus for the course Elementary Mathematics lists the following major topics.

First semester - The arithmetic of rational numbers: whole numbers, divisibility of whole numbers, rational numbers, finite continued fractions and indeterminate equations.

Second semester - Theory and practice of calculation: precise computation, elements of approximate computation, aids to approximate computation (slide rule, tables), graphical solution of equations and numerical methods of isolating roots.

Third semester - Geometry: introduction, plane figures and their characteristics, similar figures, geometric values (measures), spatial figures and their characteristics, geometric values (volume and surface area).

Fourth semester - Geometric constructions: introduction, constructions with ruler and compass, basic geometric loci of points in a plane and the method of geometric loci in the solution of construction problems, geometric transformations of the plane and the method of transformations in the solution of construction problems, algebraic methods for the solution of construction problems, solvability of construction problems with ruler and compass, loci in space.

Fifth semester - Algebra: elementary methods of the solution of algebraic equations in one unknown, combinatorial theory, binomial and polynomial theorems, polynomials in several variables, nonlinear systems of algebraic equations in several unknowns, inequalities.

Sixth semester - Irrational and transcendental equations: irrational algebraic equations in the real domain, exponential and logarithmic equations in the real domain, trigonometric equations in the real domain.

Seventh semester - Trigonometry (plane and the elements of spherical): trigonometric functions of a real argument, inverse trigonometric functions, analytic representation of trigonometric functions, applications of trigonometry to geometry, elements of spherical geometry and trigonometry.

Certain topics (continued fractions; constructability; spherical trigonometry, etc.) though not found in the secondary school program, provide the prospective teacher with valuable background and enrichment material.

The Kutuzov text Geometry has been translated into English and published by the School Mathematics Study Group for use by American teachers and students (Studies in Mathematics, Volume IV). The two Novoselov texts have been translated by various groups and individuals and are awaiting publication. These, in particular, are of interest because of the skill with which Novoselov displays the role of modern mathematical concepts at the elementary level.

Practical Training in Mathematics is a 92-hour course spanning three semesters. Actually it is comprised of three separate practical training courses - one in visual aid (devices and models) preparation, one in field measurement, and a third in techniques of calculation.

The first phase of the practical training program consists of 20 hours of instruction and workshop practice in the construction of three or four mathematical models using cardboard, wood, glass, sheet metal, wire or thread.

The second phase of the practical training program deals with field measurement.

In addition to the methods course, practical training, and the course in elementary mathematics, future Soviet mathematics teachers must enroll in a professional course entitled Special Preparation. All Soviet mathematics teachers are expected to be able to supervise at least two types of extra-class activity - usually including a mathematics club or circle. The Special Preparation course apparently is designed to provide some formal instruction along these lines.

Although the mathematics courses taught in Soviet pedagogical institutes appear to be of sound mathematical character, they are, in a sense, "professionalized" courses.

The third of the five levels of competence established by the M.A.A. (level III - Teachers of High School Mathematics - 'these teachers are qualified to teach a modern high school mathematics sequence in grades nine through twelve') requires as a minimum the following:

(a) Three courses in analysis
(b) Two courses in abstract algebra
(c) Two courses in geometry beyond analytic geometry
(d) Two courses in probability and statistics
(e) Two upper-class electives; e.g. introduction to real variables, number theory, etc. ....

(One of these courses should contain an introduction to the language of logic and sets).

There can be little doubt that Soviet mathematics teachers completing the present curricula are prepared to discharge their duties effectively." (109, p. 156-162)

9.4 THE TRAINING OF TEACHERS IN SOUTH AFRICA

9.4.1 Courses

The following courses are available to the student who wishes to qualify as a high school teacher:

(a) University teachers' diplomas which can be taken after the completion of a degree course or part of a degree course.

(b) A combined degree and teacher's course which is offered only by the four (later five) Transvaal teachers' colleges.

Since 1960 the Transvaal Teachers' Colleges have been offering modified courses lasting three years during which students can qualify as teachers for the junior secondary classes.

Students who take the combined academic (one year) and professional course of three years, study the method of teaching an academic subject up to Std. 8 in the professional part of the course or the contents of and method of teaching two school subjects (up to Std. 8) which they have passed in Std. X. If the student takes only a professional course of three years with or without specialisation, the contents of and method of teaching at least one school subject are inter alia also studied.

At the institutions of all four provincial departments of education students also qualify for teaching by taking certain specialised courses for the high school, although the basic professional training is intended only for the primary school.

Since the high school teacher must give instruction in a particular subject or subjects, it is expected that he should have a good academic grounding in the subject or subjects concerned. For this reason a degree or part of a degree is required for all diplomas. Since the academic qualifications must include subjects taught in the schools, reference is in all cases made to a prescribed degree or part of a degree.

9.4.2 Post-graduate diplomas

The Universities of Cape Town, the Orange Free State, Potchefstroom, Pretoria, Stellenbosch and Rhodes University all offer one teachers' diploma course, while the University of Natal offers three such courses. The duration of these courses is one academic year in each case. The diploma course of the University of Pretoria can also be taken extramurally and then the duration is 18 months, although the contents of the courses do not differ. The Teachers College in Pretoria also offers an extramural course of 18 months, the contents of which are the same as that for the fulltime course. (121, p. 32)

9.4.3 Undergraduate diplomas

In addition to the post-graduate diploma courses the Universities of Capetown, Natal, Rhodes and Stellenbosch also offer one teachers' diploma course for which the entrance requirements are only a prescribed part of a degree. The duration of these courses is also one academic year.

9.4.4 The combined degree and University Education Diploma

The University of Cape Town offers a combined B. Ed. course as well as a combined M. Ed. course, in which cases the candidate also obtains a teacher's diploma upon the completion of the degree course. The duration of the courses is in both cases two years, an approved degree being a requirement for admission in the first case and a master's degree, honour's degree or two bachelor degrees in the second case.

9.4.5 The University Education Diploma of the University of South Africa

The University of South Africa also offers a University Teachers' Diploma of one year, but the requirement for admission in this case is not only an approved degree but also a recognised teacher's diploma.

9.4.6 The combined degree and education course of the Teachers Colleges of the Transvaal

The four (now five) teachers' colleges in the Transvaal each offer a combined academic and professional course extending over a period of four years.

Together with the degree course taken at a university the student therefore also receives professional instruction at the teachers' college. During the last year, after the degree has already been obtained, attention is given only to professional subjects.

If a student already has a prescribed degree, he takes only the professional course which extends full-time over one year and extramurally over 18 months.

9.4.7 Three-year General Professional Course for Junior Secondary classes with specialisation in two of the following subjects:

List A

(i) Afrikaans (Higher or Lower)
(ii) English (Higher or Lower)
(iii) Mathematics
(iv) Physical Science
(v) Biology

List B

(i) History
(ii) Religious Instruction
(iii) Race Studies
(iv) Geography

Preference is given to the subjects in List A. (108, p. 11)

Apparently both the academic and the professional subjects are taught in the teachers' colleges in the Transvaal.

> (Translation) The Junior Secondary Class course was introduced in order to alleviate the shortage of Mathematics teachers. The idea is that they should give instruction in Mathematics up to and including Std. 8 only.
>
> The main stress falls primarily on the contents part of Mathematics and after that the methods and techniques of teaching etc. can be given attention. Method without any basic knowledge of Mathematics is valueless.
>
> Lecturers who also teach Mathematics I to students taking the course under the University of South Africa, very often find that the knowledge of such students is poorer than that of the J. S. C. students. The reason for this is obvious. In the case of the academic student about 80 hours are devoted to

Mathematics, while the J. S. C. student spends about 220 hours on the same work.
Whether the J. S. C. course will prove a success only time will show. An important requirement in this respect should be that only A stream pupils may take the J. S. C. course, otherwise the gap between Std. 10 and first-year Mathematics is too great. It may be dangerous to allow only a very average pupil or student to take the J. S. C. course ..... [1)]

9.4.8 Academic training

All the institutions further require that a certain number of courses in school subjects (generally 5 or 6) should be included in the degree course. In most cases at least one school subject must be taken as major subject and another school subject on the basis of two years. (121, p. 38)

9.4.9 Professional training

The professional part of the courses for high school teachers may be divided up as follows: Subjects dealing with the fundamental principles of education, the method of teaching certain school subjects, language instruction, a group of optional subjects and a large variety of ancillary subjects. Each of the institutions which train high school teachers offer subjects under the above headings, although numerous differences also exist from one to another. The professional training usually follows upon the academic training.

9.4.10 Practiceteaching

Except in the case of the University of South Africa, all the institutions require students to visit schools and to give practice lessons for fixed periods. The periods spent on practical teaching, as well as the method according to which this is done, vary in the different institutions. (121, p. 45)

9.4.11 Technical colleges

At the Pretoria Technical College a one-year professional course with specialisation in Mathematics is offered. The requirement for admission is a National Diploma in Technology or an equivalent qualification. Use is made of an experimental classroom, while criticism lessons are attended by a lecturer.

The Technical College of the Witwatersrand makes provision for a National Education Diploma in Technology. Only candidates who have a degree or diploma in engineering and two years' experience are admitted to the course. The course is given on a part-time basis and lasts two years. (123, p. 372)

9.5 THE METHODS OF THE TEACHING OF MATHEMATICS

9.5.1 The Universities

In their calendars the universities do not furnish many particulars in regard to what they offer in their Methods classes to prospective Mathematics teachers.

The University of the Orange Free State gives particulars only in respect of the first examination paper:

(a) Selections from the history of elementary Mathematics.

(b) The position and significance of Mathematics in the school curriculum.

(c) Treatment of the existing school curriculum.

(d) The method of teaching Mathematics.

[1)] Information obtained from a lecturer at a Transvaal Teachers' College.

(e) Elementary Mathematics from a higher point of view; the number concept; axiomatics in Geometry and Algebra

Rhodes University furnishes the following particulars:

(a) The principal reasons why Mathematics is taught at school. The curriculum discussed from the point of view of the teacher of secondary school Mathematics.

(b) Algebra

A branch of Algebra according to the choice of students (e.g. exponents or logarithms or equations) is examined in detail and critically scrutinized:

(i) from the teacher's point of view;
(ii) from the pupil's point of view.

The Algebra syllabus is discussed in general from the assumption that the central theme of Algebra is the equation, all the other work being seen in the light of this theme. Branches of Algebra, e.g. exponents, etc., are taught as the need for them becomes clear in the work on equations.

(c) Geometry

The practical approach to this subject is discussed. From practical problems relating to map-making, designs, the measurement of sportsfields, etc., a series of geometrical constructions is developed, e.g. the construction of a right angle, the drawing of parallel lines, the bisection of a line, etc. A number of such constructions are discussed and attention is given to the problem of when and how formal proofs with theorems and problems should be introduced. The curriculum for the Junior Certificate is dealt with fairly fully in respect of this matter. The nature of the basic assumptions in Geometry is discussed. The parallel postulate of Euclid is dealt with, as well as non-Euclidean Geometry, the Geometry of Riemann and Lobachevsky, in order to give the teacher a better idea of this aspect of the subject.

(d) Trigonometry

The value of the trigonometric ratios is discussed, as well as the different methods of introducing Trigonometry and the order in which the ratios should be taught, namely first the tangent to link up with Geometry. The principles of the theodolite and surveying are illustrated.

At the university the method of teaching each school subject is covered along three lines by the Faculty of Education:

(i) A series of lectures by one of the professors in Mathematics at the university who deals only with the philosophic aspect,

(ii) a series of lectures by one of the professors of education on general methodology in which general principles are discussed, and

(iii) a series of twelve to fourteen lectures (of one hour a week) on each subject by an in-service teacher who already has considerable experience in the tuition of the particular school subject concerned.

The University of South Africa

The Mathematics teachers who have enrolled for the U. E. D. course offered by the University of South Africa have a B. Sc. degree with Mathematics as major subject. They are persons who have been appointed in a secondary school and receive in-service training under the supervision of the principal and the inspector. They receive their correspondence lectures and study guides from the

University of South Africa and attend a winter course lasting two weeks every year in July. Their assignments are checked by the University. During the winter course they receive practical instruction in the method of Mathematics for one hour a day. This is regarded as intensive training according to a statement made by the lecturer concerned.

9.5.2 The teachers' colleges

In the Transvaal the teachers' colleges also train teachers for the secondary school. The following are the prescribed subjects in connection with the method of teaching Mathematics:

General principles

(a) The object of teaching Mathematics.

(b) Equipment of classroom (illustrations, wall charts, apparatus and films).

(c) Work and lesson schemes and organisation.

(d) Tests, examinations and timetables.

Contents and method

Basic principles in the teaching of Mathematics in the lower and higher classes in the case of -

(1) Geometry

(a) Construction and drawings to scale;

(b) Classroom technique for the teaching of

(i) Fundamental postulates and theorems in respect of the properties of angles, parallel straight lines, congruency.

(ii) Locus, common point of intersection theorems in respect of triangles and theorems on proportion.

(c) Methods for tackling, solving and setting out problems.

(2) Algebra

(a) Present-day views on the initial teaching of Mathematics.

(b) Themes; simple linear equations; simple graphs, resolution into factors, directed numbers; logarithms; functional notation, quadratic equations; variable functions, sketching of functions without the use of graph paper (e.g. $mx + c$; $ax^2 + bx + c$; p/x; $ax^3 + bx^2 + cx + d$), determination of the gradient functions and minimum and maximum values; gradient of curves.

(3) Trigonometry

The fundamental ratios in the right angled triangle; change in the value of thetrigonometric ratios for angles between 0^o and 360^o, and solution of equations and triangles.

(4) Arithmetic

Graphs, logarithms, stocks and shares; areas and volumes and other subjects in the curriculum for secondary schools.

9.5.3 The technical colleges

At the Pretoria Technical College thorough attention is given to the method of teaching Mathematics. About one-third of the time is devoted to the technique of teaching Mathematics. The remaining time is spent on the Mathematics textbook at school, the mathematical concepts and the pupils' conceptual ability, the planning of work based on the Mathematics curriculum, the drafting of Mathematics tests, mathematical proofs, analysis and synthesis in Mathematics, classroom control and organisation, the use of the library and other sources, the latest developments in the field of Mathematics, professional and trade organisations and the advantages of membership, the use of the slide rule, the history of Mathematics, deduction and induction in Mathematics, the philosophical foundations of Mathematics, the mathematical structure, the basic concepts, length, time and mass, the derived concepts, e.g. the energy concept.

9.6 WHERE THE TEACHERS RECEIVED THEIR TRAINING

Table 9.1 gives an indication of the number of teachers who are at present teaching mathematical subjects at high schools, as well as the various institutions at which they received their training.

TABLE 9.1

INSTITUTIONS AT WHICH PROFESSIONAL TRAINING WAS RECEIVED

Institution	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
Universities						
Stellenbosch	175	10.2	127	7.4	302	17.6
Orange Free State	71	4.1	60	3.5	131	7.6
Pretoria	46	2.7	37	2.2	83	4.9
Cape Town	51	3.0	31	1.8	82	4.8
Natal	37	2.2	22	1.3	59	3.5
Rhodes	35	2.0	23	1.3	58	3.3
Potchefstroom	20	1.2	33	1.9	53	3.1
South Africa	8	0.5	8	0.5	16	1.0
TOTAL	443	25.9	341	19.9	784	45.8
Training colleges						
Pretoria	75	4.4	113	6.6	188	11.0
Potchefstroom	35	2.0	72	4.2	107	6.2
Johannesburg	51	3.0	52	3.0	103	6.0
Heidelberg, Tvl.	3	0.2	70	4.1	73	4.3
Natal	6	0.3	39	2.3	45	2.6
Abroad	13	0.8	24	1.4	37	2.2
Paarl	2	0.1	32	1.9	34	2.0
Bloemfontein	3	0.2	30	1.7	33	1.9
Oudtshoorn	1	0.1	28	1.6	29	1.7
Graaff-Reinet	1	0.1	19	1.1	20	1.2
Wellington	1	0.1	11	0.6	12	0.7
Steynsburg	1	0.1	6	0.3	7	0.4
Cape Town	1	0.1	5	0.3	6	0.4
Grahamstown	1	0.1	4	0.2	5	0.3
TOTAL	194	11.6	505	29.1	699	40.9
Technical Colleges						
Witwatersrand	7	0.4	13	0.8	20	1.2
Cape Town	5	0.3	11	0.6	16	0.9
Pretoria	13	0.8	3	0.2	16	1.0
Natal	1	0.1	6	0.3	7	0.4
TOTAL	26	1.6	33	1.9	59	3.5
Elsewhere	70	4.1	98	5.7	168	9.8
GRAND TOTAL	733	43.2	977	56.6	1710	100.0

Conclusions

1. Approximately a quarter (25.9%) of the teachers are qualified in Mathematics and have also been trained at a university.

2. The University of Stellenbosch (with 17.6%) and the Pretoria Teachers' College (with 11.0%) are the only institutions which in each case have produced more than 10% of the teachers who are responsible for the teaching of mathematical subjects.

3. Centres abroad produced 2.2% of such teachers.

9.7 THE QUALIFICATIONS OF THE LECTURERS IN THE METHOD OF TEACHING MATHEMATICS

9.7.1 The universities

Table 9.2 reflects the qualifications of the lecturers in the Method of Mathematics Teaching at South African universities. No completed questionnaires were received from the University of the Witwatersrand; this table therefore does not include any data obtained from that university.

TABLE 9.2

THE QUALIFICATIONS OF UNIVERSITY LECTURERS IN THE METHOD OF MATHEMATICS TEACHING

Degree	Number of lecturers
B.Sc.	1
B.Sc., B.Ed.	1
M.Sc.	4
M.Sc., M.Ed.	1
M.A., Ph.D.	1
M.A., D.Phil.	1
Doctor in Mathematics	1
TOTAL	10

Ten lecturers are therefore concerned in the teaching of Method. All have teachers' diplomas, and only one of them does not have at least a master's degree.

The aid of three part-time lecturers is enlisted for class technique, etc. Two of these have a B.Sc. degree and one an M.A. degree. They also all have teachers' diplomas.

At some universities no complete clarity exists in regard to who really should be responsible for the instruction in the Method of Mathematics. At one of the universities (Translation): The circumstances at present show a metastable character in so far that these circumstances will in all probability be radically changed from the beginning of next year (i.e. 1963). At the moment all students with Mathematics as major subject take the Method of teaching it in the Faculty of Mathematics and Physics with the lecturers responsible for Mathematics. To obviate this splitting of the activities of the Faculty of Education there is at present a strong possibility that a lecturer will be appointed in this Faculty to take charge inter alia of the method of teaching Mathematics within the Faculty and in this way to bring about greater unity within the Faculty as well as to ensure greater co-ordination in the course of such students.[1)]

At another university the Method of teaching Mathematics is treated by a senior professor in Mathematics, and from the beginning of 1963 the number of lecture hours per week was increased from two to three.

It would appear that at some universities it is not yet quite clear who should be responsible for instruction in the Method of teaching Mathematics. Should the person be a Mathematics lecturer or should he be a lecturer in the Faculty of Education?

Koerner points out that certain dangers may arise if the Faculties of Education operate in too great isolation. "Educationists 'who worship their own gods', as one eminent dean of Education pointed out years ago, 'without admitting their colleagues of other faculties (now especially the faculty of arts and science)

1)Statement made by the head of a department.

into their congregation, are in danger of becoming fanatics'." (54, p. 205)

It is noteworthy that, except in two cases, all the full-time university lecturers have a qualification higher than a Bachelor's degree in the subject the method of which they have to teach.

9.7.2 The teachers and training colleges

Particulars were received from six institutions, four of which were in the Transvaal and two in Natal. The qualifications of the lecturers concerned are shown in Table 9.3.

TABLE 9.3

THE QUALIFICATIONS OF LECTURERS AT TEACHERS' AND TRAINING COLLEGES WHO GIVE INSTRUCTION IN THE METHOD OF TEACHING MATHEMATICS

Degree	Number of lecturers
B.Com.	3
B.Sc.	1
B.Sc., B.Ed.	1
B.Sc. (Hons.), M.Ed.	1
B.Sc., D.Ed.	1
M.Sc., M.Ed.	2
B.A.	1
B.A., B.Com.	1
B.A., D.Ed.	1
M.A.	1
TOTAL	13

All the lecturers have a teacher's diploma, but not more than four of the 13 lecturers have anything higher than a Bachelor's degree in the subject of which he must teach the method.

To have an Honours degree in Mathematics is of great importance to the lecturers in the Method of the teaching of Mathematics. The fundamentals of the subject are generally dealt with for the first time in the Honours course for Mathematics, and it is these fundamentals which are so very important. In the light of the foregoing it is clear that the great majority of these lecturers do not have adequate qualifications.

The same phenomenon is also observed in other countries. In August 1962, a report was published on the qualifications of lecturers in Psychology at teachers' training colleges in England. The qualifications of 304 lecturers were analysed, and the following conclusion was arrived at: "Taken at their face value, these figures seem to indicate that a good deal (39 per cent) of the responsibility for teaching psychology is in the hands of lecturers not formally qualified in the subject." (49, p. 15)

H.P. van Coller is strongly in favour of the teachers' colleges being connected with the universities and gives inter alia the following reasons: (Translation) A more efficient staff can be appointed at the universities because the higher status and better salary scales will draw qualified persons. And it will be possible to perform more thorough work with a smaller staff. (123, p. 407)

It is a striking fact that all the university lecturers who are charged with instruction in the Method of teaching Mathematics are qualified teachers. The question is whether these persons have adequate teaching experience. Van

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Coller's reply to this is: (Translation) The objection that professional moulding at the universities is adversely affected because most professors and lecturers have little or no teaching experience, is also removed. As soon as the posts are introduced at the universities, qualified academic persons with adequate professional experience can be appointed. (123, p. 407)

It should moreover be borne in mind that these appointments must always be made with the greatest circumspection so that there will be no complaints about the quality of the lecturers in the faculties concerned. That there have sometimes been misgivings in other countries in regard to this matter is apparent from the following quotation:

> "It is an indecorous thing to say and obviously offensive to most educationists, but it is the truth and should be said: the inferior intellectual quality of the Education faculty is the fundamental limitation of the field, and will remain so, in my judgment, for some time to come." (54, p. 7) South Africa has in general hitherto always been fortunate in having able educationists. Proof of this fact, for example, is the considerable number of university rectors who were formerly Professors of Education.

The argument is sometimes heard that it is better to have teachers trained in colleges which are directly under departmental control. In this way the authorities can ensure that the training of teachers will leave nothing to be desired. This idea should be treated with the necessary caution. "As is true of many other fields, one of the greatest obstacles to reform in Education is administrative inertia." (54, p. 7)

9.7.3 Technical colleges

Only one technical college reported that it provides for instruction in the Method of teaching Mathematics, this course being given by a person with B.A., B.Sc. and U.E.D. The technical colleges in Durban, Port Elizabeth and Johannesburg reported that they did not offer such a course.

9.8 THE ORDER OF PREFERENCE FOR THE VARIOUS SUBJECTS OF STUDY

9.8.1 The universities

Question 4 of Questionnaire N.B. 378 contained a list of the various subjects which can be dealt with in the lectures on Method and it was expected of the lecturers to indicate how much time they spent on each of these subjects.

It was of course a very difficult question to answer. Some lecturers merely indicated which subjects they treat and which they do not, while others did take the trouble to write in the number of class hours.

With the information which was furnished, it was possible to arrange the subjects in order of preference. Table 9.4 shows this order according to the information received from the universities. The right-hand column reflects the corresponding order of preference according to the lecturers at teachers' colleges.

TABLE 9.4

THE SUBJECTS IN THE METHOD OF TEACHING MATHEMATICS IN THE ORDER OF PREFERENCE GIVEN BY UNIVERSITY LECTURERS

Order of preference of the universities	Subjects	Order of preference of the teachers colleges
1	Technique of teaching Mathematics	1
2	The History of Mathematics	6
3	The axiomatic method	13
4	The mathematical concepts and the pupils' conceptual grasp	2
5	Construction of Mathematics tests	8
6	The use of the library and other sources	12
7	The Mathematics textbook in use at school	5
8	Planning of units of work based on the Mathematics syllabus	3
9	The mathematical structure	15
10	Analysis and synthesis in Mathematics	9
11	Deduction and induction in Mathematics	10
12	The mathematical proof	7
13	The philosophical bases of Mathematics	14
14	The most recent developments in the field of Mathematics	16
15	The various mathematical and professional organisations and the value of membership	19
16	Classroom management and organisation	4
17	The basic concepts: Length, time and mass	11
18	The use of the slide rule	17
19	The derived concepts, e.g. the concept of energy	18

9.8.2 The teachers' colleges

The preference list drawn up according to the replies given by lecturers at teachers' colleges is shown in Table 9.5.

TABLE 9.5

PREFERENCE LIST ACCORDING TO TIME SPENT ON THE VARIOUS SUBJECTS AT THE TEACHERS' COLLEGES

Order of preference	Subjects
1	Technique of teaching Mathematics
2	The mathematical concepts and the pupils'conceptual grasp
3	Planning of units of work based on the Mathematics syllabus
4	Classroom management and organisation
5	The Mathematics textbook in use at school
6	The history of Mathematics
7	The mathematical proof
8	Construction of Mathematics tests
9	Analysis and synthesis in Mathematics
10	Deduction and induction in Mathematics
11	The basic concepts: Length, time and mass
12	The use of the library and other sources
13	The axiomatic method
14	The philosophical bases of Mathematics
15	The mathematical structure
16	The most recent developments in the field of Mathematics
17	The use of the slide rule
18	The derived concepts, e.g. the concept of energy
19	The various mathematical and professional organisations and the value of membership

9.8.3 Discussion

If we accept that the time spent on a subject is a criterion of its importance, it will be worthwhile examining these two preference lists more closely. Only cases where the order of preference differs by five or more are considered. The figure 5 is taken quite arbitrarily to indicate a considerable difference. The conclusions must therefore also be seen in this light.

The university lecturers place "the use of the library and other sources" in the sixth position, while according to the teachers' colleges it ranks twelfth. This approach may explain a few other differences. The universities accord the "axiomatic method" position number 3, while the teachers' colleges list it at number 13. Anyone who has worked through, for example, the publications of the School Mathematics Study Group will realise that the axiomatic method has today resumed an important role and that it is in fact one of the influential elements in the reform of the teaching of Mathematics.

In general, the university lecturers give more attention to "mathematical structure" and less to "mathematical proof" (orders of preference 9 and 12, respectively). In the case of the lecturers at teachers' colleges "mathematical proof" occupies seventh position and "mathematical structure" the fifteenth. For the development of reasoning the mathematical structure should receive special attention. In this manner the teaching of Mathematics becomes a meaningful whole with the mathematical proof an important sub-unit.

If primacy is accorded to mathematical proof, the accent is allowed to fall on the solution of problems with the result that the teaching of Mathematics is reduced to the learning of certain skills. The principal aim of the teaching of Mathematics then simply becomes a desire to teach pupils how to pass an examination. It is noteworthy that the lecturers at the teachers' colleges lay such great emphasis on "planning of activities based on the Mathematics curriculum" that this occupies third place in their list as compared with position No. 8 in that of the university lecturers. That "classroom control and organisation" is item No. 4 in the case of the teachers' colleges, fits in very well in this framework. It is something which really falls under Practical Education, however, and not under the methodology curriculum. It is no doubt for that reason that it occupies the sixteenth position in the case of the university lecturers.

It is noteworthy that the "basic concepts: Length, time and mass" are placed eleventh on the list by the teachers' colleges while the university accord them seventheenth position. A large part of the time of the lecturers at teachers' colleges is spent on the training of teachers for Stds. 6, 7 and 8. Since Arithmetic constitutes an important element at this stage, it is to be expected that the concepts "length, time and mass", which play an important part in Arithmetic, will receive more attention at teachers' colleges. It would appear that the training at teachers' colleges is concentrated mainly on classroom practice.

If the prospective teachers couldbe trained at a university, (even if not for a degree course but for a diploma) they would come into contact with a totally different approach to Mathematics than is at present the case at teachers' colleges. It is this difference of approach which is of paramount importance for the development of the future Mathematics teacher.

The system according to which a person first obtains his degree and then receives professional training for one year has the following shortcomings:

> (Translation) The real and striking weakness in our academic and professional system of training future Mathematics teachers is the lack of effective preparation to teach elementary Mathematics in the secondary school. The methods for such teaching differ completely from those of the university. It is not yet adequately realised that the university method in regard to the subject is not an effective way of teaching elementary Mathematics to pupils in the secondary school. The result is that the old established methods and views have been maintained from generation to generation. For that reason it is a very difficult and long process to give practical effect to all the plans,

ideals, good intentions of new curricula and the real aims of the teaching of Mathematics because the person who has to fulfil these aims is not properly trained for this task.

We must therefore realise that there is a great need for an improvement of the Mathematics teacher's professional training. The single year of his professional preparation is too overloaded and consequently too short. It would seem that it should be extended to cover two years and that the whole of the second year should be spent only on the method of teaching one or two specific subjects. A period of in-service training (as for the medical, legal and pharmaceutical professions) is also highly desirable.

The final year of professional training after a degree has been obtained is as yet not used to full advantage by the universities. In the year of professional training the emphasis should be confined to the teaching of Mathematics. At this stage of the training of the future Mathematics teacher attention can also be given to "Practical Mathematics" which will make the subject more vital for both the teacher and the pupil. There are numerous obvious applications of school Mathematics, which can clarify this subject for the child, and with which the teacher should be equipped.[1]

The system of the University of Cape Town in which both a combined B. Ed. and a combined M. Ed. course are offered (in which cases the candidate also obtains a teacher's diploma after the completion of the degree course) offers the possibility of training the future teacher very thoroughly in both the method of his subject and in general also for the pursuit of his profession.

9.9 TRAINING IN THE METHOD OF TEACHING MATHEMATICS

9.9.1 The teachers who have received training

Tables 9.6 and 9.7 show how the teachers replied to the question as to whether they had received training in the method of teaching Mathematical subjects.

The total number of teachers in respect of whom this analysis was made was 1665.

TABLE 9.6

THE NUMBER OF TEACHERS WHO RECEIVED TRAINING IN THE METHOD OF TEACHING MATHEMATICS

Department	Qualified in Mathematics							Unqualified in Mathematics						
	Training				No reply received		Total	Training				No reply received		Total
	Received		Not received					Received		Not received				
	Number	%	Number	%	Number	%		Number	%	Number	%	Number	%	
Cape Province	194	85	29	13	5	2	228	46	16	196	68	44	16	286
Natal	40	71	15	27	1	2	56	27	27	59	60	13	13	99
Orange Free State	61	80	15	20	0	0	76	30	34	48	55	9	11	87
Transvaal	165	68	71	29	6	3	242	75	19	237	60	82	21	394
S. W. A.	8	89	1	11	0	0	9	1	10	8	80	1	10	10
Education, Arts and Science	49	58	35	41	1	1	85	15	16	61	66	17	18	93
TOTAL	517	73	166	24	13	3	696	194	20	609	63	166	17	969

1) Observations of a committee member.

TABLE 9.7

THE NUMBER OF TEACHERS WHO RECEIVED TRAINING IN THE METHOD OF TEACHING ARITHMETIC IN HIGH SCHOOL CLASSES

Department	Qualified in Mathematics							Unqualified in Mathematics						
	Training received		Training not received		No reply received		Total	Training received		Training not received		No reply received		Total
	Number	%	Number	%	Number	%		Number	%	Number	%	Number	%	
Cape Province	49	21	92	40	87	39	228	51	18	175	61	60	21	286
Natal	16	29	29	52	11	19	56	36	36	53	54	10	10	99
Orange Free State	22	29	38	50	16	21	76	28	32	43	49	16	19	87
Transvaal	95	39	114	47	33	14	242	163	41	189	48	42	11	394
South West Africa	2	22	4	44	3	34	9	2	20	7	70	1	10	10
Education, Arts and Science	14	16	46	54	25	30	85	21	23	48	52	24	25	93
TOTAL	198	29	323	46	175	25	696	301	31	515	53	153	16	969

9.9.2 Conclusions

Mathematics: About a quarter (24%) of the qualified Mathematics teachers receive no training in the method of teaching the subject. The same applies to almost two-thirds (63%) of the unqualified teachers of Mathematics. Approximately half (46%) of the teachers charged with the teaching of Mathematics received no training in the method of teaching the subject.

Arithmetic: Roughly 29% of the teachers were trained in the method of teaching this subject in high schools. There is therefore a very great need of training for in-service teachers in this important direction.

9.9.3 The effectiveness of the training

The replies of 1665 teachers were analysed in order to determine whether the training which they received in the method of teaching Mathematics was, in their opinion, sufficient.

Table 9.8 reflects the number of teachers who consider their training sufficient and also those who do not.

TABLE 9.8

THE OPINION OF TEACHERS CONCERNING THEIR TRAINING IN THE METHOD OF TEACHING MATHEMATICS

Teachers	Education, Arts and Science		Cape Province		Natal		Orange Free State		Trans-vaal		South West Africa		Total	
	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%	Number	%
Qualified in Mathematics Training														
sufficient	34	40	100	44	25	45	30	39	60	25	3	33	252	36
insufficient	20	24	96	42	18	32	35	46	145	60	4	44	318	46
No reply given	31	36	32	14	13	23	11	15	37	15	2	23	126	18
TOTAL	85	100	228	100	56	100	76	100	242	100	9	100	696	100
Unqualified in Mathematics Training														
sufficient	17	18	52	18	33	33	21	24	75	19	0	0	198	21
insufficient	28	30	94	33	19	19	23	26	156	40	2	20	322	33
No reply given	48	52	140	49	47	48	43	50	163	41	8	80	449	46
TOTAL	93	100	286	100	99	100	87	100	394	100	10	100	969	100
GRAND TOTAL	178		514		155		163		636		19		1665	

It is noteworthy that only slightly more than a quarter (27%) of the teachers are able to say that they are satisfied with the training which they received.

9.9.4 Reasons

The reasons why the teachers are not satisfied with the training which they received in the method of teaching Mathematics are set out in Table 9.9.

TABLE 9.9

THE PRINCIPAL REASONS WHY TEACHERS ARE NOT SATISFIED WITH THE TRAINING WHICH THEY RECEIVED IN THE METHOD OF TEACHING MATHEMATICAL SUBJECTS

Reasons for dissatisfaction	Qualified in Mathematics							Unqualified in Mathematics							
	Cape Province	Natal	Orange Free State	Transvaal	South West Africa	Education, Arts and Science	TOTAL	Cape Province	Natal	Orange Free State	Transvaal	South West Africa	Education, Arts and Science	TOTAL	GRAND TOTAL
1. No such training was received	10	9	12	36	0	5	72	34	11	11	44	0	17	117	189
2. The training was not practical enough	36	5	9	19	1	4	74	11	2	2	13	0	2	30	104
3. Insufficient time was allocated to the method of teaching the subject	18	0	3	35	3	8	67	9	2	3	15	0	2	31	98
4. Only training in the method of teaching primary school Arithmetic was received	1	0	1	21	0	0	23	8	1	1	21	0	1	32	55
5. Too little attention was given to the basic principles	6	1	1	12	0	0	20	2	0	0	7	0	2	11	31
6. Too wide a field was covered	5	0	3	2	1	2	13	0	1	0	8	0	0	9	22
7. The principles and methods of developing a mathematical way of reasoning were not sufficiently emphasised	6	0	1	3	0	1	11	0	0	0	4	0	0	4	15

The greatest source of dissatisfaction is the fact that so many teachers who are responsible for giving tuition in the mathematical subjects have had no training in the method of teaching these subjects.

Those who have had such training consider that the course was not practical enough and, moreover, that insufficient time was allocated to the treatment of method.

Too little attention is given to the basic principles and to the methods of developing a mathematical style of reasoning.

9.10 SUMMARY

An outline has been given of the manner in which the teachers of mathematical subjects are trained both locally and abroad. Special attention has been given to the syllabi for the Method of the teaching of Mathematics, and the approach to the subject by universities and teachers' colleges has been compared. The teachers' comments on the efficiency of the training which they have received are also set out.

An indication has been given of the institutions at which teachers who are responsible for mathematical subjects have received their training. The qualifications of the lecturers concerned in the various types of institutions are also shown.

It is obvious that the existing courses in the method of teaching mathematical subjects should be improved with respect to content, and also by the provision of well qualified and experienced teaching staff.

A high percentage of the teachers who at present teach mathematical subjects at high schools are not fully trained for such work. Attention will be given in the following chapters to the possible retraining of in-service teachers.

9.11 THOUGHTS ON THE TRAINING OF MATHEMATICS TEACHERS IN SOUTH AFRICA

The data furnished in the following chapters will show clearly that the shortage of teachers with training of at least two Mathematics courses towards a degree is so great that there is little hope of sufficient teachers with even these minimum qualifications becoming available within the foreseeable future.

> (Translation) In view of the shortage of manpower, especially in the field of the natural sciences in South Africa, and the consideration of the conditions prevailing in our country, there appears to be little hope that all Mathematics teachers will possess a higher academic qualification than B.Sc. within the foreseeable future. In fact, even the B.Sc. degree appears to be a somewhat optimistic expectation in so far as the immediate future is concerned. We must therefore of necessity become reconciled to the idea that a B.Sc. degree together with a professional certificate will still be the standard equipment of our secondary teachers for a long time to come.
>
> It would therefore be more realistic rather to ask how the present B.Sc. degree plus the U.E.D. can be arranged to ensure maximum efficiency. (104, p. 130)

(It is of course also possible to obtain a B.A. degree with Mathematics as major subject. And at some universities the relevant high school teachers diploma is indicated by the abbreviation H.E.D. or S.E.D. Where reference is made to the B.Sc. degree and U.E.D., the B.A. degree and these diplomas are also included.)

> (Translation) A solution which is frequently suggested is that the B.Sc. curriculum should be arranged in such a manner that it offers the desired preparation for the prospective teacher. There are serious objections, however, to such a course. In the first place it is not fair to the subject that it should be taught with some ulterior aim. In the undergraduate studies in Mathematics the interests of Mathematics itself should be put first, there being great danger that, if the choice of subjects in the curriculum is influenced by the student's future career, justice will not be done to the subject itself. A second objection is that those very parts of Mathematics which are concerned with the fundamentals of the subject, can be dealt with satisfactorily only if the student has reached a certain degree of maturity. The student must reach a certain level before he can really appreciate anything of this nature. (104, p. 131)

In a reasonably complete B.Sc. course we can regard the first year as a rounding off of the work of the secondary school, plus an introduction into infinitesimal calculus. In the following two years considerable additional analysis, in the real and complex sphere, say as far as Cauchy's theorem as well as linear Algebra, differential equations, and vector and tensor calculation, and, if at all possible, differential geometry should also be included. If a student has to gain a thorough knowledge of all the foregoing within three years, not much room, if any, is left for further additions.

If cognisance is taken of what the future teacher requires, i.e. after those parts of Mathematics which have a direct influence on the school curriculum, then the following is of particular importance.

In the teaching of Geometry it is essential that the teacher should have a thorough knowledge of the fundamentals of this subject. He should be fully acquainted with a work such as Hilbert's Grundlagen der Geometrie or Van der Waerden's De Logische Grondslagen der Euklidische Meetkunde. A further knowledge of non-Euclidean Geometry will also be very useful.

For the teaching of Algebra a knowledge of groups, rings, bodies etc. as far as the quotient body of an integrity field is of great value. It is essential furthermore that the teacher should have a thorough knowledge of the development and expansion of the number concept. In this connection mention may for example be made of Landau's Grundlagen der Analysis. In view of the fact that the completeness of the system of real numbers is so important, one wonders whether a little functional analysis would not also be appropriate. Since figures and sets play such a fundamental role in the definition of a function, the question arises whether the teacher should not also know more about the Theory of Sets. If there is any time left, an introduction into symbolic logic may also be very useful, and finally every teacher should, after all, also know something about the history of his subject. (104, p. 132)

It is also desirable that all Mathematics teachers should, in the course of their training, become thoroughly acquainted with Statistics, irrespective of whether or not room is found for Statistics in the high school curriculum.

> (Translation) We therefore have a whole list of subjects which cannot possibly all be squeezed into undergraduate studies, quite apart from the fact that the method of teaching the subject has not even been mentioned. It would appear that the only solution is that these subjects should be studied during the year set aside for the education diploma. If thorough work is to be done, it will be necessary to allocate at least five periods a week to Mathematics itself throughout the U.E.D. year. These periods do not include the time spent on the method of the subject, although the two will of course sometimes be intermingled.
>
> The objection may understandably be raised that the implementation of these ideas would not leave enough time for the study of Education itself. This is a real difficulty, but the fact remains that the teacher's approach to his subject must be from a more advanced level than that of the pupil. He must on no account be merely a few steps ahead of his pupils but must be able to view their work from a much higher standpoint. Without the proposed additional study he will not be able to do so. Everybody realises that a good teacher of a particular subject must be a master thereof; otherwise the educationists would be wasting their time on him. No study of Education and Psychology can compensate for the defective knowledge of a subject, and for that reason it is strongly advocated that greater stress in the training of our secondary teachers should be laid on acquiring a thorough knowledge of a subject. (104, p. 132)

In view of May's remark that the present Mathematics courses at the universities still concentrate too strongly on the requirements of engineers, the question arises whether the universities cannot give consideration to the introduction of a separate B.A. Mathematics course for Mathematics teachers and others studying the social sciences and humanities. In such a course there need be no hurry to include differential equations as early as possible for the sake of those taking Physics and Applied Mathematics. Instead attention can be given particularly to the fields of study which are of importance to future Mathematics teachers, economists, sociologists, and others interested in the social sciences. Whereas B.Sc. Mathematics is aimed more at application in the physical sciences, B.A. Mathematics should be intended more for application in the humanities. The development of such a curriculum falls outside the scope of this investigation, but it may with advantage be tackled by persons qualified to do so. In the meantime it is desirable that a measure of differentiation should take place on the undergraduate level. It may be possible to distinguish between those students who will later become Mathematics teachers and those who receive training in other directions.

It is furthermore worthy of consideration that the B.Ed. courses for

Mathematics teachers should also be directed towards the subject.

In addition to existing training of secondary teachers, institution of integrated courses at universities also merits attention. Provision should be made for courses which will enable the prospective teacher to proceed with both his academic and his professional studies immediately after his first year at university. Such an arrangement will make his studies more purposeful and is similar to that followed in the study of medicine and engineering. The teacher's diploma can be obtained after three years and with a further year of study the student can also satisfy the requirements for a Bachelor's degree. (Another possibility is that both the degree and the teacher's diploma can be gained simultaneously at the end of the fourth year.)

> "It is essential that university teachers of mathematics co-operate in the instruction of teachers in training colleges or in the University Education Diploma courses. Mathematics teachers in training should be made familiar with the set theoretic approach and will have to know something about the philosophy of mathematics, something of the value of a minimum logical calculus. Another important aspect of teacher training should be the emphasis on continuity from the beginnings of arithmetic in junior school through high school mathematical courses and on through the university. All teachers involved in this development should be in communication with one another. Junior school arithmetic teachers, high school mathematics teachers, university mathematics teachers - all should be intimately associated somewhere along the line. And note in particular that even if the modern set theoretic approach is a long time in coming, it is nevertheless possible to lay the foundations before its arrival. For example, junior school teachers could vary the format of the conventional arithmetic problems so that the pupils would become familiar with the vertical arrangement of an addition sum and the horizontal, equation form of the same problem. The difference between numbers and numerals could also be made clear." (62, p. 34)

One of the shortcomings of the teaching of Mathematics at high schools is the fact that teachers are trained to give instruction in a very rigid syllabus. "This curriculum is immutable and time-consuming,and no teacher finds time to digress a moment from his planned task. His job is to teach manipulatory skills and the way he does it is by drill. It is simply not possible for him to know how his teaching will benefit the pupil when he later studies mathematics at the university."

> "The fault lies with the rigid syllabus, the human limitations of the teacher, and the pupil's lack of enthusiasm for any effort which will not show in his matriculation examination."

Although the majority of the teachers responsible for the teaching of mathematical subjects have less than twenty years' service it is nevertheless necessary that the teachers' knowledge of modern Mathematics should be brought up to date. The fact that mathematics periodicals are read by such limited numbers makes refresher courses even more essential. What aspects should receive attention in such courses?

> "Making use of set theory as a unifying concept and bearing in mind the modern axiomatic basis of algebra require two new things of the teacher. First, he must have a sound knowledge of elementary set theory and, second, he must have a sound working knowledge of the axiomatic basis of modern algebra. All our teachers are likely to be lacking in these two respects. To dispel any psychological barrier that is likely to be built in self-defence, let me say that these topics are not beyond the reach of the competent mathematics teacher. Set theory is a delightful subject with an immediate appeal to students, and the unifying nature of modern algebraic concepts should be an encouragement well worth the effort required to master them." (62, p. 34)

CHAPTER 10

THE TEACHERS AT PRESENT TEACHING MATHEMATICAL SUBJECTS IN HIGH SCHOOLS

10.1 WHAT IS EXPECTED OF A MATHEMATICS TEACHER?

10.1.1 General

(Translation) In discussing the mathematics teacher it should be borne in mind that such a person should be both teacher and mathematician. Everyone choosing this vocation must have a love of children and of Mathematics The love of children is not easily acquired and no training, no matter how good, will guarantee it. This love is innate, and all who feel that they do not possess it would be wise to stay out of the teaching profession.

Love for an appreciation of mathematics can only persist in a person who is actively engaged in it. A teacher should read and study mathematics and not use it only in his daily school round or else the subject loses all its appeal and vitality. The repetition of the same manipulations and calculations year after year must eventually bore him. If he is bored with mathematics then how much more boring his pupils will find it. (104, p. 129)

The Mathematics teacher must be both carefully and purposefully trained to do his work, and he must show a permanent and even an increasing interest in his subject.

10.1.2 Division of the High School

Is it necessary that teachers in the lower and in the higher classes of high school should possess the same qualifications? Should there not be a difference in respect of the requisite qualifications of the teachers for the higher and for the lower classes?

In Europe the position in general is as follows:

"Generally speaking the secondary schools are divided into a lower cycle of three years and an upper cycle of three years." (76, p. 146)

10.1.3 The lower classes

"There are two approaches to teaching in the lower cycle. One is to extend the elementary training programme by attending a university for a minimum of two additional years, specializing in at most two subjects and passing required examinations. The other is to attend a university for three years, studying two subjects - e.g. mathematics and physics - and the necessary courses in pedagogy. One can, on passing the examinations, obtain a first licence for teaching in the secondary schools in the lower cycle. By further study a teacher can advance to teaching in the upper cycle." (76, p. 146)

"If certified elementary school teachers wish to teach mathematics in the middle school or the lower cycle of the secondary school, they must extend their training by attending a university. There they specialize in mathematics and pass an examination on the equivalent of a two-year programme in that field. No teacher can teach beyond this level unless he or she completes a university degree or licence in mathematics." (76, p. 144)

10.1.4 The higher classes

"To teach in the upper cycle of the secondary school, the requirement is, without exception, graduation from a university (or the equivalent by state examination.) However this graduation does not represent the same degree or kind of mathematics study in all countries. This varies to a large extent from country to country." (76, p. 146)

10.1.5 Conclusion

It is clear that at least two university courses in a mathematical subject are necessary before one is qualified to teach the subject at a high school. In South Africa this minimum qualification is also required, but it is difficult to apply this in practice because of the present shortage of teachers.

That these requirements are not unreasonable, appears from the following quotation:-

> (Translation) In order to remain alive to his subject, the teacher must have reached a high level of proficiency and as Mathematics is such an old and all-embracing science, it is virtually self-evident that he should have obtained at least a B.Sc. Honours, or an M.Sc. degree. A comparison with other countries confirms this conclusion. For the past couple of centuries the continent of Europe has quite definitely been in the lead in the field of Mathematics and here it has for many years been the case that the Mathematics teacher has had to reach at least that minimum level in his subject. In the Netherlands, for example, the doctoral examination is still regarded as the admission requirement to secondary teaching. (104, p. 130)

10.2 THE ARITHMETIC TEACHER

What requirements should a teacher satisfy in order to be considered a properly qualified Arithmetic teacher for standards 6, 7 and 8? This is one of the most difficult questions to answer. Some people are of the opinion that the same requirements should apply as for Mathematics teachers, namely at least two university courses in Mathematics. On this matter the last word has not yet been said. "I wish to repeat emphatically that the more mathematical a background a teacher of arithmetic possesses, the better; but to suggest that the ideal content preparation for the teacher of arithmetic lies in the study of algebra, geometry and trigonometry is as questionable as to suggest that the ideal preparation for a good tennis player lies in practising ping-pong and badminton.

> "There are far too many administrators and general education specialists now who contend, in effect, that if the prospective teacher knew enough arithmetic to pass out of the eighth grade, then she knows enough arithmetic to go back and teach that subject at any grade level below the ninth.
>
> "The mature and serious study of the content of arithmetic is not only worthy of a place on the college programme of a student preparing to enter elementary teaching, but, I contend, it represents the most valuable single investment of that student's mathematics-course-taking time." (64, p. 572-3)

10.3 TEACHERS CONSIDERED TO BE QUALIFIED AND TO BE UNQUALIFIED IN MATHEMATICS

As mentioned in Chapter 1, teachers who possess the following qualifications are considered, in this report, as being qualified in Mathematics:

The teacher should have successfully completed two university courses in one or more of the following subjects:

Mathematics : Academic or engineering course
Applied Mathematics : Academic or engineering course
Mathematical Statistics:
Statistical Methods
Advanced Technical Certificate II or a National Diploma in Mathematics.

Professional educational qualifications are not considered at this stage.

Seven hundred and thirty-five of the teachers who returned completed questionnaires, complied with these requirements in full, while 986 teachers had to be considered as "unqualified in Mathematics."

10.4 AGE DISTRIBUTION

The age distribution of the teachers in reflected in Table 10.1.

TABLE 10.1

AGE DISTRIBUTION OF TEACHERS IN MATHEMATICAL SUBJECTS

Experience	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
19 years and younger	0	-	2	0.2	2	0.1
20 - 24 years	117	15.9	154	15.6	271	15.8
25 - 29 years	127	17.3	188	19.1	315	18.3
30 - 39 years	157	21.4	191	19.3	348	20.2
40 - 49 years	163	22.2	217	22.0	380	22.0
50 - 59 years	128	17.4	153	15.5	281	16.3
60 - 64 years	26	3.5	37	3.8	63	3.7
65 years and older	15	2.0	26	2.6	41	2.4
Age not stated	2	0.3	18	1.9	20	1.2
TOTAL	735	100.0	986	100.0	1721	100.0

In the survey a little more than a third of the teachers (34.2%) were younger than 30 years of age, while 6.1% were 60 years old or more.

10.5 THE NUMBER OF YEARS EXPERIENCE

In Table 10.2 an analysis in shown of the teachers who were teaching mathematical subjects during 1962, according to the number of years of experience in teaching Mathematics, Arithmetic and Mechanics.

TABLE 10.2

TEACHERS ACCORDING TO YEARS EXPERIENCE IN TEACHING MATHEMATICAL SUBJECTS

Experience	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
0 - 2 years	170	23.1	313	31.6	483	28.2
3 - 5 years	98	13.3	205	20.7	303	17.6
6 - 10 years	92	12.5	168	17.3	260	15.1
11 - 19 years	172	23.5	121	12.3	293	17.0
20 - 29 years	112	15.2	91	9.2	203	11.8
30 - 39 years	77	10.5	53	5.4	130	7.5
40 years and longer	13	1.9	20	2.0	33	1.9
Not given	1	-	15	1.5	16	0.9
TOTAL	735	100.0	986	100.0	1721	100.0

10.5.1 <u>Qualified in Mathematics</u>

Almost two-thirds (63.6%) of these teachers had more than five years experience behind them. More than a quarter of the teachers had been 20 years or longer in service. If it is true that a university training of 20 years standing may no longer be accepted as being up to date, unless it is supplemented and brought up to date, then 27.6% of the Mathematics teachers who are considered to be qualified should undergo retraining in Mathematics on the grounds of this hypothesis. (79, p. 238)

10.5.2 Unqualified in Mathematics

It is significant that almost a third (31.6%) of the teachers who are required to teach Mathematics without the necessary mathematical qualifications, have from nil to two years experience only in teaching the subjects. More than half (52.3%) have five years experience at the most. It is difficult to see how justice can be done to the mathematical subjects under such circumstances.

10.6 THE POSTS FILLED BY THE TEACHERS

In Table 10.3 an indication is given of the number of teachers filling different posts. The numbers only refer to those teachers from whom completed questionnaires were received.

10.6.1 Persons qualified in Mathematics

If it be accepted that those persons who concealed their status were assistant teachers, then 62.1% of the persons who answered the questionnaires were assistant teachers.

The fact that about equal numbers of principals, vice-principals and senior teachers teach the mathematical subjects is not of full significance because the post of senior teacher does not exist in some provinces. It is significant that a large number of principals and vice-principals have to teach Mathematics.

TABLE 10.3

THE DISTRIBUTION OF MATHEMATICS TEACHERS ACCORDING TO THE POSTS THEY FILL

Post occupied	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
Principal	85	11.1	54	5.5	139	8.1
Vice-principal	91	12.5	50	5.1	141	8.2
Senior teacher or departmental head	87	11.8	54	5.5	141	8.2
Lecturer (in technical college)	18	2.5	4	0.4	22	1.3
Assistant teacher	414	56.6	708	71.7	1122	65.1
Grade not mentioned	40	5.5	116	11.8	156	9.1
TOTAL	735	100.0	986	100.0	1721	100.0

10.6.2 Unqualified teachers in Mathematics

The teachers were requested to indicate on Questionnaire N.B. 377 which mathematical subjects they taught mainly. In Table 10.4 an indication is given of the manner in which these subjects are shared between teachers who are qualified in Mathematics and other mathematical subjects.

TABLE 10.4

THE TEACHERS OF THE VARIOUS MATHEMATICS SUBJECTS

Subject	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
Arithmetic	41	5.6	305	30.9	346	20.5
General Mathematics	102	13.9	250	25.4	352	20.8
Mathematics for Stds. 6, 7 and 8	121	16.5	222	22.5	343	20.3
Mathematics for Stds. 9 and 10	415	56.5	120	12.2	535	31.6
Did not answer	56	7.5	89	9.0	145	6.8
TOTAL	735	100.0	986	100.0	1721	100.0

The qualified Mathematics teachers who teach mainly Arithmetic are only 5.6% of the total whereas nearly a third, (30.9%), of the teachers who are not qualified in Mathematics teach Arithmetic. Nearly an eighth (12.2%) of the unqualified teachers have to teach Mathematics, for the most part, in Standards 9 and 10.

As may be expected, there is a tendency to use teachers with lower qualifications in Arithmetic and Mathematics in the junior classes. They are expected to lay the foundations upon which the teachers with higher qualifications have to attempt to build in the higher standards.

10.7 FURTHER QUALIFICATIONS OF THE TEACHERS WHO ARE QUALIFIED IN MATHEMATICS

The qualifications of the teachers appear in Table 10.5.

TABLE 10.5

ACADEMIC AND PROFESSIONAL EDUCATIONAL QUALIFICATIONS OF THE MATHEMATICALLY QUALIFIED TEACHERS

Academic			Professional Educational		
Degree	Number	%	Diploma	Number	%
B.A.	74	10.1	University:		
B.Sc.	429	58.3	H.E.D., U.E.D., S.E.D.	411	55.9
Two of the following degrees			U.P.T.D., H.P.T.D.	9	1.2
B.A., B.Sc., B.Com., L.L.B.	7	1.0	Training College:		
			T.E.D., T.H.E.D., N.S.T.D.,		
B. Com.	18	2.4	01, 02, H.P.T.C.	209	28.4
Honours degree	21	2.9	Technical College:		
M.Sc., M.Com., M.A., M.Ed.	61	8.2	Diploma	31	4.2
B.Ed.	43	5.9	Non-educational diplomas	4	0.5
National Engineering Diploma	1	0.1	Other	71	9.8
Doctor's degree	4	0.5			
Other (e.g. only two degree courses)	77	10.6			
TOTAL	735	100.0	TOTAL	735	100.0

Conclusion

With the exception of four,the qualified teachers all have a teacher's certificate.

Only 86 of the 735 qualified teachers of Mathematics have an Honnours or higher degree; i.e. 11.7 per cent of the total. Six hundred and fifty-seven (89.4%) are graduates.

In Table 10.6 the number of persons who have acquired degrees and diplomas other than the Bachelor's degree are shown according to the field in which they have obtained a degree or diploma.

Table 10.7 contains the number of teachers who teach Mathematics and who have a degree in Education in mathematical subjects. The number of B.Ed. degrees does not agree with that shown in the previous table because a mathematical orientation is specified.

If there should be complaints that Mathematics teachers do not study further in Mathematics, further study for an educational degree should not be blamed as the cause.

TABLE 10.6

FIELD IN WHICH DEGREE (OTHER THAN BACHELOR'S) HAS BEEN ACQUIRED BY QUALIFIED MATHEMATICS TEACHERS

Mathematics

Degree or diploma	Academic course	Engineering course	Specialization course
Honours	5	2	
Master	14		
Doctor	1		
A. T. C. I		5	
A. T. C. II and National Diploma	1	25	6
TOTAL	21	32	6

Applied Mathematics

Degree or diploma	Academic course	Engineering course	Mathematical Statistics
Honours			1
Master	3	1	2
Doctor		1	
A. T. C. I			
A. T. C. II and National Diploma		10	2
TOTAL	3	12	5

Not much interest is taken in post-graduate study in a mathematical direction.

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TABLE 10.7

DEGREES IN EDUCATION ACQUIRED BY TEACHERS WHO TEACH MATHEMATICAL SUBJECTS

Degree acquired	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
B. Ed.	27	3.7	8	0.8	35	2.0
M. Ed.	8	1.1	4	0.4	12	0.7
Doctor in Education	1	0.1	0	-	1	-
Other	699	95.1	974	98.8	1673	97.3
TOTAL	735	100.0	986	100.0	1721	100.0

10.8 ESTIMATE OF THE NUMBER OF TEACHERS WHO TEACH MATHEMATICAL SUBJECTS

10.8.1 The number of teachers

The above tables refer to teachers who completed and returned Questionnaire N.B.377. In the following paragraphs some estimates are given which are based on data supplied by principals of schools in reply to Questionnaire N.B.375. In Tables 10.8 to 10.12 the number of teachers, the number of Mathematics teachers and the number of school principals who teach Mathematics in their schools are set out. As some of the questionnaires were not returned, estimates were made to give a fuller picture. The method of making this estimate was set out in Chapter 1.

TABLE 10.8

THE NUMBER OF MATHEMATICS TEACHERS AT PUBLIC HIGH SCHOOLS IN THE TRANSVAAL

Schools		Teachers			School principals still teaching		
Language medium	Number	Number	Qualified in Mathematics		Mathematics	General Mathematics	Arithmetic
			Number	%			
According to questionnaires received							
Afrikaans	76	2505	155	6.2	7	1	0
English	39	1219	94	7.7	11	2	4
Parallel	14	447	24	5.4	2	1	0
TOTAL	129	4171	273	6.5	20	4	4
According to estimate							
Afrikaans	81	2679	166	6.2	7	1	0
English	44	1315	101	7.7	12	2	4
Parallel	18	572	31	5.4	2	1	0
TOTAL	143	4566	298	6.5	21	4	4

TABLE 10.9

THE NUMBER OF MATHEMATICS TEACHERS AT PUBLIC SCHOOLS IN THE CAPE PROVINCE

Schools		Teachers			School principals still teaching		
Language medium	Number	Number	Qualified in Mathematics Number	Qualified in Mathematics %	Mathematics	General Mathematics	Arithmetic
According to questionnaires received							
High Schools							
Afrikaans	109	1784	153	8.6	31	20	9
English	35	658	90	13.7	10	9	2
Parallel	26	435	45	10.3	5	5	0
TOTAL	170	2877	288	10.0	46	34	11
Secondary schools							
Afrikaans	29		10		9	11	7
English	3		0		0	1	0
Parallel	8		2		4	3	4
TOTAL	40	222	12	5.0	13	15	11
According to estimate							
High Schools	207	3464	347	10.0	55	41	13
Secondary schools	53	269	15	5.0	16	19	16
TOTAL	260	3733	362	9.7	71	60	29

It was not possible to determine how many teachers were actually concerned with the secondary departments since the total numbers of teachers attached to both primary and secondary departments of schools were shown on the questionnaires. The numbers given are estimates.

TABLE 10.10

THE NUMBER OF MATHEMATICS TEACHERS AT PUBLIC SCHOOLS IN NATAL

Schools		Teachers			School principals still teaching		
Language medium	Number	Number	Qualified in Mathematics		Mathematics	General Mathematics	Arithmetic
			Number	%			
According to questionnaires received							
High Schools							
Afrikaans	4	125	4	3.2	0	0	0
English	18	475	32	6.7	5	1	3
Parallel	9	222	19	8.6	1	0	0
Secondary schools							
English and Parallel	4	60	6	10.0	4	0	1
TOTAL	35	882	61	6.9	10	1	4
According to estimate							
High schools							
Afrikaans	5	156	5	3.2	0	0	0
English	22	628	42	6.7	7	1	4
Parallel	10	247	21	8.6	1	0	0
Secondary schools							
English and Parallel	4	60	6	10.0	4	0	1
TOTAL	41	1091	74	6.8	12	1	5

TABLE 10.11

THE NUMBER OF MATHEMATICS TEACHERS AT PUBLIC SCHOOLS IN THE ORANGE FREE STATE

Schools		Teachers			School principals still teaching		
Language medium	Number	Number	Qualified in Mathematics		Mathematics	General Mathematics	Arithmetic
			Number	%			
According to questionnaires received							
High schools							
Afrikaans	15	294	42	14.3	4	0	1
Parallel	5	172	8	4.7	3	0	1
TOTAL	20	466	50	10.7	7	0	2
Secondary departments of schools with primary and secondary classes							
Afrikaans	35	624	34	5.6	9	2	1
Parallel	10	188	13	6.9	1	0	0
TOTAL	45	812	47	5.8	10	2	1
According to estimate							
High schools	27	603	65	10.7	9	0	3
Secondary departments	53	963	56	5.8	12	2	1
TOTAL	80	1566	121	7.7	21	2	4

TABLE 10.12

THE NUMBER OF MATHEMATICS TEACHERS AT VOCATIONAL HIGH SCHOOLS

Schools		Teachers			Principals who teach		
Type	Number	Number	Qualified in Mathematics		Mathe-matics	General Mathe-matics	Arith-metic
			Number	%			
According to questionnaires received							
Housecraft Schools	6	64	0	-	0	0	0
Commercial and Technical	5	128	20	15.6	1	1	0
Commercial	8	160	5	3.1	1	0	0
Technical	11	398	52	13.1	0	0	0
Industrial	12	237	7	3.0	0	0	0
Agricultural	5	70	2	2.9	0	0	0
Special	2	33	1	3.3	0	0	0
TOTAL	49	1090	87	8.0	2	1	0
According to estimate							
Housecraft Schools	9	105	0	-	0	0	0
Commercial and Technical	9	251	39	15.6	2	2	0
Commercial	14	274	9	3.1	2	0	0
Technical	17	593	78	13.1	0	0	0
Industrial	17	294	9	3.0	0	0	0
Agricultural	15	312	9	2.9	0	0	0
Special	2	33	1	3.3	0	0	0
TOTAL	83	1862	145	7.8	4	2	0

10.8.2 The number of posts for mathematical subjects

Table 10.13 compares the number of posts for the mathematical subjects with the number of teachers who are qualified in Mathematics. Use is made of estimated numbers.

TABLE 10.13

THE ESTIMATED NUMBER OF POSTS FOR MATHEMATICAL SUBJECTS AND THE AVAILABLE NUMBER OF TEACHERS QUALIFIED IN MATHEMATICS (1962)

Schools	Number of posts					Number of teachers		
Language medium/Type	Number	Arithmetic	General Mathematics	Mathematics	Total	Qualified in Mathematics	Not teaching a mathematical subject	Available
Transvaal								
Afrikaans	81	145	144	173	462	166	39	127
English	44	54	91	107	252	101	22	79
Parallel	18	32	28	43	103	31	7	24
TOTAL	143	231	263	323	817	298	68	230
Cape Province								
High	207	234	273	227	734	347	41	306
Secondary	53	24	37	14	75	15	1	14
TOTAL	260	258	310	241	809	362	42	320
Natal								
Afrikaans	5	22	4	14	40	5	1	4
English	22	87	13	83	183	42	4	38
Parallel	10	36	2	33	71	21	2	19
Secondary	4	10	0	5	15	6	1	5
TOTAL	41	155	19	135	309	74	8	66
Orange Free State								
High	27	66	35	46	147	65	9	56
Secondary	53	77	39	63	179	56	4	52
TOTAL	80	143	74	109	326	121	13	108
Vocational High Schools								
Housecraft schools	9	10	0	0	10	0	0	0
Commercial & Technical	9	4	2	37	43	39	4	35
Commercial	14	19	4	18	41	9	2	7
Technical	17	0	21	44	65	78	8	70
Industrial	17	9	12	12	33	9	1	8
Agricultural	15	9	18	13	40	9	8	1
Special	2	2	0	1	3	1	0	1
TOTAL	83	53	57	125	235	145	23	122

10.8.3 The estimated shortage of Mathematics teachers

If it be assumed that a teacher in Arithmetic, Mathematics and General Mathematics should have successfully completed a minimum of two degree courses in Mathematics, or have passed A. T. C. II or a National Diploma in Mathematics, the shortage of such teachers will be as indicated in Table 10.14.

TABLE 10.14

THE ESTIMATED SHORTAGE OF TEACHERS WHO ARE QUALIFIED IN MATHEMATICS (1962)

Schools		Posts	Teachers available	Shortage	
Department	Medium/Type			Number	%
Transvaal	Afrikaans	462	127	335	72.5
	English	252	79	173	68.7
	Parallel	103	24	79	76.7
	TOTAL	817	230	587	71.8
Cape of Good Hope	High	734	306	428	58.3
	Secondary	75	14	61	81.3
	TOTAL	809	320	489	60.4
Natal	Afrikaans	40	4	36	90.0
	English	183	38	145	79.2
	Parallel	71	19	52	73.2
	Secondary	15	5	10	66.7
	TOTAL	309	66	243	78.6
Orange Free State	High	147	56	91	61.9
	Secondary department	179	52	127	70.9
	TOTAL	326	108	218	66.9
Education, Arts and Science	Housecraft schools	10	0	10	100.0
	Commercial and Technical	43	35	8	18.6
	Commercial	41	7	34	82.9
	Technical	65	70	-5	-7.7
	Industrial	33	8	25	75.8
	Special	3	1	2	66.7
	TOTAL	195	121	74	37.9

The technical schools have no shortage but apparently have a surplus of teachers for Mathematics.

Class II schools of the Transvaal Education Department and agricultural high schools are not included because insufficient data were available at the time.

The table makes no provision for teachers of Mechanics and Commercial Arithmetic.

A disturbing shortage of teachers with the necessary professional qualifications and training is apparent.

An obvious reason for this great shortage of Mathematics teachers is the low percentage of Afrikaans-speaking pupils who take Mathematics up to Standard 10. Recruits for the teaching profession are largely drawn from the Afrikaans-speaking members. After the professions for which Standard ten Mathematics is a requirement have absorbed their proportion of the mathematically trained matriculants, very few are left for teaching.

10.9 THE REASONS WHY SOME MATHEMATICS TEACHERS DO NOT TEACH MATHEMATICS

Accordingto Table 10.13 there are 154 teachers with the necessary qualifications in Mathematics who nevertheless do not teach Mathematics, General Mathematics or Arithmetic. Seeing that there is a terrific shortage of qualified Mathematics teachers, it is most important to know the reason for this.

In Table 10.15 the most important reasons are indicated. The number of teachers who do not teach a mathematical subject on account of their incompetency or because they teach a science subject or because of administrative duties, or for some other reason, are indicated in the table.

TABLE 10.15

THE NUMBER OF MATHEMATICS TEACHERS WHO FOR SOME REASON OR OTHER DO NOT TEACH MATHEMATICAL SUBJECTS (1962)

School group	Number of schools	Number of teachers who for given reasons do not teach a mathematical subject				
		Incompetency	Teach Science	Administrative duties	Other reasons	Total
Transvaal						
Afrikaans medium	81	8	21	5	5	39
English medium	44	0	22	0	0	22
Parallel medium	18	0	7	0	0	7
TOTAL	143	8	50	5	5	68
Cape Province						
High	207	0	34	2	5	41
Secondary	53	0	0	1	0	1
TOTAL	260	0	34	3	5	42
Natal						
Afrikaans medium	5	0	0	0	1	1
English medium	22	0	4	0	0	4
Parallel medium	10	1	1	0	0	2
Secondary	4	0	1	0	0	1
TOTAL	41	1	6	0	1	8
Orange Free State						
High	27	0	6	3	0	9
Secondary departments	53	0	2	1	1	4
TOTAL	80	0	8	4	1	13
Vocational High Schools						
Commercial & Technical	9	0	4	0	0	4
Commercial	14	0	2	0	0	2
Technical	17	0	3	5	0	8
Industrial	17	0	0	1	0	1
Agricultural	15	0	4	4	0	8
TOTAL	72	0	13	10	0	23

The Mathematics teachers who do not teach a mathematical subject are largely used to teach a science, probably Physical Science. This is understandable, since many teachers take Mathematics and Chemistry or Mathematics and Physics as their major subjects for degree purposes.

10.10 THE NATURE OF THE APPOINTMENTS OF TEACHERS WHO TEACH MATHEMATICAL SUBJECTS

In Table 10.16 an indication is given of both the sex and the nature of the appointment of the qualified and the unqualified teachers of Mathematics.

TABLE 10.16

THE NATURE OF THE APPOINTMENT OF THE TEACHERS OF MATHEMATICAL SUBJECTS

Nature of appointment	Qualified in Mathematics				Unqualified in Mathematics				Grand Total
	Male	Female	Total	%	Male	Female	Total	%	
(a) Permanent	531	71	602	81.9	567	88	655	66.4	1257
(b) Temporary, but eligible for permanent appointment	21	16	37	5.0	64	31	95	9.6	132
(c) Retired teacher	22	4	26	3.5	37	6	43	4.4	69
(d) Married women, not eligible for permanent appointment	-	33	33	4.5	-	105	105	10.7	138
(e) Without recognised teachers'diploma	19	4	23	3.2	15	1	16	1.6	39
(f) No information given	-	-	14	1.9	-	-	72	7.3	86
TOTAL	593	128	735	100.0	683	231	986	100.0	1721
As a percentage of total number of teachers	34.5	7.4			39.9	13.4			

Conclusions

If the 86 cases for which the data supplied was inadequate are left out of consideration, it is found that:

(1) Of the men 3.1% and 1.6% of the women are without a teacher's diploma.

(2) Of the teachers who teach mathematical subjects, 74.4% are men and 20.8% are women. (The 86 form the remaining 4.8%).

(3) Of all the teachers 97% are in possession of a teacher's diploma.

(4) Married women constitute 6.3% of the 1721 teachers. If account be taken of the fact that since 1961 twenty per cent of the teachers' posts in the Transvaal Education Department have been available for married women, then 6.3% is a comparatively small amount.

In Table 10.17 an indication is given of the manner in which the Arithmetic posts are filled in those schools from which completed questionnaires were received.

In Table 10.18 an indication is given of the manner in which the posts for General Mathematics are filled at the different schools. In Tables 10.17, 10.18 and 10.19 no distinction is made between teachers who are qualified as Mathematics teachers or not but only whether they are qualified as teachers or not.

In Table 10.19 an indication is given of the way in which the Mathematics posts in the different groups of schools were filled. Only those schools which answered the questionnaires are shown. Only the manner of appointment is indicated in this table. The teachers were not necessarily trained for teaching Mathematics.

TABLE 10.17
HOW THE ARITHMETIC POSTS WERE FILLED

School groups	Teachers: Permanently appointed		Temporary but eligible for permanent appointment		Without teacher's diploma		Married women and retired teachers		Total
	M	F	M	F	M	F	M	F	
Transvaal									
Afrikaans medium	86	7	21	5	5	2	3	19	148
English medium	12	6	5	5	0	0	2	20	50
Parallel medium	16	5	1	2	0	0	1	2	27
TOTAL	114	18	27	12	5	2	6	41	225
Percentage	50.7	8.0	12.0	5.3	2.2	0.9	2.7	18.2	100.0
Cape Province									
High schools									
Afrikaans medium	96	15	2	3	10	0	4	10	140
English medium	17	11	0	0	0	0	2	7	37
Parallel medium	27	6	0	0	2	0	1	1	37
Secondary									
Afrikaans medium	9	2	2	0	2	0	0	1	16
Parallel medium	2	0	0	0	0	0	0	1	3
TOTAL	151	34	4	3	14	0	7	20	233
Percentage	64.8	14.6	1.7	1.3	6.0	0	3.0	8.6	100.0
Natal									
Afrikaans medium	13	1	0	0	0	0	1	0	15
English medium	35	23	2	3	5	1	3	4	76
Parallel medium	16	10	2	1	0	0	1	4	34
Secondary	6	2	1	2	0	1	0	0	12
TOTAL	70	36	5	6	5	2	5	8	137
Percentage	51.1	26.3	3.6	4.4	3.6	1.5	3.6	5.9	100.0
Orange Free State									
High schools									
Afrikaans medium	19	3	0	1	1	0	4	3	31
Parallel medium	16	0	1	0	2	0	3	0	22
Secondary schools									
Afrikaans medium	34	3	3	2	7	3	0	2	54
Parallel medium	10	7	3	0	0	0	0	4	24
TOTAL	79	13	7	3	10	3	7	9	131
Percentage	60.4	9.9	5.3	2.3	7.6	2.3	5.3	6.9	100.0
Vocational High School									
Housecraft schools	0	2	0	0	0	0	0	3	5
Commercial and Technical	1	0	0	0	0	0	0	0	1
Commercial	11	0	0	0	1	0	0	0	12
Technical	1	1	0	0	1	0	0	0	3
Industrial	4	2	0	0	0	0	0	1	7
Agricultural	2	0	0	0	0	0	0	0	2
Special	1	0	0	0	0	0	0	1	2
TOTAL	20	5	0	0	2	0	0	5	32
Percentage	62.5	15.6	0	0	6.3	0	0	15.6	100.0

10.11 THE RELATION BETWEEN THE PERCENTAGE OF PUPILS WHO TAKE MATHEMATICS IN STANDARD 10 AND THE NUMBER OF TEACHERS AVAILABLE

In Table 10.20 the percentage of pupils who take Mathematics in Standard 10 are compared with the available permanently appointed male teachers for the subject. The numbers in the Table are obtained from the completed questionnaires which were returned.

It is clear that the percentage of pupils who take Mathematics in Standard 19 is quite independent of the number of teachers of the subject.

TABLE 10.20

THE NUMBER OF MATHEMATICS PUPILS IN STANDARD TEN FOR EACH PERMANENTLY APPOINTED MALE TEACHER WHO TEACHES MATHEMATICS

High school group	Percentage Mathematics pupils in Standard 10	Number of Mathematics pupils in Standard 10	Number of permanently appointed male teachers of Mathematics	Number of Mathematics pupils per male Mathematics teacher
Transvaal				
Afrikaans medium	53.5	2694	115	23
English medium	81.8	2265	40	57
Parallel medium	46.5	588	24	25
Cape Province				
Afrikaans medium	44.3	1423	73	19
English medium	65.4	1032	28	37
Parallel medium	71.4	625	22	28
Natal				
Afrikaans medium	65.7	140	9	16
English medium	86.7	697	46	15
Parallel medium	74.7	297	15	20
Orange Free State				
High schools				
Afrikaans medium	47.5	326	17	19
Parallel medium	64.4	212	11	19
Secondary departments				
Afrikaans medium	53.8	538	26	21
Parallel medium	58.9	66	6	11
Vocational high schools				
Commercial and Technical	67.4	87	13	7
Commercial	13.8	108	9	12
Technical	100.0	264	28	9
Industrial	37.7	23	8	3
Agricultural	33.3	36	2	18

10.12 DISCUSSION

10.12.1 Introduction

Tables 10.16 to 10.19 present a picture of the teachers who are concerned with the teaching of Mathematics. The picture is however only significant when the data is compared with the number of pupils who take Mathematics in Standard 10.

According to Table 10.19 there are 115 permanently appointed male teachers for Mathematics in the Transvaal Afrikaans medium high schools as against 15 retired teachers and married women.

In the Transvaal English medium high schools there are 40 male permanently appointed teachers for Mathematics, compared with 39 retired teachers and married women. There are thus about equal numbers in the two groups while in Afrikaans medium schools a ratio of 115:15 pertains. Notwithstanding this only 53.5% of the Afrikaans Standard 10 pupils are in the Mathematics classes as against 81.8% amongst the English-speaking children.

The present shortage of manpower makes it absolutely necessary that more pupils should matriculate in Mathematics. The survey has indicated that it is not necessary to wait until there are enough teachers of Mathematics available. On the contrary the maximum number of pupils should now take the subject in order to avoid an even more serious shortage of Mathematics teachers in the future.

10.12.2 The shortage of Mathematics teachers

People who are not interested in the professional teaching of the subject will deny that there is a shortage of Mathematics teachers as long as anyone can be found to fill each teaching post for a Mathematics class.

In actual fact the state of affairs is most disturbing. The principal of a large high school in the Transvaal writes that the problem of the teaching of Mathematics at his school is indeed serious. For the past five years he has had a new senior teacher for this subject each year. Some have resigned whilst others have gone to Natal.

> "Twice it has taken two weeks for the Department to find me a teacher in spite of notice given six weeks before the end of the year, i.e. it has taken two weeks after the beginning of the first term. My present senior Maths teacher has not taught Senior Maths for twenty years! My matric results are only average due to failures in Mathematics and Science - particularly Maths, as there is a correlation between the two."

Another principal writes that after having been principal of a large high school on the Witwatersrand for ten years he has never had a qualified Mathematics teacher on his staff. The present Standard 10 teacher has no degree having completed a first year course in Mathematics only.

It is very difficult to obtain properly qualified Mathematics teachers to fill vacancies. Some teachers are qualified in both Mathematics and Science and Physical Science teachers are virtually unobtainable. Even when they can be found, they have to be given extraordinary privileges such as exemption from extramural activities and free lodging in the school hostels.

Some schools have had to do without a qualified Mathematics teacher since 1952 while the services of qualified Mathematics teachers have been available to other classes for only a year or two.

As a result of the shortage of mathematicians it has become customary for the schools to nominate teachers of Mathematics who have the flimsiest of qualifications. As long as a candidate has had a university course in which the name Mathematics appears, this has been considered sufficient qualification for the school committee concerned to make a recommendation for his appointment. It is these teachers who will create a problem when the Mathematics course is extended or changed.

In order to encourage teachers to improve their qualifications, the authorities should give earnest attention to the payment of better salaries to teachers who have higher qualifications in the subjects which they teach. The emoluments should be such that an efficient teacher will be happy to remain a language teacher, a history teacher or a mathematics teacher and need not aspire to a principalship or vice-principalship as the only possibility for promotion.

The teaching of Arithmetic gives rise to considerable problems. It frequently happens that teachers with no training in the subject teach it. The lack of the necessary background and of enthusiasm is a considerable drawback.

One principal has had Arithmetic taught to Standard 6 by three teachers who held the degree of B. Comm. He is most unhappy about this arrangement.

According to the tables, there are hardly sufficient qualified Mathematics teachers in some of the provinces to give Mathematics to all the Mathematics classes. The consequence is that General Mathematics is taught by persons who are not qualified to teach the subjects. The misconception exists that Arithmetic can be taught by any teacher at a school. Now that people have to teach Mathematics under the pseudonym of "General Mathematics", the trouble is even worse. Where specialized Arithmetic teachers for the primary schools are to be found in this atomic age is a mystery.

10.12.3 How to overcome the shortage

In the appointment of principals and vice-principals, preference is frequently given to teachers of Mathematics. According to Table 10.7 about 55 principals in the Cape Province and a quarter of the principals of the English medium high schools in the Transvaal teach Mathematics themselves. Moreover, use is made largely of temporary assistants and of teachers without the necessary qualifications.

10.12.4 Suggested solutions

The absolute minimum requirement for a qualified teacher in mathematical subjects is two degree courses in Mathematics. A school principal writes as follows:

> "What hope have I of trying to persuade my son to go in for teaching when, with a plain B. Sc., he can command an initial salary of R160 per month in industry with the possibility, if he uses his brains and opportunities, to command a salary in ten years time, bigger than I have reached after nearly forty years of teaching? It is not merely paying the new teacher a higher salary. Something must be done for the teacher already in service who for so long and so little has kept our shaky educational edifices upright. These stout pillars are going one by one and there are no replacements."

There has been a suggestion that the maximum salary of a senior assistant (departmental head) in Mathematics should be R4,800 per year.

In order to ease the reigning need which is caused by the gross shortage of Mathematics teachers, pressure is being brought to bear for the immediate institution of training courses during vacations. At the same time it is thought that teachers who give up their holidays to become better qualified should have this recognised financially if necessary.

10.12.5 How can the immediate demand for Mathematics teachers be met?

In order to meet the immediate demand for Mathematics teachers, men and women are needed who

(i) are devoted to teaching and who would prefer to teach Mathematics and Arithmetic classes;

(ii) have sufficient knowledge of the subject through academic study;

(iii) possess the temperament to cope with the routine of teaching (e.g. preparation, correcting written work, setting up and correction of tests, the accurate assessment of the progress of the pupils and the diagnosis of their problems and difficulties and the like);

(iv) have a general knowledge of the methods of teaching and a wide knowledge of the methods and techniques of the teaching of Mathematics and Arithmetic;

(v) are in a position to inspire children when teaching them and to arouse a love for their subject.

The successful teacher of Mathematics and Arithmetic must attain an exceptionally high standard. The recruitment of young men and women with the necessary capacity to take on this work, thus becomes even more difficult by virtue of the fact that schools are forced to place the teaching of mathematical subjects in the hands of teachers who have neither the knowledge nor the desire to teach these subjects. Such teachers are the cause of pupils acquiring a dislike for the subjects and in this way potential teachers are lost to the profession.

Whatever is done to ease the situation, it must be expected that the shortage of well qualified Mathematic teachers will continue. This problem constitutes merely a portion of the larger problem of manpower in the Republic. An attempt to persuade more school girls to become Mathematics teachers may alleviate the shortage. Experience has shown that in the field of Mathematics, women are by no means less capable than men.

> "One way to maintain high scholarship and to ignore the shortage of teachers is, of course, to admit to Mathematics study only that number of children for which qualified teachers exist. This, however, would not produce the increased mathematically trained manpower so sorely needed by any technological society.
>
> "The technological, scientific, and industrial expansion of our society will continue during the next decade, and meeting the mathematical needs of this society becomes a very important problem for all countries to consider." (76, p. 150)

CHAPTER 11

THE TRAINING OF TEACHERS IN SERVICE

11.1 GENERAL INTRODUCTION

> "Some time ago, I met an old friend, a mathematics teacher. Years ago we were students together at the university. He is now a very capable teacher. He said: 'I see you advocate a new program in mathematics instruction in schools'. 'Yes', I said, 'But if this program is introduced in our schools, I cannot teach mathematics any more'." (79, p. 232)

This sad story shows how important the re-education of Mathematics teachers has become.

In the previous chapter the qualifications of the teachers who taught Mathematics and Arithmetic at the high schools were considered. It is clear that many teachers will require further training, because they have not had sufficient training (at least on paper) in order to teach these subjects efficiently.

Secondly, the introduction of a new program for the teaching of Mathematics at high schools will necessitate the retraining of a great number of teachers.

> "The shocking lack of responsibility in keeping abreast of new developments in Mathematics and education is a serious flaw in maintaining an adequate educational programme. All countries should seek ways of helping teachers to continue studying and to constantly improve their teaching of mathematics." (76, p. 151)

The necessity for the retraining of teachers is pointed out by Professor O. Ore: "A university training which is 20 years old, is too old if not supplemented and renewed." (79, p. 238)

11.2 A COMPARISON OF TEACHERS WITH LESS THAN TWENTY YEARS SERVICE, AND THOSE WITH TWENTY YEARS SERVICE AND MORE

11.2.1 Re-training

With reference to the further training or retraining of teachers, the first question that arises is as to which teachers are most in need of such training. Some people are of the opinion that the training of teachers was better in the past and that the older teachers have had so many years of teaching experience that it can hardly be expected of them to attend vacation courses.

On the other hand the opinion has been expressed that all university training of twenty or more years ago is out of date. The old brigade may be good teachers, but not all of them kept abreast of the latest developments in the field of Mathematics.

The younger generation of teachers would wish to benefit from the experience of the older generation by discussing mathematics in the light of modern views with them.

All teachers, young and old, who have not had sufficient training in the Method of teaching Mathematics would benefit by a course in method.

11.2.2 Groups of teachers

As the requirements of the "older" and the "younger" groups of teachers differ from one another, it is appropriate to determine how many there are in each group. The data in respect of 1691 teachers who completed questionnaires have been analysed. In 1962, 1320 teachers had had nineteen years or less teaching experience in these particular subjects, while 352 teachers had twenty years or more experience. Eighteen teachers failed to report for how

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long they had taught their subjects. The manner in which the teachers are distributed in the various education departments is indicated in Table 11.1.

TABLE 11.1

NUMBER OF TEACHERS TEACHING MATHEMATICS ACCORDING TO EDUCATION DEPARTMENT, AND PERIOD OF SERVICE

Education departments	Service period				Total
	19 years and less		20 years and more		
	Number	%	Number	%	
Cape of Good Hope	377	73.8	134	26.2	511
Natal	110	71.9	43	28.1	153
Orange Free State	124	77.0	37	23.0	161
Transvaal	511	81.2	118	18.8	629
South West Africa	18	94.7	1	5.3	19
Education, Arts and Science	160	90.9	16	9.1	176
Private and other schools	20	83.3	4	17.7	24
TOTAL	1320	78.9	353	21.1	1673

More than three quarters (78.9%) of the teachers had less than 20 years experience in teaching by the year 1962. The great need for education in Mathematics is amongst the teachers who have not taken Mathematics at a university and amongst those who have had 20 or more years of service.

11.2.3 The Method of teaching Mathematics

The next question concerns the number of "older" and of "younger" teachers who need training in the Method of teaching Mathematics. The numbers are shown in Table 11.2.

It appears to be mainly the teachers with less than twenty years service who have had no training in the Method of teaching Mathematics.

There is a very large group of teachers who should benefit by attending vacation courses in the method of this subject.

11.2.4 The Method of teaching high school Arithmetic

In Table 11.3 an indication is given of the number of teachers who have had training in the Method of teaching Arithmetic to high school pupils.

A little more than a quarter of the teachers had undergone training in the Method of teaching high school Arithmetic. There is thus a particular need for vacation and other courses for teachers teaching this important subject.

11.2.5 The Method of teaching Mechanics

Only 48 teachers had had training in the Method of teaching Mechanics. Of those 20 were in the Transvaal, while 13 were in the Department of Education, Arts and Science. Thirteen of these teachers had twenty years service or more, while 35 had less than twenty years service.

11.2.6 Teachers' views on the adequacy of their training in the method of teaching mathematical subjects

The number of teachers who were satisfied with the training which they had had in the method of teaching mathematical subjects is indicated in Table 11.4.

TABLE 11.2

TRAINING IN THE METHOD OF TEACHING MATHEMATICS

Number of years service of teachers	Teachers						
	Training				No reply		Total
	Received		Not received				
	Number	%	Number	%	Number	%	
Cape of Good Hope							
Less than twenty years	161	42.7	176	46.7	40	10.6	377
Twenty years and more	79	59.0	48	35.8	7	5.2	134
TOTAL	240	47.0	224	43.8	47	9.2	511
Natal							
Less than twenty years	45	40.9	52	47.3	13	11.8	110
Twenty years and more	21	48.8	21	48.8	1	2.4	43
TOTAL	66	43.1	73	47.7	14	9.2	153
Orange Free State							
Less than twenty years	67	54.0	48	38.7	9	7.3	124
Twenty years and more	24	64.9	13	35.1	0	-	37
TOTAL	91	56.5	61	37.9	9	5.6	161
Transvaal							
Less than twenty years	186	36.4	251	49.1	74	14.5	511
Twenty years and more	54	45.8	53	44.9	11	9.3	118
TOTAL	240	38.2	304	48.3	85	13.5	629
South West Africa							
Less than twenty years	9	50.0	9	50.0	0	-	18
Twenty years and more	0	-	0	-	1	100.0	1
TOTAL	9	47.4	9	47.4	1	5.2	19
Education, Arts and Science							
Less than twenty years	55	34.4	90	56.2	15	9.4	160
Twenty years and more	9	56.2	5	31.2	2	12.6	16
TOTAL	64	36.3	95	54.0	17	9.7	176
GRAND TOTAL	710	43.1	766	46.5	173	10.4	1649

TABLE 11.3

TRAINING IN THE METHOD OF TEACHING HIGH SCHOOL ARITHMETIC

Number of years service of teachers	Teachers						
	Training				No reply		Total
	Received		Not received				
	Number	%	Number	%	Number	%	
Cape of Good Hope							
Less than twenty years	70	18.6	202	53.6	105	27.8	377
Twenty years and more	29	21.6	64	47.8	41	30.6	134
TOTAL	99	19.4	266	52.0	146	28.6	511
Natal							
Less than twenty years	37	33.6	57	51.8	16	14.6	110
Twenty years and more	14	32.6	25	58.1	4	9.3	43
TOTAL	51	33.3	82	53.6	20	13.1	153
Orange Free State							
Less than twenty years	36	29.0	70	56.5	18	14.5	124
Twenty years and more	14	37.8	11	29.7	12	32.5	37
TOTAL	50	31.1	81	50.3	30	18.6	161
Transvaal							
Less than twenty years	211	41.3	240	47.0	60	11.7	511
Twenty years and more	46	39.0	59	50.0	13	11.0	118
TOTAL	257	40.9	299	47.5	73	11.6	629
South West Africa							
Less than twenty years	4	22.2	11	61.1	3	16.7	18
Twenty years and more	0	-	0	-	1	100.0	1
TOTAL	4	21.1	11	57.8	4	21.1	19
Education, Arts and Science							
Less than twenty years	28	17.5	87	54.4	45	28.1	160
Twenty years and more	6	37.5	6	37.5	4	25.0	16
TOTAL	34	19.3	93	52.9	49	27.8	176
GRAND TOTAL	495	30.0	832	50.5	322	19.5	1649

TABLE 11.4

TEACHERS WHO CONSIDERED THAT THE TRAINING WHICH THEY UNDERWENT IN THE METHOD OF TEACHING MATHEMATICAL SUBJECTS WAS ADEQUATE

Years service as teacher of a mathematical subject	Teachers						
	Training				No reply		Total
	Received		Not received				
	Number	%	Number	%	Number	%	
Cape of Good Hope							
Less than twenty years	103	27.3	141	37.4	133	35.3	377
Twenty years and more	48	35.8	49	36.6	37	27.6	134
TOTAL	151	29.5	190	37.2	170	33.3	511
Natal							
Less than twenty years	38	34.5	28	25.5	44	40.0	110
Twenty years and more	20	46.5	9	20.9	14	32.6	43
TOTAL	58	37.9	37	24.2	58	37.9	153
Orange Free State							
Less than twenty years	36	29.0	42	33.9	46	37.1	124
Twenty years and more	15	40.5	16	43.2	6	16.3	37
TOTAL	51	31.7	58	36.0	52	32.3	161
Transvaal							
Less than twenty years	101	19.8	240	47.0	170	33.2	511
Twenty years and more	33	28.0	59	50.0	26	22.0	118
TOTAL	134	21.3	299	47.5	196	31.2	629
South West Africa							
Less than twenty years	3	16.7	6	33.3	9	50.0	18
Twenty years and more	0	-	0	-	1	100.0	1
TOTAL	3	15.8	6	31.6	10	52.6	19
Education, Arts and Science							
Less than twenty years	43	26.9	44	27.5	73	45.6	160
Twenty years and more	8	20.0	4	25.0	4	25.0	16
TOTAL	51	29.0	48	27.3	77	43.8	176
GRAND TOTAL	448	27.2	638	38.7	563	34.1	1649

About a quarter (27.2%) of the teachers are of the opinion that they received adequate training in the method of teaching mathematical subjects.

More of the older teachers were satisfied with their training than was the case amongst the younger ones. A possible reaon is the tendency to use unqualified young teachers to fill vacancies where qualified Mathematics teachers are not available.

It is clear that the further training of serving teachers should receive the most earnest attention of all the education departments.

11.3 THE WILLINGNESS OF TEACHERS TO UNDERGO FURTHER TRAINING

11.3.1 Vacation courses

By means of Questionnaire N. B. 377 a question was put to the teachers regarding the conditions under which they would be prepared to attend vacation courses in the teaching of Mathematics. The number of teachers who answered on certain lines is shown in Table 11.5.

TABLE 11.5

THE NUMBER OF TEACHERS WHO WOULD ATTEND VACATION COURSES IN THE TEACHING OF MATHEMATICS UNDER CERTAIN CONDITIONS

Conditions for attendance at vacation course	Teachers					
	Qualified in Mathematics		Unqualified in Mathematics		Total	
	Number	%	Number	%	Number	%
1. No reply	222	31	375	38	597	35
2. Re-imbursement of travelling and subsistence expenses	221	31	255	26	476	28
3. Will attend course uncontionally	150	21	143	15	293	18
4. Receipt of departmental recognition	80	12	173	18	253	15
5. The opportunity of as many informal contacts as possible with lecturers	35	5	37	3	72	4
TOTAL	708	100	983	100	1691	100

11.3.2 Conclusion

Although only 65% of the teachers showed sufficient interest in the possibility of such a course to answer the question, it is nevertheless significant that it was the qualified teachers of Mathematics who showed most interest and that about a fifth of them (21%) were quite prepared to attend such courses, without specifying any conditions.

The re-imbursement of travelling and subsistence costs would be an encouragement to a considerable number of teachers (28%) to attend a vacation course. In a large and geographically widely spread-out country such as South Africa this is naturally a very important factor. The receipt of departmental recognition is a matter regarding which the unqualified teachers (18%) felt more strongly than those qualified in Mathematics (12%). This is quite understandable since all teachers feel a desire to be qualified for the work which they undertake.

The institution of vacation courses to enable unqualified teachers to qualify themselves gradually and to enable qualified Mathematics teachers to improve their knowledge and bring it up to date, is deserving of the most earnest attention of the authorities in all education departments and of all teachers of Mathematics. "Let us remember that in Denmark now 50 per cent of <u>all</u> mathematics teachers have taken a 14 day summer course in modern mathematics." (79, p. 241)

11.3.3 How should the re-training of teachers take place?

In the further training of teachers in service, the universities should play a most important role.

> "It is felt that the retraining of teachers should be the product of close collaboration between university mathematicians, educators and secondary

school teachers. In this way, it would be possible for teachers who take courses to:

(a) see how the mathematics developed in recent years should be closely related with what goes on in the classroom;

(b) take part in experimental work in teaching secondary school mathematics from a modern point of view; and

(c) sense the genuine need for further study in subjects of 'modern mathematics'."

"Such co-operation would also tend to direct the teaching of modern algebra, algebraic geometry, topology etc. not towards the preparation of specialists and research workers, but towards a better understanding of what must be taught in the secondary school." (76, p. 96-7)

In Roumania and the Ukraine it is expected that high school teachers should attend a free refresher course every five years.

"It is also of interest to note that the German Federal Republic and several countries in Eastern Europe have special institutes for the further training of teachers, in the form of courses, conferences, seminars, etc., and that in France secondary teachers may be seconded to a national scientific centre for the purpose of doing scientific or teaching research." (74, p. 39)

11.4 THE EXTENT TO WHICH TEACHERS MAKE USE OF JOURNALS AND THE NEED FOR SOUTH AFRICAN PROFESSIONAL JOURNALS

11.4.1 The existing journals

The journals read by more than ten teachers who teach mathematical subjects in high schools, are shown in Table 11.6.

TABLE 11.6

THE JOURNALS WHICH ARE READ BY THE NUMBER OF TEACHERS SHOWN

Journals	Teachers		
	Qualified in Mathematics	Unqualified in Mathematics	Total
1. Provincial teachers' journals	83	124	207
2. Journal for science teachers	62	16	78
3. Archimedes	19	19	38
4. Journal for technical and vocational education	14	2	16
5. Bantu education journal	5	8	13
6. Journals of the British Mathematical Societies	9	3	12

The position may have improved considerably through the appearance of the journal "Spectrum". In 1962 the interest in reading professional journals of the teachers who taught mathematical subjects in the high schools was very little indeed. The question may well be put as to whether professional journals would be of any use to these teachers and what reason is to be found for this very limited interest.

11.4.2 The significance of professional journals and membership of societies for teachers in Mathematics

The renowned American educationalist, Howard F. Fehr, writes as follows: "All over the country various groups, large and small, are embarking on projects to produce a better mathematics program. Will they succeed? The answer lies in the minds of the classroom teachers of mathematics. Unless a teacher is a scholar in his field, one who knows and can speak with authority, he will be unable to judge the merit and feasibility of the programs mentioned before, or of any other new program. The good mathematics teacher must become a respected scholar. To do this he must keep up to date by reading, study, attendance at professional meetings, and taking in-service courses in new developments in mathematics, its applications, and its teaching. No teacher can do this when daily he has five classes to teach, a study hall to supervise, and extra-curricular duties, including perhaps a P.T.A. meeting to attend, plus family responsibilities, some of the last rather heavy. The teacher must demand removal of excessive teaching assignments and administrative and nonteaching duties; he must demand released time and travel funds for attendance at professional meetings; he must demand an adequate salary to live comfortably." (35, p. 18)

11.4.3 The need for South African Mathematics journals for teachers and lecturers

The teachers were asked whether there was a need for a South African journal for teachers and lecturers in Mathematics. The number of teachers who considered that there was indeed such a need are shown in Table 11.7.

TABLE 11.7

THE NEED FOR A MATHEMATICS JOURNAL FOR TEACHERS AND LECTURERS

Answers of the teachers who consider	Teachers: Qualified in Mathematics		Teachers: Unqualified in Mathematics	
	Number	%	Number	%
There is a need for a journal	634	90	807	82
There is no need for a journal	31	4	21	2
No answer	43	6	155	16
TOTAL	708	100	983	100

The great majority of teachers feel the need for a South African Mathematics journal for teachers and lecturers.

11.5 TEXTBOOKS ON THE TEACHING OF MATHEMATICS

11.5.1 The teachers' opinions

By means of Questionnaire N.B. 377, the teachers were asked whether there were enough textbooks in their own language at their disposal to teach the subject satisfactorily and to extend their own background in the subject.

In Table 11.8 the reaction of the teachers in respect of textbooks in Afrikaans is analysed.

TABLE 11.8

THE TEACHING OF MATHEMATICS - THE TEXTBOOKS AVAILABLE IN AFRIKAANS

Opinion regarding available textbooks	Adequate		Inadequate		Total
	Number	%	Number	%	
Qualified Mathematics teachers	374	74.4	129	25.6	503
Unqualified Mathematics teachers	484	80.4	118	19.6	602
TOTAL	858	77.6	247	22.4	1105

In Table 11.9 the reaction of the teachers regarding textbooks in English is shown.

TABLE 11.9

THE TEXTBOOKS AVAILABLE IN ENGLISH ON THE TEACHING OF MATHEMATICS

Opinion regarding available textbooks	Adequate		Inadequate		Total
	Number	%	Number	%	
Qualified teachers	381	90.5	40	9.5	421
Unqualified teachers in Mathematics	362	87.9	50	12.1	412
TOTAL	743	89.2	90	10.8	833

Some teachers submitted answers in respect of both Afrikaans and English textbooks.

11.5.2 Conclusion

It is clear that the great majority of the teachers are under the impression that they have enough textbooks available in their own language to teach the subject adequately and to extend their own mathematical background. The greatest number of teachers (25.6%) who were of the opinion that the available books in their own language were inadequate, were the qualified Afrikaans-speaking Mathematics teachers. As far as is known there is one Afrikaans textbook on the method of teaching Mathematics namely Alletson, D. C.; Die Onderwys Van Elementêre Wiskunde met Spesiale verwysing na Suid-Afrikaanse Skole[1]. About a quarter (25.6%) of the qualified Mathematics teachers and a fifth (19.6%) of the unqualified teachers in Mathematics feel this lack. These percentages would probably have been larger if the question had been put more clearly.

11.6 MEMBERSHIP OF PROFESSIONAL SOCIETIES

11.6.1 The South African Mathematical Association

In Table 11.10 an indication is given of the number of teachers who were members of this association during 1962.

TABLE 11.10

MEMBERSHIP OF THE SOUTH AFRICAN MATHEMATICAL ASSOCIATION

Teachers	Qualified in Mathematics		Unqualified in Mathematics	
	Number	%	Number	%
Members	31	4	18	2
Not members	639	90	877	89
Did not answer	38	6	88	9
TOTAL	708	100	983	100

Very few teachers are members of the South African Mathematical Association. In the same survey it was established that only seven teachers are members of the South African Statistical Association.

1) (Translation The teaching of elementary mathematics with special reference to South African schools.) London, G. Bell and Sons Limited, 1962.

11.6.2 Teachers' Associations

The number of teachers who were members of a teacher's association in 1962 is shown in the Table 11.11.

TABLE 11.11

MEMBERSHIP OF A TEACHER'S ASSOCIATION

Teachers	Qualified in Mathematics		Unqualified in Mathematics	
	Number	%	Number	%
Members	567	80	734	75
Not members	130	18	200	20
Did not answer	11	2	49	5
TOTAL	708	100	983	100

The number of teachers who attended Mathematical meetings of their teachers' associations is shown in Table 11.12.

TABLE 11.12

THE ATTENDANCE AT MATHEMATICAL CONFERENCES ORGANIZED BY TEACHERS' ASSOCIATIONS

Teachers	Qualified in Mathematics		Unqualified in Mathematics	
	Number	%	Number	%
Attend meetings	353	50	318	32
Do not attend meetings	268	38	505	51
Did not answer	87	12	160	17
TOTAL	708	100	983	100

About half of the qualified and a third (32%) of the unqualified Mathematics teachers attend the Mathematics meetings arranged by their teacher's associations. The procedure for bodies which wish to promote the teaching of mathematics (the South African Mathematical Association and others) is obviously to make use of the facilities offered by the teachers' associations in order to reach the teachers.

11.6.3 What professional associations can do

The professional associations may play a large role as will appear from an interesting development which has taken place in the United Kingdom: "A diploma awarded by England's Mathematical Associations". (32, p. 311) The first examination took place in November 1961 and annually thereafter.

> "The Diploma was designed to widen the education of mathematics teachers by encouraging both present and intending teachers to go further and more deeply into several branches of mathematics to see this subject in a wider context both topically and historically."

> "The examination consists of four papers of three hours each. The subjects are: (1) pure mathematics (two papers); (2) elementary mechanics and elementary statistics (a combined paper); and (3) the history and ideas of mathematics." (32, p. 311) Provision is also made for optional subjects.

The extent to which these examinations receive departmental recognition is not quite clear. Direct salary increases will not necessarily be awarded, but in the promotion of staff members this type of voluntary study should surely play some part and may on occasion tilt the scales in their favour.

The good work which is done by a body such as the "National Council of Teachers of Mathematics" in the United States, "WIMECOS" and "LIWENAGEL" in the Netherlands is worthy of recommendation.

11.6.4 Teachers unqualified in Mathematics

It should be made possible for unqualified Mathematics teachers who are required to teach mathematical subjects, to qualify in Mathematics. This should serve as a reward for the fact that they are required to teach a subject for which they have not been trained. Study leave and vacation courses are the obvious solution. Such vacation courses should have the same content and standard as normal degree courses in the subjects and should be tested by means of the same examinations. As far as the content is concerned, it should be possible to make other provisions for teachers of Arithmetic, so that the Mathematics courses which they follow will have greater relevance to their work.

11.7 FURTHER TRAINING

A committee, of which Dr. A. L. Kotzee, Professional Assistant, Transvaal Education Department, was Chairman, instituted an investigation and made recommendations concerning refresher courses and further training of teachers.

The result of the work of this Committee appears in (Translation) The Report of the Interdepartmental Committee on Refresher Courses and Further Training of Teachers, dated 20th December, 1963.

A second report, dated 26th February, 1964 deals with the following aspects:

(a) Types of courses

(b) Compensation to teachers.

These reports pave the way for the further training of teachers and the prospect of holding refresher courses and give the opportunity for taking the initiative to the education departments.

CHAPTER 12

SUMMARY AND RECOMMENDATIONS

12.1 MATHEMATICS

Mathematics is more than a collection of calculations according to arbitrarily determined rules. It is also more than a collection of ever increasingly refined and involved techniques which enables man to control material, the earth and the space about the earth.

Mathematics is an instrument which has been created by the human mind, but which can shape the mind which creates it in such a way that new insight is obtained.

It is the science of number and space, a sphere which is carefully investigated by the mind which believes in an orderly creation. This mind itself plays a part in the creation, but develops more as it acquires the capacity to understand the discrete and the continuous.

Without number and space the world of experience is inconceivable and also indescribable. Number and space are fundamental to human observation and perception.

Man thinks mathematically, yet his thoughts are not bounded by number and space. By mathematical thought he penetrates the secrets of the material world and of forces in the different forms in which they appear.

12.2 THE AIM AND VALUE OF MATHEMATICS TEACHING

Mathematics is an art which is very closely connected with physical science. For the mental development of man it is of the greatest importance, while the scientific application thereof is invaluable. This applies also to Arithmetic which is really a part of Mathematics. Whenever Mathematics is discussed in general in this report, it is intended that Arithmetic should be included.

Mathematics is indispensable for the practical work of the architect, the quantity surveyor, the engineer and the technician and for the study of the pure and applied sciences such as mechanics, physics, astronomy, land surveying and chemistry.

In the biological and economic sciences Mathematics has also come to the fore to an increasing extent. Biometrics and Econometrics are very rapidly developing areas of study in which the mathematical content makes ever-increasing demands upon the persons engaged in them.

This application of Mathematics continues to expand more and more even in the direction of the social sciences. In the sphere of Psychology, Psychometry has already developed its own structure, while the study of Sociology demands Mathematics to an ever increasing extent.

The aim of the investigation was to determine the present state of the teaching of Mathematics, General Mathematics and Arithmetic in the South African high schools. The survey was limited to the White population group and concerns more specifically public high schools.

In order to determine the aim of Mathematics teaching in high schools, a distinction was drawn between the following three types of pupil:

Pupils who eventually go to university.

Pupils who complete the high school course, without going to university afterwards.

Pupils who leave school before they have completed the full high school course.

For all these pupils one finds the following common aims:

To promote the ability to think clearly and to make logical deductions;

to train pupils to calculate correctly, and

to provide a basic training for a future career.

For those pupils who will go to a university at a later stage, the university lecturers consider mathematical concept formation as the most important aim, while teachers consider the cultivation of a love for and an interest in Mathematics as more important.

For those pupils who complete their high school course without going to university, the aim of mathematical concept formation is according to university lecturers and teachers, only number five in order of importance while the acquisition of factual knowledge in connection with every day life is at least of equal importance.

The pupil who leaves school without completing the full school course has to make his way in the world without having been equipped to the same extent as his fellows who stay at school for a longer time. These pupils require practical knowledge of Mathematics, and so the emphasis in their teaching should fall on that aspect.

It is therefore recommended that a distinction should be drawn between a general aim in teaching Mathematics to all high school pupils and the particular aim for each group.

The general aim is to promote the ability to think clearly and to make logical deductions.

For those pupils who will later go to university and more particularly for those who will study this subject further, mathematical concept formation is a particular aim.

The course studied by those pupils who will leave school earlier, should be mainly on a practical level.

12.3 THE MATHEMATICS PUPIL

It is significant that in general a higher percentage of English-speaking pupils are found to be taking Mathematics in Std. 10 than is the case amongst the Afrikaans-speaking pupils. It is clear that this considerable difference between the percentage of Afrikaans-speaking pupils and English-speaking pupils who take Mathematics at school requires further research.

The percentage of pupils who take Mathematics in Std. 10 also differs from province to province. As it is to be expected that the intelligence of the pupils and their aptitude for Mathematics will not vary much from province to province, there is probably a considerable loss of talent. In the light of the present manpower shortage, more particularly in those spheres where mathematical schooling is a requirement, this state of affairs is disturbing.

A possible reason why pupils do not take Mathematics is the fact that other subjects may be taken as alternative subjects in place of Mathematics and Applied Mathematics. The investigation has indicated that a liberal policy in the schools where pupils may take one of a number of subjects instead of Mathematics, places Mathematics in a less favourable position. The more alternative subjects there are, the fewer pupils take Mathematics as a subject. In the Afrikaans medium schools particularly pupils are exposed to the temptation to take another subject in the place of Mathematics.

In the schools where Arithmetic is taught, this subject is compulsory up to Standard 8. The schools in which this is not the case may be considered as exceptions. In some schools Arithmetic is taught in Standard 6, while in other standards it forms a part of the General Mathematics syllabus.

It is most important that the love for and interest in Mathematics should be consciously cultivated in pupils. A Mathematics journal for pupils would fill a great need. Whether such a journal should be specifically designed for pupils only or for pupils, students and teachers should be investigated further. The extent to which the new journal "Spectrum" has met this need has yet to be established.

There is also a need for a series of short treatises by prominent mathematicians which may pay particular attention to Mathematics for high school pupils.

Interest in Mathematics may also be awakened by the foundation of Mathematics clubs and by holding Mathematics competitions throughout the length and breadth of the country. The interested support of authorities will be a great asset to this movement.

On the average there are just under 30 pupils in the classes for Mathematics, General Mathematics and Arithmetic. At the same time there are schools which have as many as forty pupils in a class, particularly in the Transvaal and the Cape Province. The great majority of the classes in Stds. 9 and 10 have fewer than 30 pupils.

University lecturers draw attention to the following shortcomings amongst their first-year university students: In the first place a lack of mathematical insight and understanding, secondly a lack of the capacity to think logically and study independently, and thirdly a deficiency in mathematical skill.

<u>It is therefore recommended</u> that the National Bureau of Educational and Social Research should determine by means of further research why fewer Afrikaans-speaking pupils take Mathematics than do English-speaking pupils. It is a matter of urgent importance that there should be a greater number of Mathematics pupils and a greater number of Mathematics teachers.

It is further recommended that the Bureau should undertake a similar investigation in respect of the subject of Arithmetic in primary schools. This further research should take place under the guidance of an advisory committee similar to that which undertook responsibility for the present investigation.

It is recommended that a <u>Publications Committee</u> should be set up. The terms of reference to that committee should be as follows:

> To provide journals with mathematical material for the pupils;
>
> to investigate the possibility and desirability of setting up a special Mathematics journal for pupils and to act in the light of the findings;
>
> to undertake to have short treatises written by prominent mathematicians in order to draw attention to appropriate mathematical literature and subject matter, and
>
> to organize countrywide competitions between schools and between individuals in a manner similar to the Mathematics Olympiads in the Netherlands.

<u>It is recommended</u> that the proposed Publications Committee should receive the most sincere support and genuine interest, both financial and otherwise, of the authorities.

Pupils with the necessary aptitude should be encouraged in a systematic way to take Mathematics at school, while the appropriate training of teachers for that subject should receive equally serious attention.

12.4 THE MATHEMATICS SUBJECT MATTER

There is a world-wide interest in bringing the subject matter in Mathematics in schools up to date. Where Algebra and Arithmetic have largely consisted of "problem mathematics", a clearer structure is now developing. The "axiomatic method" is no longer limited to the Euclidian mathematics and space mathematics, but is carried right through to the ways in which the study of number may be presented in the classroom.

New courses are being drawn up and it is noteworthy that pupils understand this new subject matter far more easily than was anticipated by their teachers.

Basic concepts such as "sets" are emphasised, without the set theory itself, becoming part of the school Mathematics. The idea is that concepts such as sets, ratio, variation and function should purposefully become part of the mental equipment of the pupil.

If the new approach to Mathematics teaching be accepted as a matter of policy, public opinion must not be left out of consideration. Provision should be made for the enlightenment of parents as well as for teachers.

The new school textbooks from overseas create the impression of a new mathematical language. New value is assigned to precision in mathematical language. The greater the degree of language purity in mathematical concepts and terms, the better are these understood by pupils.

The introduction of concepts which are new to the pupils sometimes demands lengthy preparation, since the pupils have to undergo different stages of development. Other concepts may be intuitively clear and the learning thereof much easier.

According to some writers the development of the pupils requires a six year course consisting of three years naïve Mathematics, two years critical Mathematics and one year abstract Mathematics. As the present investigation was limited to high schools, it was not possible to determine whether the teaching of Algebra and Geometry should begin in Standard 5 or not. Some teachers believe that pupils are ready to begin Algebra and Geometry at that stage.

The problem regarding subject matter which should be taught to pupils in the different stages of their development is one which is engaging the attention of research workers and there is a tendency to replace some of the traditional boundaries by others.

Account should be taken of the fact that Mathematics has developed at a considerably quicker pace during the twentieth century and that in addition to extension in breadth there has been considerable increase in the depth of the study.

Investigation has shown that subjects which have been introduced into the South African Mathematics syllabi in the recent past meet with general approval, namely arithmetical and geometrical progressions, analytical geometry and the trigonometrical ratios of the sum and difference of two angles.

The importance of inequalities is being emphasised particularly by university lecturers in the mathematical subjects.

Applied Mathematics or Mechanics is taught in a limited number of schools only. It is possibly time that the meaning, nature and content of this subject should be investigated more fully. Regarding the place of Statistics in school, the last word has not been said, while the problem of a separate Commercial Mathematics should receive attention. The problems which arise in connection with the training of programmers for electronic computers are also being considered.

It is thus recommended that a Committee for Applied Mathematics in schools should be appointed and that this Committee, in co-operation with the Joint Matriculation Board, should pay attention to the different aspects of Mathematics as an applied science, and more particularly to the place of Commercial Mathematics, Statistics and Theory of Probability, Mechanics and Programming in schools. The teaching of Arithmetic in high schools should also be considered in this connection.

12.5 THE SYLLABI FOR MATHEMATICS, GENERAL MATHEMATICS AND ARITHMETIC

The investigation has shown that the syllabi have not kept pace with the most recent developments in the field. A serious gap is to be seen in the fact that the basic concepts are not sufficiently emphasised in the syllabi. The consequence is that teachers are in almost complete disagreement in regard to which concept should ultimately be considered as the central fundamental concept, while some teachers are even in doubt in regard to the meaning of a mathematical concept. Concepts such as sets, ratio, variable, one-to-one correspondence, and function deserve far more attention.

The Arithmetic syllabus is so overloaded with commercial subject matter that purposeful concept formation is frequently neglected. The danger exists that the subject Arithmetic will be swamped by the applications of Arithmetic and that concepts such as the natural numbers, whole numbers, rational numbers, real numbers, sets and ratios will pass unnoticed.

In some Algebra syllabi too much prominence is given to equations and more emphasis should be placed on the set and function concepts.

Particularly for the more gifted pupil, logarithms and Trigonometry should provide more opportunity for the formation of concepts by the treatment of the logarithmic series and more extended consideration of trigonometrical ratios. The Euclidian Geometry could probably be presented in a more modern form. The most important of all, however, is that the idea of the development of a mathematical structure should not be lost, since the study thereof is of the greatest importance for the development of deductive thought on the part of the pupils.

The common basic syllabus of the Joint Matriculation Board should make provision for a considerable degree of uniformity in the Mathematics syllabi for Standards 9 and 10.

There are, however, certain deviations. The syllabi of the Transvaal Education Department do not provide for Arithmetic in Standards 9 and 10, with exception of the use of logarithms. In this way thorough axiomatic development of the number concept is neglected, which is considered to be a serious omission.

Both the Transvaal and Natal Education Departments offer special courses for pupils who intend to go to the university. The Transvaal course offers elementary calculus over and above the requirements of the Joint Matriculation Board.

The Transvaal and Natal Education Departments offer shortened courses for pupils who wish to complete Standard 10 but who do not qualify for university admission. Between these two syllabi there are some differences. The Natal syllabus offers the possibility of a course which may be called Mathematical Arithmetic. It is possible to indicate the development of the number concept from natural numbers to real numbers and to illustrate this by means of the number continuum. This is missing in the Transvaal Standard 10 syllabus. On the other hand the Transvaal syllabus also makes provision for Differential Calculus for pupils who are not proceeding to a university.

The custom of commencing Algebra and Geometry in serious vein in Standard 6 seems to be acceptable, as some teachers are of the opinion that these subjects may even be taught in Standard 5.

In Standards 7 and 8 the same instruction appears to be given in the various education departments. The Transvaal "C" stream syllabus offers a little less than that of the other syllabi, while the syllabus in the Cape Education Department makes provision for even less than that.

The Geometry syllabi for Standards 7 and 8 differ to a certain extent. Pupils who move from one province to another will have to combat certain problems of adjustment although the differences are not very fundamental.

With the exception of the Natal syllabus and the Transvaal "C" stream syllabus, a certain amount of Trigonometry is offered as early as Standard 8.

The conclusion is that there is partial uniformity between the syllabi of the different education departments. The reasons why there are deviations could not be established with any degree of certainty.

The syllabi for Additional Mathematics in the Natal Education Department and that of the Joint Matriculation Board do not agree exactly, since portions of the Natal Mathematics syllabus already appear in the Advanced Grade.

The gap between school and university cannot be ascribed primarily to the content of the school syllabi.

Given the necessary attention to concept formation, the same syllabi will contribute largely to bringing the school and the university nearer to one another, and to solving the problem of adjustment between these two types of institutions.

That the syllabi of the various education departments are of such a nature that they fulfil the aims and the anticipated educational objectives adequately cannot be maintained without reservation. It is a disturbing fact that most of our teachers are not au fait with the aims and the envisaged educational values of the syllabi. In many instances it is most difficult to discover these aims in the syllabi, since the formulation thereof is not always quite clear.

It is therefore recommended that a Syllabus Committee be appointed. This committee may be representative of the various education departments and the universities. The terms of reference of the committee may include the following:

The setting up of a reasonably broad guide to accompany the syllabi. In this guide the aim and central theme and concept should be clearly indicated with an explanation of how the syllabi may be adapted to the requirements of the various pupils. There should be a distinction drawn between the following types of pupils: Pupils who wish to continue with the subject at university, pupils who wish to go to university without continuing with the study of Mathematics, pupils who wish to complete their high school career without going to the university thereafter and pupils who do not wish to complete the full high school course. In the guide the various concepts may be indicated and the manner in which the subject may be presented should be adapted to the development of the pupils' capacity to understand concepts. It will be particularly instructive if the development of the concepts can be introduced to undergraduate and even post-graduate Mathematics.

It is recommended that the suggested Syllabus Committee should co-operate with the Joint Matriculation Board to set up uniform syllabi with due regard to geographical and socio-economic differences which appear in the country. As these differences are not provincially determined, the setting up of such syllabi should be possible.

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12.6 THE METHODS OF TEACHING

For teaching General Mathematics an average of 5.8 periods per week are allocated, for Mathematics in Standard 6, 3.7 periods per week, Standard 7, 4.8 periods per week, Standard 8, 4.9 periods per week and in Standards 9 and 10, 6.4 periods per week. The number of periods per week is not the same for all education departments nor for all schools.

For Arithmetic teaching in Standard 6 an average of 4.4 periods per week are allocated, in Standard 7 an average of 3.5 periods per week and in Standard 8 an average of 3.4 periods per week.

Some teachers teach Mathematics in no less than seven classes of the same standard. The majority of the teachers have to repeat the same lesson only once. Investigation was also made of the amount of time the teachers devoted to the correction of Arithmetic and Mathematics homework. This differs considerably from teacher to teacher, which may be ascribed to the fact that not all teachers are engaged in teaching Arithmetic or Mathematics exclusively.

The great majority of the teachers are not in favour of lengthening the school day so that "homework" may be done during school hours under supervision.

The most important measure which is taken to ensure that pupils work correctly, is the regular checking of the work.

The majority of teachers do not make use of objective tests while more than 60% of the teachers experience a need for such measuring instruments. Tests are necessary for the measurement of basic knowledge, concept formation, skill and the attitude towards the subject.

More than three-quarters of the teachers confirm that they frequently make use of drill methods and aver that the kind of question which appears in the School Leaving Certificate Examination encourages the drill method in their work.

Almost half of the teachers are of opinion that the gap between Mathematics in the high school and that in the beginning of the first year in the university is too wide.

According to the majority of the teachers the university Mathematics course is presented at too fast a tempo to begin with, and there are too great differences in methods of teaching.

It is therefore recommended that the proposed Publications Committee should propagate the use of objective tests, so that greater use may be made of such testing materials[1]. The proposed Syllabus Committee should give attention to the problem of the transition from school to university in respect of the teaching of Mathematics and also investigate the extent to which the talents of the good teacher are utilised to the advantage of the maximum number of pupils. It is recommended that this research be undertaken by the Bureau. The possible use of a closed television circuit should also be investigated. It is further recommended that should the investigation justify this, suitable persons should be sent abroad.

12.7 THE TEXTBOOKS FOR MATHEMATICS, GENERAL MATHEMATICS AND ARITHMETIC

In the South African high schools a great variety of textbooks are used. Most of the teachers are satisfied with the textbooks which they use. Some of the teachers who answered questionnaires indicated that they were the authors of Mathematics textbooks. Of the 45, 30 may be considered to be qualified in Mathematics, while 15 had successfully completed less than two

[1] A catalogue is obtainable from the National Bureau of Educational and Social Research, Private Bag 122, Pretoria.

university courses in a Mathematical subject.

Nearly all the teachers who were impressed by the necessity for modernising the Mathematics teaching were of the opinion that a set of experimental textbooks would play an important part in such a process.

By "experimental textbooks" is meant those textbooks which could be released to schools in a tentative form to be used in an experimental way. The comments of the teachers and the pupils would be obtained and a new set would thereafter be compiled. This process would be repeated until satisfactory end products are obtained.

The majority of the teachers who were qualified in Mathematics were in favour of the compilation of such a textbook by a committee consisting of teachers and university lecturers. This committee could at the same time write teachers' manuals to accompany the textbooks.

These experimental textbooks would be able to serve as examples of how the mathematical concepts should be developed in high schools, an aid in the training of teachers, an example of the building up of a mathematical structure and a model for future authors of textbooks. Moreover these experimental textbooks might serve as interim textbooks until similar books became available through commercial sources.

The majority of the teachers were in favour of having experimental textbooks published by a committee which was representative of different bodies interested in the teaching of Mathematics.

It is therefore recommended that a Textbook Committee should be instituted. This Committee would consist of teachers and university lecturers and should be in a position to give full-time attention for a given period to the writing of school textbooks for Mathematics and Arithmetic. The members would also be expected to write the accompanying manuals for the teachers.

These textbooks should be such that the teaching of Mathematics in schools, seen from the professional point of view, would be placed on a modern basis.

If need be, the books may initially be regarded as experimental textbooks and presented in a tentative form, on the understanding that the members of the committee would be given the opportunity from time to time to bring about the necessary revision.

It is further recommended that adequate government support be made available for this project.

12.8 EXAMINATIONS

It is clear that the present system of examination, particularly that of external examinations, has many shortcomings. At the moment there are six different examining bodies and a variety of examinations: University Entrance, Standard 10 Examination, Ordinary Grade, Advanced Grade, Additional Mathematics. Examinations have increased without the standard thereof having improved. The most extraordinary demands are made on the Moderator of the Joint Matriculation Board in an attempt to ensure that these different examinations set a uniform standard. Moreover it has thus far not been possible to determine what the reliability and the validity of the examinations are. The discrimination index of the different examination questions has never been calculated. Consequently the final examination in the high schools is a measuring instrument of which the value has not yet been fully determined and which, because of its particular form, is difficult to determine.

It is possible that the different education departments will keep their own school leaving examinations.

It is recommended that every education department should conduct only one school leaving examination (Standard 10 or Ordinary Grade) so that all pupils who complete their high school career will go through the same mill, and that the proposed Syllabus Committee will pay attention to the mode of examining and will determine in which way the work of the moderators of the Joint Matriculation Board can be lightened. This Committee may then consider whether the examination system could possibly be so changed that examination papers which are specially designed for university admission will be such that they can be easily corrected and item analysis, validity and reliability determinations may take place. Furthermore, investigation may be made into the degree to which such (additional) examination papers may be used to determine the standard of the departmental examination and whether a reasonably valid and reliable measure of university entrance qualification may be set up in this way. Also deserving of attention is the suggestion that the contents of these suggested examination questions shall remain confidential after the examination has been taken. In this way a certain item must be used more than once in the examinations and be utilised as a basis of comparison to determine the standard of the candidates from year to year. This method will also determine whether the evil of examination drill can be effectively combatted. It is further recommended that in examinations more emphasis be placed upon insight than upon skill in manipulation.

12.9 THE TRAINING OF MATHEMATICS TEACHERS

Mathematics teachers are presented with a tremendous undertaking, namely the drastic modernisation of the Mathematics teaching in high schools. The training of properly qualified teachers of the subject constitutes the most important factor in all teaching of Mathematics.

The present serious shortage of teachers who are qualified in Mathematics must be taken into account. The problem is how to ensure that potential Mathematics teachers are enabled to qualify as such without having their study careers held up by factors which are not directly or indirectly of significance in relation to their future teaching careers.

Everybody is agreed that the standard of the traditional academic undergraduate Mathematics course should be maintained, but there remains the question regarding the effectiveness thereof. Is it not necessary for teachers to have a training which is of the same value but of another kind? It is necessary that future teachers should be brought into contact with modern Mathematics as speedily as possible.

In South Africa the pattern of training of teachers becomes more involved almost daily. In the Transvaal a Mathematics teacher may receive his training either at a university, or partly at a university and partly at a college of education or wholly at a college of education. In the latter case he will follow a three year specialisation course (Junior Secondary Classes) which qualifies him to be a Mathematics teacher up to Standard 8.

Some aspirant teachers receive their training in the Method of teaching Mathematics at a university and some at a college of education. Some of those who receive training in the method of teaching at a university have a member of the Department of Mathematics as lecturer, while others have a member of the Faculty of Education for the sake of unity within the faculty. Slightly more teachers have had their professional training in universities than in a training college or college of education.

When the qualifications of the lecturers in the method of teaching are analysed, the universities make a better impression than do the colleges of education. The impression is gained that the universities do a more efficient job with fewer staff than the colleges of education.

The training in the colleges of education is apparently directed largely at classroom practice; at the universities the idea of a mathematical structure comes more into its own.

It should be noted that about a quarter of the teachers in service who are qualified in Mathematics have had no training in the Method of teaching Mathematics. Even those teachers who have had training in Method were not wholly satisfied with the nature of the training which they had received. They found that the course was not practical enough and that insufficient time was devoted to it.

It is recommended that a committee should be appointed to give further attention to problems relating to the training of teachers in Mathematics and Arithmetic. The following matters should receive attention:

The determination of the minimum qualifications of the following three groups of teachers, namely teachers in Arithmetic, Mathematics for Standards 6, 7 and 8, and Mathematics for Standards 9 and 10.

The undergraduate training of these teachers and the manner in which their special requirements may be met at that stage.

The professional training with particular attention to the Method of teaching Mathematics, the aims of Mathematics teaching, the central fundamental concepts and the implementation of modernised syllabi.

The institutions where the various stages of the training should take place: university, college of education or technical college.

If the training in the Method of Mathematics is to take place in a university, should this be in the Department of Mathematics or in the Faculty of Education?

12.10 TEACHERS IN MATHEMATICAL SUBJECTS

It is expected of a Mathematics teacher to be a teacher as well as a mathematician, a person who has a love for children and who is actively concerned with his subject.

A teacher with two degree courses in a mathematical subject which he has successfully completed is considered in South Africa to be a qualified Mathematics teacher. In some overseas countries an honours degree or a master's degree is considered to be the necessary qualification for a teacher of Mathematics. In this discussion the local requirement of at least two successfully completed degree courses is taken as the criterion. Regarding the minimum qualification for a teacher in Arithmetic there seems to be no clarity.

A little more than a third of the teachers who teach the mathematical subjects were under thirty years of age at the time of the survey (June 1962). Almost two-thirds of the teachers who are qualified in Mathematics had more than five years experience. About a third of the teachers without the required qualifications in Mathematics had at the most two years experience in the teaching of these subjects.

A sixth of the persons who taught mathematical subjects, occupied a position of principal or vice-principal. The majority (56.5%) of the qualified teachers in Mathematics taught Mathematics in Standards 9 and 10. Of the teachers who could not be considered to be mathematicians, about 30% teach largely Arithmetic and a further quarter teach mainly General Mathematics.

In the Transvaal the number of qualified Mathematics teachers was less than the number of Mathematics posts; this meant that even if all qualified Mathematics teachers were used exclusively for teaching Mathematics, there would be no teachers remaining for the teaching of General Mathematics and Arithmetic. There were not even enough to fill all the Mathematics posts[1].

In the Cape, conditions are more favourable. The number of qualified teachers was sufficient (theoretically) to fill all the Mathematics posts and

[1] Table 10.13.

in addition a quarter of the posts for General Mathematics. Three-quarters of the posts for General Mathematics and all the posts for Arithmetic would then be filled by teachers without the necessary qualifications in Mathematics.

In Natal there were just sufficient qualified teachers for half the Mathematics posts. The other half of these posts and all the posts for General Mathematics and Arithmetic were filled by teachers who could not be considered as mathematicians.

In the Orange Free State there were just enough qualified Mathematics teachers to fill the Mathematics posts. The other mathematical subjects would of necessity be taught by teachers without mathematical qualifications.

In the technical high schools there was no shortage of qualified Mathematics teachers. In the other vocational high schools there were appreciable shortages.

From the above it is clear that it has been assumed that a qualified Mathematics teacher would preferably be placed in a Mathematics post; if these posts were all filled, then in a post for General Mathematics and only thereafter in an Arithmetic post. By this it is not meant to imply that this is the existing practice or should be the practice, but this assumption is made with the sole aim of drawing comparisons and of obtaining some idea of the shortfall in this way.

The investigation brought to light the fact that the percentage of pupils who take Mathematics up to Standard 10 is not dependent on the available permanently appointed teachers. There are apparently other factors which decide the issue.

It is therefore recommended that the proposed Committee for the Training of Teachers in Mathematical Subjects should draw up a comprehensive plan. This plan should indicate how many Mathematics teachers should be trained annually, so that in due course all these posts be filled by persons with the necessary qualifications. This training could include teachers in service as well as recruits.

It is further recommended that this Committee should see that the aims of teaching Mathematics and mathematical concepts receive the necessary attention during the training period, and that the way is prepared for the implementation of new syllabi.

12.11 THE TRAINING OF TEACHERS IN SERVICE

The proposed modernisation of Mathematics teaching in high schools makes the retraining of qualified teachers of Mathematics essential.

In due course the teachers who teach mathematical subjects without having the necessary qualifications therefore should be required either to undergo further training or to give instruction only in those subjects for which they have been trained. (In the interim their assistance is indispensable.)

The retraining of teachers should take place on a carefully planned basis. At the outset a start should be made with teachers who have had twenty or more years' service; they will not be more than four hundred in number. Then other groups of teachers, grouped according to the number of years' service may be approached. Training may take place in both Modern Mathematics and in the Methods of teaching Mathematics.

To all the teachers who teach mathematical subjects the question was put regarding the conditions under which they would be prepared to attend vacation courses in Mathematics and Mathematics teaching. Although only 65% of the teachers were sufficiently interested in the possibility of such a course to answer the question, it is noteworthy that it was just those qualified Mathematics teachers who showed the greatest interest, and that about one-fifth of them were unreservedly and unconditionally prepared to attend courses.

The reimbursement of travel and subsistance costs will be a great encouragement for many teachers to attend courses. In a vast country such as South Africa, it stands to reason that this is a most important factor.

The receipt of departmental recognition was of more consequence to the unqualified teachers than to the qualified teachers, for understandable reasons.

When State-subsidised vacation courses are offered, the lecturers and the subject matter will have to be chosen most carefully to ensure that such courses answer their purpose.

As far as the reading of professional journals is concerned, there is much room for improvement. It has yet to be determined whether the appearance of "Spectrum" has improved the present state of affairs. One of the aims of vacation courses should be to stimulate teachers in their desire to read professional literature. The teachers can ultimately be reached only through their teachers' associations, since very few of them are interested in professional associations.

It is therefore recommended that the proposed Committee for the Training of Teachers in the mathematical subjects should advise the education departments in connection with the arrangement for vacation courses and should supply practical assistance. It is recommended that this committee should go out of their way to ensure that the courses so offered will enable teachers who teach the subject without the necessary qualifications to qualify in Mathematics and to receive such training in the method of teaching the subject as is desirable.

Courses should also be arranged to enlighten teachers in regard to the newest developments and trends in mathematical matters, to bring to their attention the aims of the syllabi, to emphasise the central fundamental concepts and to implement the new syllabi smoothly.

It is further recommended that the education departments should exercise adequate control over the content as well as the presentation of the syllabi.

12.12 CONTINUATION OF THE WORK

The Advisory Committee concerned with this investigation made certain recommendations. Direct action may be the result of some of these recommendations after they have been approved. Part of the work may possibly be continued more effectively under the guidance of expert committees.

The appointment of the following committees for the promotion of Mathematics teaching is recommended:

A Publications Committee.

A Committee for Applied Mathematics in School.

A Syllabus Committee.

A Textbook Committee and

A Committee for the Training of Teachers in Mathematics and Arithmetic.

It is further recommended that when the committee members attend meetings they should be recompensed in respect of travelling and subsistance expenses. It should also be possible for committee members to be set free from their other duties from time to time in order to devote themselves full-time to the execution of certain tasks.

The work of these committees will set in motion certain research and other activities. The question arises where this work can be undertaken.

Use has been made of the services of the National Bureau for Educational and Social Research in the execution of this investigation and the preparation of the report. It is recommended that theBureau should continue these activities in co-operation with the proposed five committees.

The duties of the proposed committees may in the interim be delineated as follows:

The Publications Committee:

(a) The provision of mathematical literature for existing journals.

(b) The consideration of the institution of a special Mathematics journal for pupils and matters which arise therefrom.

(c) The publication of short articles written by eminent mathematicians in order to draw attention to mathematical subject matter and literature.

(d) The organisation of country-wide mathematical competitions between schools and individuals on the same basis as the Mathematics Olympiads in the Netherlands.

(e) The enlightenment of teachers and parents in connection with the modernisation of Mathematics teaching.

(f) Research in connection with the best use of media such as radio, films, television, the public press and programmed teaching.

(g) The arrangements for visits of overseas experts locally and local experts overseas.

(h) The provision of professional reading matter for teachers.

(i) The propagation of the use of objective tests by teachers in the schools.

The Committee for Applied Mathematics in school:

(a) Research in connection with the existence of the present Mechanics course in high schools.

(b) Determining the necessity for the teaching of Statistics and Theory of Probability, and the programming of electronic computers at school.

(c) Research in connection with the place of Commercial Mathematics and Arithmetic in high schools.

The Syllabus Committee:

(a) The compilation of a guide to teachers in such a way that a explicit aim and clear development of concept may be effected within the framework of the existing syllabi, and the correct subject matter may be presented at the right time to pupils who take Mathematics with different objects in view.

(b) The institution of uniform syllabi for the whole country, having regard to the geographical and socio-economic differences which appear in the various regions (not necessarily provinces).

(c) Further research in connection with the transition between school and university as far as Mathematics teaching is concerned and in connection with the problem of using the talents of the better teachers to the maximum benefit of the greatest number o. pupils, for example, through the use of a closed television circuit.

(d) Research in connection with the methods of examination in order to determine the best way in which the work of the Moderator of the Joint Matriculation Board may be facilitated, and how the examination papers may be so arranged that marking may be done in a simple way, item analysis, validity and reliability determinations may be carried out and the relative standard of examination of the different departments may be determined from year to year. In addition note should be taken of the desirability of examining in such a way that the emphasis is laid more upon insight than upon skill in manipulation.

The Textbook Committee:

(a) The compilation of textbooks and accompanying guides for teachers which will be applied experimentally in the first instance.

(b) The final form of these books after further information has been obtained.

(The textbooks should be such that teaching the mathematical subjects in school should be modern when looked at from a mathematical point of view and should compare favourably with similar books from overseas.)

The construction of the mathematical structure should be clearly emphasised in textbooks.

Textbooks should also be such as to stimulate the pupils to independent study.

Committee for the training of teachers of Mathematics and Arithmetic:

(a) The determination of the desirable minimum qualifications for Arithmetic teachers, Mathematics teachers for Standards 6, 7 and 8 and Mathematics teachers for Standards 9 and 10.

(b) The development of proposals in connection with the undergraduate and professional training of these teachers and possible differentiation at the undergraduate level.

(c) The drawing up of the curriculum, the length of the training and the type of institution where the training should take place.

(d) The organisation of the retraining and the training of teachers in service in co-operation with the teachers' associations, the education departments and educational institutions.

(e) Stimulation of teachers to study professional reading matter.

(f) The committee should ensure that in the training of Mathematics teachers, the aims of Mathematics teaching, the central fundamental concepts and the implementation of new syllabi are given the necessary attention.

APPENDIX 1

ALGEBRA FOR STANDARDS 7 AND 8

In the Transvaal the work is divided into three courses, namely the University Entrance Course, the Standard Ten Course and the Standard Eight Course. These are indicated shortly here as the A, B and C groups respectively.

This syllabus is to be found in the proposed syllabus for Mathematics for the Secondary Schools (Stds. VI, VII and VIII) published by the Transvaal Education Department in November 1958.

The syllabus for the Orange Free State is found in the pamphlet: Syllabus for the Secondary School (Stds. 6, 7 and 8), General Mathematics published by the Education Department of the Orange Free State.

In Natal the work is divided into two groups, i.e. that for the advanced grade and that for the ordinary grade which is indicated as V and W in this portion. This syllabus is found in the handbook: Junior Certificate Examination which was published by the Natal Education Department in 1964.

The syllabus for the Cape is found in the handbook: Junior Secondary Course which was published by the Department of Public Education of the Cape of Good Hope in 1953.

The work which appears in the syllabuses of the various provinces in respect of the various groups is shown below.

		Transvaal			O.F.S.[1]	Natal		Cape
		A	B	C		V	W	
1.	The use of symbols to generalize arithmetical calculations	x	x	x	-	x	x	-
2.	Problems which lead to equations of the first degree with one variable	x	x	x	x	x	x	x
	Test of solution	x	x	x	-	-	-	-
3.	Equations of the first degree with one unknown	x	x	-	x	x	x	x
	Test of solution	x	x	-	-	-	-	-
	Where co-efficients are letter numerals	-	-	-	x	-	-	-
	Easy literal equations	-	-	-	-	x	-	-
	Easy literal equations with one unknown	-	-	-	-	-	x	-
4.	Substitution	x	x	x	-	x	x	-
5.	Positive and negative numbers	x	x	x	x	x	x	x
	in four main calculations	x	x	x	x	x	x	x
	and raising the power.	x	x	-	-	-	-	-
	Number zero	-	-	-	-	x	x	-
6.	Simple brackets, removing and inserting	x	x	x	x	x	x	x
7.	Equations with negative numbers, and problems	x	x	-	-	-	-	-
8.	Addition and subtraction of polynomials	x	x	-	x	x	x	-
9.	Multiplication:							
	Polynomials with two or three terms	x	x	-	-	-	-	-
	Not more than two variables	-	-	-	-	x	-	-

[1]Orange Free State.

		Transvaal			O.F.S.[1]	Natal		Cape
		A	B	C		V	W	
	Division:							
	Polynomials by binomials	x	x	-	-	-	-	-
	Not more than two variables	-	-	-	-	x	-	-
10.	Multiplication:							
	Principal standard forms:							
	$(a + b)^2$	x	x	x	x	x	x	-
	$(a - b)^2$	x	x	x	x	x	x	-
	$(a + b)\ (a - b)$	x	x	x	x	-	-	-
	$(x + a)\ (x + b)$	x	x	-	x	-	-	-
	$5a(a^2 - 3ab + 2ac)$	-	-	x	-	-	-	-
	$(a + b)^3$	-	-	-	-	x	-	-
11.	Factors of the following types:							
	$ax + ay$	-	-	-	x	-	-	-
	$ax + ay + az$	x	x	x	-	x	x	-
	$x(a + b) + y(a + b)$	x	x	x	-	-	-	-
	$x^2 + px + q$	x	x	x	-	x	x	-
	$a^2 - b^2$	x	x	x	x	x	x	-
	$a^3 - b^3$	x	-	-	-	x	x	-
	$a^3 + b^3$	x	-	-	-	x	x	-
	$ax + bx + ay + by$	x	x	x	x	x	x	-
	$6x^2 + 5xy - 21y^2$	x	x	-	x	x	x	-
	$a^2\ x^2 \pm 2abx + b^2$	x	x	-	-	-	-	-
	$a^2 - b^2$ including four terms	x	x	-	-	-	-	-
12.	More difficult equations of the first degree with one unknown and problems leading thereto	x	x	x	x	-	-	-
13.	Statistical graphs (line) Sectorgraphs	x	x	x	x	x	x	-
	Column graphs	-	-	x	-	-	-	-
	Curves	-	-	-	x	-	-	-
14.	Graph of $mx + c$	x	x	x	x	-	x	-
15.	Graph of $ax + by + c = 0$	-	-	-	-	x	-	-
	Graphs of two simultaneous equations of the first degree	x	x	x	x	x	x	-
	Intersection	x	x	x	x	x	x	-
16.	Equations of the type $ax + by = c$ where a, b and c are numbers. Two equations with two unknowns	x	x	x	x	x	x	x
	of the type: $\frac{a}{x} + \frac{b}{y} = c$	x	-	-	-	-	-	-
	Three equations with three unknowns	x	-	-	-	-	-	-
17.	Revision and extension of symbolic representation	x	x	-	-	-	-	-
18.	More difficult substitutions	x	x	x	-	-	-	x
	Changing the subject of a formula	x	x	x	-	x	x	x
19.	Problems which lead to equations of the type $ax + by = c$	x	x	x	x	x	x	x

[1] Orange Free State.

		Transvaal			O.F.S.[1]	Natal		Cape
		A	B	C		V	W	
20.	Graph mx + c:							
	Gradient and intercept on Y-Axis	x	x	-	x	-	-	-
	Calculation of a straight line	-	-	-	-	x	-	-
	through two points	x	x	-	-	-	-	-
21.	H.C.F. and L.C.M. by factors	x	x	-	x	x	x	-
22.	Simplification of fractions by means of							
	factors	x	x	-	x	x	x	-
	Four main calculations with fractions	x	x	-	-	x	x	x
23.	Square root by means of							
	factors	x	x	-	-	x	x	-
	division	x	x	-	-	-	-	-
24.	Drawing the graph $ax^2 + bx + c$	x	-	-	-	-	-	-
25.	Solution of equations of the second degree							
	by means of factors	-	-	-	-	x	x	-

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[1] Orange Free State.

APPENDIX 2

GEOMETRY FOR STD. 6

Transvaal

The remarks made in the algebra section are also applicable here.

			A	B	C
1.		Geometrical concepts of line and point	x	x	x
	(a)	Lines as boundaries between surfaces	x	x	x
	(b)	Points as intersections of two straight lines, two arcs or a straight line and an arc	x	x	x
	(c)	The measurement and marking off of lengths	x	x	x
	(d)	Addition and subtraction of lengths inter alia as in the perimeter of triangles and quadrilaterals	x	x	x
2.		Circles			
	(a)	Centre, radius, diameter, chord, arc, segment, sector (no definition)	x	x	x
	(b)	Regular hexagon	x	x	x
	(c)	Concentric and eccentric circles: Common areas of circles which intersect	x	x	x
3.		Angles			
	(a)	Size and meaning of an angle	x	x	x
	(b)	Opposite angles	x	x	x
	(c)	Rotation	x	x	x
	(d)	The straight angle	x	x	x
	(e)	The right angle	x	x	x
	(f)	If one straight line meets or cuts another straight line	x	x	x
	(g)	The use of a protractor	x	x	x
	(h)	The eight main points of a compass	x	x	x
	(i)	Recognition of acute, obtuse and reflex angles	x	x	x
	(j)	The sum of the angles of a triangle	x	x	x
4.	(a)	Congruency (3 sides)	x	x	x
	(b)	To bisect an angle	x	x	x
	(c)	To erect a perpendicular at a given point on a straight line	x	x	x
	(d)	Squares and rectangles. The properties of the diagonals of these figures	x	x	x
	(e)	Nature and properties of isosceles and equilateral triangles	x	x	x
	(f)	Angles 60^{o}, 90^{o}, 30^{o} and 45^{o}	x	x	x
		Construction of angles up to 360^{o} and combinations thereof	x	-	-
	(g)	To bisect a straight line	x	x	x
	(h)	To construct a perpendicular to a line from a point outside that line	x	x	x
	(i)	The altitudes of a triangle pass through one point	x	x	-
	(j)	The circumscribed circle of a triangle	x	x	-
	(k)	The medians of a triangle pass through one point	x	x	-
	(l)	Easy drawings to scale and problems of height and distance	x	x	-
	(m)	Congruency with two sides and the included angle	x	x	-
	(n)	Congruency with two angles and the corresponding side	x	x	x
	(o)	Congruency with a right-angle, the hypotenuse and another side	x	-	-
	(p)	Testing the theory of Pythagoras by construction and measurement	x	-	-

Orange Free State

The remarks made in the Algebra section are also applicable here.

1. Knowledge of the following concepts:

   (a) Point, straight line;
   (b) Angle and the different kinds of angle; right angle, acute angle, obtuse angle, reflex angle, supplementary and complementary angles;
   (c) Triangle and the different kinds of triangles; Isosceles triangle, equilateral triangle, right angled triangle, etc.;
   (d) Perpendicular;
   (e) Parallel lines.

2. The following constructions:

   (a) Construction and measurement of straight lines and angles.
   (b) Bisection of straight lines and angles.
   (c) The construction with the aid of a compass of angles of 90^o, 45^o, 60^o, 30^o.
   (d) The construction of a perpendicular to a given straight line.

      (i) From a point on the line;
      (ii) From a point outside the line;

   (e) The construction of an angle equal to a given angle.
   (f) The construction of a line parallel to a given straight line.
   (g) The construction of a triangle when the following are known;

      (i) Two sides and the included angle.
      (ii) One side and the two adjoining angles.
      (iii) Three sides.
      (iv) Two sides and a right angle.

3. The concepts of locus and drawing, amongst others, the locus of a point which moves:

   (a) at a fixed distance from a given fixed point;
   (b) at a fixed distance from a given straight line;
   (c) equally distant from two given fixed points;
   (d) equally distant from two given straight lines.

4. Easy drawings according to scale including cases where the angles of elevation are used or where angles relative to the points of the compass are included.

Natal

The remarks made on the algebra section are also applicable here.

1. Meaning of point, line, straight line, surface, area, solid.

2. Measurement of straight lines in inches and tenths of an inch, (estimation of the lengths as accurately as possible to the second decimal place) and in centimetres and millimetres to the nearest millimeter.

3. Angles: Definition. Types (revolution, straight angle, right angle, acute angle, obtuse angle, reflex angle). Meaning of arm, apex, perpendicular.

4. Adjacent angles; The sum of the adjacent angles on a straight line.

5. Supplementary and complementary angles.

6. Vertically opposite angles. Equality of vertically opposite angles.

7. Parallel straight lines: Definition. Construction by use of a ruler and set square.

8. Circle: Definition. Construction. Meaning of circumference, radius, diameter, arc.

9. Rectangle: properties and construction.

10. Triangle: Definition. Sum of the angles of a triangle = 180°. Classification according to the sides (equilateral, isosceles, scalene) and according to the angles (acute angled, obtuse angled, right angled).

Construction of triangles, given the following:

(i) Three sides
(ii) Two sides and the included angle
(iii) One side and the angle at its extremities.

11. Quadrilaterals. Definition. Special quadrilaterals (parallelograms, rectangle, square, rhombus, trapezium.

12. Further constructions (by using a ruler and compasses only).

To construct an angle of 60°.
To bisect a given angle.
To construct an angle of 30°.
To construct an angle of 90°.
To bisect a given straight line.
To construct an angle equal to a given angle.

13. Properties of parallel straight lines. Construction of parallel straight lines by using a ruler and compasses only.

14. Scale Drawings.

15. Points of the compass. North, South, East, West, North East, South East, North West, South West.

Cape Province

There is no Geometry taught here except the usual geometry found in an Arithmetic Syllabus.

APPENDIX 3

GEOMETRY FOR STDS. 7 AND 8

Remarks made at the beginning of Appendix 1 also apply here.

Where proofs are not required, an asterisc is marked.

		Transvaal			O.F.S.[1)]	Natal		Cape
		A	B	C		V	W	
1.	Measurement of straight lines and angles	-	-	x	-	x	x	x
2.	Construction of perpendiculars with the aid of a set square	-	-	-	-	x	x	-
	Construction of triangles and rectangles	-	-	-	-	x	x	x
	Construction of parallel lines by moving a set square	-	-	x	-	x	x	x
	Determination of heights or distances by means of scale drawings	-	-	-	-	x	x	-
	Construction including angular directions with the four points of the compass as reference points	-	-	x	-	x	x	x
3.	To bisect a straight line	x	x	x	-	x	x	-
4.	To bisect an angle	x	x	x	-	x	x	x
5.	To construct a perpendicular to a given straight line at a point on that line	x	x	x	-	x	x	x
6.	To construct a perpendicular to a given line from a given point outside that line	x	x	x	-	x	x	x
7.	To construct angles of 60°, 30°, 45°, etc. without a protractor	x	x	x	-	-	x	-
8.	To construct an angle equal to a given angle	x	x	-	-	-	-	x
9.	To draw parallel lines using a compass	x	x	x	-	x	x	-
10.	To construct regular hexagons, octagons, pentagons, etc.	-	-	-	-	-	-	x
	(i) which fit into a circle	x	x	x	-	x	x	-
	(ii) with a given side	x	x	-	-	-	-	-
11.	Construction of triangles	x	x	x	-	-	-	x
12.	Construction of rectangles	x	x	-	-	x	x	-
13.	Construction of triangles with areas equal to those of given triangles or rectangles	x	x	-	-	x	x	-
14.	Division of a straight line into a number of equal parts	x	x	x	x	x	x	x
15.	Scale drawings	x	x	x	-	-	-	x
16.	Drawing a perpendicular at the end of a straight line by using the theorem of Pythagoras	-	-	-	-	-	-	x

1)Orange Free State

		Transvaal			O.F.S.[1]	Natal		Cape
		A	B	C		V	W	
17.	To construct a circle which passes through three points not in a straight line	-	-	-	x	-	-	-
18.	To determine the centre of a given circle	x	-	-	-	-	-	-
19.	To draw a tangent to a circle at a given point on the circumference of that circle	x	-	x	x	x	x	-
20.	To draw a tangent to a circle from a point outside the circle	x	-	x	x	x	x	-
21.	To construct the inscribed, circumscribed and escribed circles of a given triangle	x	-	x	-	-	-	-
22.	The construction of the medians and perpendicular bisectors of the sides of a triangle	-	-	x	-	-	-	-
23.	The following geometrical loci:							
	(a) At a constant distance from a given point	x	x	x	x	-	-	-
	(b) At a constant distance from a given straight line	x	x	x	x	-	-	-
	(c) At a constant distance from the circumference of a given circle	x	x	x	-	-	-	-
	(d) So that it remains equidistant from two parallel lines	x	x	x	x	-	-	-
	(e) So that it remains equidistant from two given points	x	x	x	x	x	x	x
	(f) The converse of (e)	-	-	-	-	x	x	-
	(g) So that it remains equidistant from two intersecting lines	x	x	x	x	x	x	x
	(h) The converse of (g)	-	-	-	-	x	x	-
	(i) So that it is the vertex of a triangle of given surface area on a given base	x	x	-	-	-	-	-
	(j) So that a given straight line will subtend a right angle at the point	x	-	-	x	-	-	-
	(k) So that it is the same distance from a given point and a given straight line	x	-	-	-	-	-	-
	(l) So that the sum of its distances to two given points remains constant	x	-	-	-	-	-	-
24.	The construction of parallelograms	-	-	x	-	-	-	-
	THEOREMS AND PROBLEMS							
25.	If one straight line meets another straight line, two angles are formed which are together equal to two right angles and the converse.	x	x	-	x	x	x	x
26.	If two straight lines cut one another the pairs of vertically opposite angles are equal	x	x	-	x	x	x	x
27.	If a pair of straight lines are cut by a transverse line, those lines are parallel if							

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		Transvaal A	Transvaal B	Transvaal C	O.F.S.[1]	Natal V	Natal W	Cape
	(a) A pair of corresponding angles are equal	**x**	**x**	-	**x**	**x**	**x**	-
	(b) A pair of alternate angles are equal	x	x	-	**x**	**x**	**x**	-
	(c) A pair of interior angles on the same side of the transversal are supplementary	**x**	**x**	-	**x**	**x**	**x**	-
28.	When two parallel straight lines are cut by a transversal							
	(a) the pair of corresponding angles are equal	**x**	**x**	-	**x**	**x**	**x**	**x**
	(b) the pair of alternate angles are equal	x	x	-	**x**	**x**	**x**	**x**
	(c) the interior angles on the same side of the transversal are supplementary	x	x	-	**x**	**x**	**x**	-
29.	Straight lines which are parallel to the same straight line are parallel to one another	**x**	**x**	-	**x**	**x**	**x**	-
30.	The angle between two straight lines is equal to the angle between lines drawn parallel to these straight lines and in the same direction	-	-	-	-	**x**	**x**	-
31.	The sum of the angles of a triangle is equal to two right angles	x	x	-	x	x	x	x
32.	If one side of a triangle is produced, the exterior angle which is formed is equal to the sum of the interior opposite angles	**x**	**x**	-	x	x	x	-
33.	If two angles of a triangle are respectively equal to two angles of another triangle then the third pair will also be equal	**x**	**x**	-	-	-	-	-
34.	In a right angle triangle the right angle is the greatest angle and the sum of the other two angles is equal to one right angle	**x**	**x**	-	-	-	-	-
35.	If two sides and the included angle of a triangle are equal to two corresponding sides and the included angle of another triangle, the two triangles are congruent	**x**	**x**	-	**x**	**x**	**x**	**x**
36.	If one side and two angles of a triangle are equal to the corresponding side and angles of another triangle, the two triangles are congruent	**x**	**x**	-	**x**	**x**	**x**	**x**
37.	If two sides of a triangle are equal, the angles opposite those two sides are also equal and the converse	x	x	-	x	x	x	x
38.	If two sides of a triangle are unequal, the angles opposite those two sides are also unequal with the larger angle opposite the longer side and the converse							

[1] Orange Free State

		Transvaal			O. F. S.[1]	Natal		Cape
		A	B	C		V	W	
39.	Any two sides of a triangle are together longer than a third side	-	-	-	-	x	x	-
40.	If the three sides of a triangle are respectively equal to the three sides of another triangle, then the two triangles are congruent	x	x	-	x	x	x	x
41.	If the hypotenuse and one other side of a right angled triangle are equal to the corresponding sides of another right angled triangle, the two triangles are congruent	x	x	-	x	x	x	x
42.	Of all the straight lines which may be drawn from a given point to a given straight line, the perpendicular is the shortest	x	x	-	x	x	x	-
	Converse	-	-	-	-	x	x	-
43.	The pairs of opposite sides of a parallelogram are equal, and the converse	x	x	-	x	x	x	-
44.	The pairs of opposite angles of a parallelogram are equal, and the converse	x	x	-	x	x	x	-
45.	The diagonals of a parallelogram divide it into two congruent triangles	x	x	-	x	x	x	-
46.	The diagonals of a parallelogram bisect one another, and the converse	x	x	-	x	x	x	-
47.	Definition of a right angle and its derived properties	-	-	-	x	-	-	-
48.	Definition of a rhombus and its derived properties	-	-	-	x	-	-	-
49.	Definition of a square and its derived properties	-	-	-	x	-	-	-
50.	The theorem of Pythagoras	-	-	-	x	x	x	x
	Converse	-	-	-	x	x	x	x
51.	The area of a parallelogram is equal to the area of a rectangle with the same base and between the same parallels	x	x	-	-	-	-	-
52.	The area of a rectangle is the product of two adjoining sides	-	-	-	-	x	x	-
53.	The area of a parallelogram is the product of its base and its perpendicular height	x	x	-	-	x	x	-
54.	The area of a triangle is half the product of its base and its perpendicular height	x	x	-	-	x	x	-
55.	The area of a trapesium	-	-	-	-	x	x	-

[1] Orange Free State

		Transvaal			O.F.S.[1]	Natal		Cape
		A	B	C		V	W	
56(a)	Parallelograms on the same base and between the same parallels are equal in area	**x**	**x**	-	-	**x**	**x**	-
	Converse	-	-	-	-	**x**	-	-
(b)	As above, for triangles	-	-	-	-	x	-	-
57.	If a pair of opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram	x	x	-	x	x	x	-
58.	The straight line which is drawn through the midpoint of one side of triangle, parallel to a second side bisects the third side	**x**	**x**	-	x	x	**x**	-
	Converse	**x**	**x**	-	x	x	**x**	-
59.	If three or more parallel lines make equal intercepts on a transversal they make equal intercepts on any other transversal	**x**	**x**	-	**x**	**x**	**x**	-
60.	The straight line joining the centre of a circle with the midpoint of a chord is perpendicular to the chord	-	-	-	x	x	x	-
	Converse	-	-	-	x	x	x	-
61.	In a circle the angle subtended by an arc at the centre is twice the angle subtended at the circumference by the same arc	-	-	-	x	x	x	-
62.	The perpendicular bisector of a chord passes through the centre of a circle	-	-	-	-	**x**	**x**	-
63.	Only one circle can be drawn passing through three given points which do not lie in the same straight line	-	-	-	-	**x**	**x**	-
64.	Angles in the same segment of a circle are equal	-	-	-	x	x	x	-
65.	The angle in a semi-circle is a right angle	-	-	-	x	**x**	**x**	**x**
66.	If a tangent is drawn to a circle, the radius to the circle at the point of contact is perpendicular to the tangent	-	-	-	**x**	**x**	**x**	**x**
67.	Converse	-	-	-	**x**	**x**	**x**	-
68.	Two tangents from the same point outside a circle drawn to that circle are equal	-	-	-	**x**	-	-	-
69.	Symmetry	-	-	-	-	-	-	x
70.	Types of triangle	-	-	-	-	-	-	x
71.	The meaning of the term similar triangles i.e. similar triangles are equiangular and their corresponding sides are proportional	-	-	-	-	-	-	**x**
72.	The area of a triangle is equal to half the area of a parallelogram on the same base and of the same perpendicular height	-	-	-	-	-	-	**x**

[1] Orange Free State

		Transvaal			O.F.S.	Natal		Cape
		A	B	C		V	W	
73.	The calculation of areas of parallelograms and trapeziums	-	-	-	-	-	-	x
74.	The calculation of the volume of a prism on a triangular base	-	-	-	-	-	-	x
75.	Calculation of the circumference and area of a circle, the curved surface of a cylinder and the volume of a cylinder	-	-	-	-	-	-	x
TRIGONOMETRY								
1.	The properties of similar triangles	-	-	-	x	-	-	x
2.	The sine, cosine and tangent of an acute angle	x	x	-	x	-	-	-
3.	Graphic representation of the change in value of these functions from 0° to 90°	x	x	-	x	-	-	-
4.	Determination of the value of these functions by construction and measurement	x	x	-	-	-	-	-
5.	Tables of the functions mentioned in (2)	x	x	-	x	-	-	x
6.	Determination of simple heights and distances	x	x	-	x	-	-	x

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APPENDIX 4

COMPARISON OF THE STANDARD SIX SYLLABI IN ARITHMETIC

		Cape and S.W.A.[3]	Natal	O.F.S.[1]	B-stream Trans-vaal	E. A &[2] Science
1.	Notation, oral number work, whole numbers	-	x	-	-	x
	Square root by factors	-	x	x	-	x
2.	Money, weights and measures, time and content, shop transactions. Invoices	x	x	-	-	x
3.	British money system. Long Division and multiplication. Conversion of British Money into South African money and the converse	-	x	-	-	-
4.	H.C.F. by factors and alternate division. L.C.M. by factors	-	x	x	x	-
5.	Common and decimal fractions	x	x	x	x	-
6.	Percentages	x	x	x	-	x
7.	Short methods of multiplying and dividing by multiples and submultiples of 10, 100 and 1000	-	x	-	-	-
8.	Ratio and proportion by the method of fractions	x	x	x	x	x
9.	Averages	-	-	-	x	x
10.	Simple interest.	-	x	-	x	x
	Refer to post-office and other savings banks, Union Loan Certificates, Building Societies, etc.	-	x	-	-	-
11.	Profit and loss, expressed as a percentage of the cost price only.	-	-	-	-	x
	Cost price and selling price.	-	x	-	-	-
12.	Commercial transactions.	-	x	-	-	-
	Discount rates	-	x	-	-	-
	Hire Purchase	-	x	-	-	-
13.	Measurement (a) Perimeters	-	x	-	Ring	x
	(b) Areas including triangles	-	x	-	Parm.	x
	(c) Volume of a prism	-	x	-	x	Cylinder
14.	Rules for divisibility by 2, 5, 3, 4, 8, 9, 11	-	-	x	x	-
15.	Metric units. Length, area and volume	-	-	x	x	-
16.	Statistical Graphs. Reading of data from scale drawings	x	-	-	-	-
17.	Fractions and simplification of fractions	-	-	-	x	-
18.	Conversion of one unit to another	-	-	-	x	-

[1] Orange Free State

[2] Education, Arts and Science

[3] South West Africa

		Cape and S.W.A.[3]	Natal	O.F.S.[1]	B-stream Transvaal	E.A &[2] Science
19.	Money (a) Calculations	-	-	-	x	-
	(b) Costing (fractions of a Rand)	-	-	-	x	-
20.	Approximations and short methods in the four basic calculations	-	-	-	x	-
21.	Discount and Commission	-	-	-	x	-

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[1] Orange Free State

[2] Education, Arts and Science

[3] South West Africa

APPENDIX 6

COMPARISON OF THE STANDARD 8 SYLLABI IN ARITHMETIC

		Cape and S.W.A.[3]	Natal	O.F.S.[1]	B-stream Trans-vaal	E. A. &[2] Science
1.	Number theory:					
	(a) Short methods of calculation	-	x	-	-	-
	(b) Long and short tots	-	x	-	-	-
	(c) Four main calculations with common and decimal fractions	x	x	x	x	x
	(d) Recurring decimals - elementary	-	x	-	-	-
	(e) Approximations	x	-	-	x	x
	(f) Formulae: Change of subject	-	-	-	x	-
	(g) Exponents: The laws	x	-	-	x	-
	(h) The use of logarithms in division, multiplication and raising the power	-	-	-	x	-
2.	L. C. M. and H. C. F. by factors and division	-	x	x	-	-
3.	Square roots: Factors and division	-	-	x	-	x
4.	Ratio and proportion	-	-	x	-	x
5.	Averages and mixtures	-	x	-	-	x
6.	Time, measures and weights, also in the metric system	-	-	x	-	x
7.	Simple interest	x	x	x	x	x
8.	Percentages (Education, Arts and Science	x	x	x	x	x
	also the 1/3, 1/10, 1/100 rule)	-	-	-	-	x
9.	Money (a) Foreign systems of money	x	x	-	x	-
	(b) Rates of exchange	x	x	-	x	x
10.	Commercial and Savings Bank accounts	-	x	-	-	-
	(a) Daily balances and overdrawn accounts	x	x	-	x	-
	(b) Compound interest (long method)	x	x	x	x	x
	(c) Compound interest (by formula)	x	x	x	x	-
	(d) Compound interest (read from tables)	-	x	-	x	-
11.	Insurance	-	x	-	-	-
12.	Commerce and industry					
	(a) Manufacturer, wholesale merchant, retail merchant	-	-	-	x	-
	(b) Commercial discount and cash discount	x	-	-	x	-
	(c) Profit and loss	x	x	x	x	x
	(d) Partnerships	-	x	x	x	x
	(e) Companies, co-operative societies	-	-	-	x	-
	(f) Shares and stock exchange quotations	-	-	-	x	-
	(g) Dividends	-	-	-	x	-
	(h) Payment by cheque, postal order, postal note, promissory notes	x	x	-	x	-
	(i) Bill of exchange and discounting	-	x	-	x	x
	(j) Depreciation	-	-	-	x	-
	(k) Insolvencies	-	x	-	x	-
13.	Pensions	-	-	-	x	-

[1] Orange Free State

[2] Education, Arts and Science

[3] South West Africa

		Cape and S. W. A.[3]	Natal	O. F. S.[1]	B-stream Trans-vaal	E. A &[2] Science
14.	Municipal services and revenue					
	(a) Assessment rates	x	x	-	x	x
	(b) Water and lights, transport, recreation	x	-	-	-	x
	(c) Penalties and licences	x	-	-	x	x
15.	Provincial and National Revenue	x	x	-	x	x
16.	Loans: Municipal and Government Stock					
	(a) Stock market quotations	-	-	-	x	-
	(b) Buying and selling, brokers, stamp duty and transfer	-	-	-	x	-
17.	Graphs based on this work	-	x	-	-	-
18.	Solids					
	(a) Revision of previous work	x	x	x	x	x
	(b) Area of triangle by S-formula	-	x	-	x	x
	(c) Prism: rectangular and triangular	-	x	x	x	x
	(d) Area and volume of the circle, cone, sphere and cylinder	-	-	x	-	x
	(e) Wood in a rectangular box	-	-	x	-	-
	(f) Use of formulae $\pi(R^2 - r^2)h$					
	(g) Pyramid	-	-	-	-	x

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[1] Orange Free State

[2] Education, Arts and Science

[3] South West Africa

APPENDIX 7

THE MATHEMATICS SYLLABI FOR STANDARDS 9 AND 10

Under the heading of Natal, the 0 indicates that the subjects appear in the Additional Mathematics or Advanced Grade Mathematics Syllabus. Under the heading of the Joint Matriculation Board, the 0 indicates that the subjects appear under the Additional Mathematics Syllabus.

Under the heading of the Transvaal, the 0 indicates that the subjects are only intended for candidates for Matriculation Exemption. * indicates that theorems should be learnt. The others may be accepted as axioms.

	Subject	J.M.B.	Cape S.W.A.	Natal	O.F.S.	Tvl.	E. A & Science
Arithmetic							
1.	Number: whole numbers, fractions, rational, irrational, real numbers	x	x	x	-	-	x
2.	Representation of numbers by points on a straight line	x	-	x	-	-	x
3.	The four fundamental processes with fractions and decimal fractions	x	-	x	-	-	x
4.	Lengths, areas and volumes: Triangle, parallelograms, trapezium, circle, cylinder, prism, pyramid, cone and sphere. (In the last six cases no proof is necessary for the formulae)	x	x	x	x	-	x
5.	Simple and compound interest where logarithmic calculations are necessary	x	*	x	-	x	x
6.	Rationalisation of simple root forms e.g.: $\frac{1}{\sqrt{5}}$	0	-	-	-	-	x
Algebra							
1.	Algebraic notation	x	x	x	x	x	x
2.	Subtraction. Addition, multiplication and division						
	Products by inspection	x	x	x	x	x	x
	Impossibility of division by zero	x	x	x	x	x	x
3.	Use of brackets. Rules for brackets and signs	x	-	x	x	x	x
4.	Meaning of: term, factor, co-efficient, degree of expression	x	x	x	x	x	x
5.	The function concept and independent and dependent variables	x	-	x	x	x	x
6.	Linear second and third degree expressions in one variable	x	x	x	x	x	x
7.	Homogeneous expressions in two variables	x	-	x	x	x	x
8.	Functional notation e.g. f(x), f(x, y)	x	-	x	x	x	x
9.	The calculation of functions for numerical values of variables	x	-	x	x	x	x

	Subject	J. M. B.	Cape S. W. A.	Natal	O. F. S.	Tvl.	E. A & Science
10.	The remainder theorem with applications	x	x	x	x	x	x
11.	The solution of $ax^3+bx^2+cx+d=0$, where at least one factor is obtained by means of the remainder theorem, (a, b, c, and d have numerical values)	-	-	-	-	x	-
12.	The graphs of						
	(i) $ax+b$ (ii) $x^2+y^2=a^2$ (iii) a^x	x	x	x	x	x	x
	(iv) ax^2+bx+c and (v) $xy=c$	x	x	x	x	x	x
	(vi) ax^3+bx^2+cx+d (in all these instances, a, b, c, and d have numerical values)	-	-	-	-	0	-
13.	(a) Elementary deductions from graphs	x	x	x	x	x	x
	(b) With the inclusion of inequalities	x	x	-	x	x	x
	(c) Finding the equation for the parabola if 3 points are given on the curve	-	-	-	-	x	-
14.	(a) Definition of the tangent to a curve at a point on the curve	-	-	-	-	x	-
	(b) The slope of a curve at a given point on the curve considered as the slope of the tangent at that point	-	-	-	-	x	-
15.	The average slope of a curve between two points on the curve	-	-	-	-	x	-
16.	Discussion of the slope of a curve at any point on the curve and determination of the tangential function by means of the h-method where h is very small, by referring to x^2 and $2x^2+3$ etc. (Use the capital D notation for the tangential function.)	-	-	-	-	x	-
17.	Find the equation of a tangent to a curve at a given point on the curve	-	-	-	-	0	-
18.	(a) Geometrical demonstration that the slope of a curve at a point of maximum or minimum value is zero	-	-	-	-	x	-
	(b) Determination of the co-ordinates of the points at which a quadratic function has its maximum or minimum value: (the abscissa by equating the gradient function to zero, and the ordinate by substitution)	x	x	x	x	x	-
	(c) Find the maximum and minimum values of ax^3+bx^2+cx+d (a, b, c, and d have numerical values). More elaborate sketching of cubics	-	-	-	-	0	-
19.	(a) Simple examples of factorization	x	x	x	x	x	x
	(b) H. C. F. and L. C. M. by means of factors	-	x	-	-	x	-
	(c) Square roots by means of factors	-	-	0	x	x	-
20.	Identities and their properties	x	-	x	x	-	x
21.	Equations. Roots of equations	x	x	x	x	x	x

	Subject	J.M.B.	Cape S.W.A.	Natal	O.F.S.	Tvl.	E. A & Science
22.	The solution of linear equations in two or three unknowns	x	x	x	x	x	x
23.	The quadratic expression ax^2+bx+c and reduction to the form of complete squares	x	x	x	x	x	x
24.	(a) The roots of types: $ax^2+bx+c=0$	x	x	x	x	x	x
	(b) Find the equation if the roots are given	-	-	-	-	x	-
25.	(a) The nature of the roots and the determination of the sum and the product of the roots of $ax^2+bx+c=0$	x	x	x	x	0	x
	(b) The determination of the other root if one is given	-	-	-	-	0	-
26.	Simultaneous equations with two unknowns, one of the first and one of the second order	x	x	x	x	x	x
27.	To express, in a given functional relation, one of the quantities in terms of the other quantities (change of the subject)	x	-	x	x	x	x
28.	The meaning of a^n where n is a natural number and proofs of the laws:						
	(i) $a^m \times a^n = a^{m+n}$	x	x	x	x	x	x
	(ii) $a^m \div a^n = a^{m-n}$ $m > n$	x	x	x	x	x	x
	(iii) $(ab)^n = a^n x b^n$	x	x	x	x	x	x
	(iv) $(a^m)^n = a^{mn}$	x	x	x	x	x	x
	where m and n are natural numbers						
29.	Extension of the definition of a^n where (i) n=0, (ii) n is negative	x	x	x	x	x	x
	(iii) n is any rational number	x	x	x	x	x	x
	(iv) the use of fractional and negative exponents in simple examples of division and multiplication	-	-	-	-	x	-
30.	The fundamental properties of roots namely						
	(i) $\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$	x	x	x	x	x	x
	(ii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	x	x	x	x	x	x
	(iii) $(\sqrt[m]{a})^n = \sqrt[m]{a^n}$	x	x	x	x	x	x
31.	Like and unlike root-forms and reduction to like forms	x	x	x	-	-	x
32.	Application of the exponential laws to examples with numerical and symbolic exponents (i) multiplication, division, raising the power and extracting roots, including the simplification of forms such as for e.g. $\frac{\sqrt{(a^2-b^2)^3} \times \sqrt[6]{(a-b)^5}}{\sqrt[3]{(a^2-b^2)^2}}$	-	-	-	-	-	0

	Subjects	J.M.B.	Cape S.W.A.	Natal	O.F.S.	Tvl.	E. A & Science
	(ii) The solution of exponential equations for e.g. $3.3^{2x} - 28.3^{x} + 9 = 0$	-	-	-	-	-	0
33.	Discussion of the solution of $a^x = b$ where a and b are given numbers and a is greater than 0	x	x	x	x	x	x
34.	The properties of logarithms and the use of logarithms and antilogarithm tables (propositions regarding the change of base are not required)	x	x	x	x	x	x
35.	Logarithms to the base a ($a > 0$ and $a \neq 1$) Definition	x	x	x	x	x	x
36.	The graph of log x	-	-	-	x	-	-
37.	The theory of logarithms including the change of base with formal proof	-	-	0	-	-	-
38.	Problems based on equations of two unknowns	-	x	x	-	x	-
39.	Simple examples of ratio and proportion with the examples	x	x	-	x	x	x
40.	Variation including easy examples of more than two variables	0	-	-	-	-	-
41.	The meaning of homogeneity, symmetry, cyclic order, with simple applications of these concepts	0	-	0	-	-	-
42.	Arithmetic and geometric series. Finding (i) nth term of a series	x	x	0	x	x	x
	(ii) Sum of the first n terms of a series	x	-	0	x	x	x
	(iii) The Arithmetic or geometric average of two numbers	x	x	0	x	x	x
	Geometry						
	The following concepts and facts are introduced intuitively: point, straight line, angle, right angle, straight angle, revolution, degree, acute angle, obtuse angle, reflex angle, adjacent angles, supplementary and complementary angles, equality of lines and angles, parallel lines, triangle and circle (For examination purposes proofs are required for theorems indicated by an asterisc. The remainder may be regarded as axiomatic)	x	-	x	x	x	x
1.	If a straight line stands on another straight line, the sum of the adjacent angles formed is equal to 180°, and the converse	x	x	x	x	x	x
2.	If two straight lines intersect, the vertically opposite angles are equal	x	x	x	x	x	x

	Subjects	J.M.B.	Cape S.W.A.	Natal	O.F.S.	Tvl.	E. A & Science
3.	If two parallel straight lines are cut by a third straight line, (a) the corresponding angles are equal (b) the alternate angles are equal (c) the sum of the interior angles on the same side of the transverse line are equal to 180 degrees and conversely	x	x	x	x	x	x
4.*	The exterior angle of a triangle is equal to the sum of the interior opposite angles and the three angles of a triangle are together equal to 180^{o}	x	x	x	x	x	x
5.	If two sides and the included angle of a triangle are equal to two sides and the included angle of another triangle, the two triangles are congruent	x	x	x	x	x	x
6.	If two angles of a triangle are respectively equal to two angles of another triangle and one side of the one is equal to the corresponding side of the other, the triangles are congruent	x	x	x	x	x	x
7.	If three sides of a triangle are respectively equal to three sides of another, the triangles are congruent	x	x	x	x	x	x
8.	If the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right angled triangle, the triangles are congruent	x	x	x	x	x	x
9.*	The angles at the base of an isoceles triangle are equal and the converse	x	x	x	x	x	x
10.	Any two sides of a triangle are together greater than the third side	x	x	x	x	x	x
11.	If two sides of a triangle are unequal, the angle opposite the longer side will be larger than the angle opposite the shorter side	x	x	-	x	x	x
12.*	The opposite sides and angles of a parallelogram are equal and each diagonal bisects the area of the parallelogram	x	x	x	x	x	x
13.*	If a pair of opposite sides of a quadrilateral are parallel and equal, the quadrilateral is a parallelogram	x	x	x	x	x	x
14.*	The diagonals of a parallelogram bisect one another	x	x	x	x	x	x
15.*	A straight line drawn through the midpoint of one side of a triangle, parallel to a second side bisects the third side and is equal to half of the second side	x	x	x	x	x	x

	Subjects	J.M.B.	Cape S.W.A.	Natal	O.F.S.	Tvl.	E. A & Science
16.*	The line joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	x	x	-	x	x	x
17.	If three or more parallel straight lines cut off equal portions of a transverse line, they will cut off equal portions of any intersecting line	x	x	-	x	x	x
18.*	The line joining the centre of a circle and the midpoint of a chord is perpendicular to the chord, and conversely	x	x	-	x	x	x
19.*	The angle subtended by the arc of a circle at the centre is double the angle subtended at a point on the circumference of the circle	x	x	-	x	x	x
20.*	(a) The angle in a semicircle is a right angle	x	x	x	x	x	x
	(b) As a geometrical locus	-	-	-	-	x	-
21.*	(a) Angles in the same segment of a circle are equal and conversely	x	x	x	x	x	x
	(b) As a geometrical locus	-	-	-	-	x	-
22.	The sum of a pair of opposite angles of a cyclic quadrilateral is 180° and the converse	x	x	x	x	x	x
23.	The area of a triangle is half the base times the height	x	x	-	x	-	x
24.	Triangles on the same base (or equal bases) and between the same parallels are equal in area (this is also true for parallelograms)	x	x	-	x	x	x
25.	(a) The area of parallelograms, rectangles and trapezium	x	x	-	x	-	x
	(b) Definition of similarity	x	x	x	x	x	x
26.*	Any line parallel to the side of a triangle divides the other two sides proportionately and conversely	x	x	x	x	x	x
27.*	Corresponding sides of equiangular triangles are proportional and conversely	x	x	x	x	x	x
28.*	The theorem of Pythagoras and the converse. Definition of a tangent as the limiting position of a chord when	x	x	x	x	x	x
	the two points of intersection coincide	x	x	-	x	x	x
29.	A straight line drawn through any point of a circle perpendicular to a radius at that point is a tangent to the circle and conversely	x	x	-	x	x	x
30.*	The angle between a tangent to a circle and a chord through the point of contact are equal to the angle in the opposite segment, and conversely	x	x	x	x	x	x

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	Subjects	J.M.B.	Cape S.W.A.	Natal	O.F.S.	Tvl.	E. A & Science
31.*	If two chords of a circle intersect at a point inside or outside a circle, the rectangle contained by the segments of the one equal to the rectangle contained by the segments of the other, and conversely	x	-	x	x	x	x
32.*	If from a point outside a circle a secant and a tangent to the circle are drawn, the rectangle contained by the segments of the secant is equal to the square on the tangent and conversely	x	-	x	x	x	x
33.	The concept of locus. Simple cases of loci in which the straight line and circle are involved	x	x	x	x	-	x
34.*	The following cases of the concurrency of lines should be treated:						
	(a) the internal bisectors of the angles of a triangle	x	x	x	x	0	x
	(b) the perpendicular bisectors of the sides of a triangle	x	x	x	x	0	x
	(c) The medians of a triangle	x	x	x	x	0	x
	(d) The altitudes of a triangle	x	x	x	x	0	x
NOTE:	A sound knowledge of the concepts which are fundamental to the above theorems is of the greatest importance. Only simple work and problems and an equal quantity of bookwork will be required for examination purposes. Problems of loci will only be of an elementary nature. (The teaching of Euclidian Geometry should take place in three stages:	x	-	-	x	-	x
	1. The experimental stage 2. The deductive stage 3. The systematic stage)	x	-	-	-	-	x
35.	If two angles of a triangle are respectively equal to two angles of another triangle, they will be similar triangles	-	-	-	-	x	x
36.	Draw the (a) circumscribed (b) inscribed	-	x	x	-	-	-
	(c) escribed circles of a triangle	-	x	-	-	-	-
37.	On a given chord of a circle draw a segment of a circle containing a given angle	-	x	-	-	-	-
38.	Draw the fourth porportional of three given straight lines	-	x	x	-	-	-
39.	Draw the third porportional of two given straight lines	-	x	x	-	-	-
40.	To divide a straight line (a) internally and (b) externally in a given ratio	-	-	0	-	-	-
41.	Draw the middle proportional of two straight lines	-	-	x	-	-	-

	Subjects	J.M.B.	Cape S.W.A.	Natal	O.F.S.	Tvl.	E. A & Science
42.	Draw the external common tangent to two circles	-	-	0	-	-	-
43.	Draw the internal common tangent to two circles	-	-	0	-	-	-
44.	(a) To draw a tangent from a point to the circumference of a circle	-	x	-	x	x	-
	(b) From a point outside the circle draw tangents to the circle	-	x	-	x	x	-
	(c) The tangents in (b) are equal to one another	-	-	-	-	x	-
45.	Draw a square equal in area to a given rectangle	-	x	-	-	x	-
46.	Construct a rectangle on a given line equal in area to a given square	-	-	-	-	0	-
47.	The perpendicular is the shortest distance from a given point to a straight line	-	-	x	-	-	-
48.	A tangent to a circle is perpendicular to the radius drawn to the point of contact	-	x	-	-	x	-
49.	If two circles touch one another the point of contact and their centres are in a straight line	-	-	0	-	-	-
50.*	The exterior angle of a cyclic quadrilateral is equal to the angle in the opposite segment, and conversely	-	x	-	x	x	-
51.	Equal chords of a circle are equidistant from the centre of the circle, and conversely	-	-	-	-	x	-
52.*	If a perpendicular is drawn from the right angle of a right angle triangle to the hypotenuse, it divides the triangle into two triangles which are similar to one another and to the original triangle	-	-	x	-	x	-
53.*	Ratio and proportion.	x	x	x	x	x	x
	The areas of triangles with equal altitudes are in the same ratio as their bases	-	x	-	-	0	-
54.*	If two triangles have one angle equal and the sides including the equal angles are proportional the triangles are similar	-	x	-	-	x	-
55.*	The areas of similar triangles are proportional to the squares of the corresponding sides	-	-	-	-	0	-
56.	Simple illustrations of						
	(a) a perpendicular to a surface	-	x	-	-	-	-
	(b) An angle between a straight line and the surface	-	x	-	-	-	-
	(c) An angle between two surfaces	-	x	-	-	-	-

	Subjects	J.M.B.	Cape S.W.A.	Natal	O.F.S.	Tvl.	E. A & Science
Trigonometry							
1.	Definitions of the trigonometric ratios for acute angles and elementary relations between the trigonometrical ratios $0^\circ - 180^\circ$	-	x	-	-	St. X	-
	$0^\circ - 360^\circ$	x	-	x	x	x	x
2.	Four figure tables of the ratios and their logarithms	x	x	x	x	x	x
3.	Solution of right angled triangles	x	x	x	x	x	x
4.	(a) Right angled cartesian ordinates	x	x	x	x	x	x
	(b) The convention signs	-	-	x	-	-	-
5.	Elementary relations between the ratios:						
	(i) Sin $(90 \pm x)$	x	x	x	x	x	x
	(ii) Cos $(180 \pm x)$	x	x	x	x	x	x
	(iii) Tan $(360-x)$ expressed in the ratios of x where x is an acute angle	x	x	x	x	x	x
6.	Simple indentities in which the relations of the trigonometrical functions occur	x	x	0	x	0	x
7.	Graphs of the functions: sin x, cos x and tan x (a) from $0^\circ - 180^\circ$	-	-	-	-	St. X	-
	(b) from $0^\circ - 360^\circ$	x	x	x	x	-	x
	(c) from $-90^\circ - 360^\circ$	-	-	-	-	0	-
8.	Solution of simple trigonometrical equations	x	x	-	x	0	x
9.	Constructions based on trigonometrical ratios of angles of (a) $0^\circ - 180^\circ$	-	-	x	-	St. X	-
	(b) $-90^\circ - 360^\circ$	-	-	-	-	0	-
10.	Elementary knowledge of negative angles from 0° to -90°	-	-	0	-	0	-
11.	Expressions for tan $(90 \pm x)$, Sin $(180 \pm x)$, Cos $(180-x)$, Tan $(180 \pm x)$, Sin $(360-x)$, Cos $(360-x)$, Tan $(360-x)$ in terms of x where x is an acute angle	-	x	-	-	-	-
12.	The expression of all the functions in terms of one function	-	-	-	-	x	-
13.	Find numerical values of all the functions for the angles 0°, 30°, 45°, 60° and 90°	-	-	-	-	x	-
14.	The triangular formulae namely:						
	(i) area = $\frac{1}{2}$ ab Sin c	x	x	x	x	x	x
	(ii) $\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$	x	x	x	x	x	x
	(iii) a = b Cos C + c Cos B	-	x	-	-	-	-
	(iv) $a^2 = b^2 + c^2 - 2bc \text{ Cos A}$	x	x	x	x	x	x
	(v) $1 + \text{Cos A} = \frac{(b+c+a)(b+c-a)}{2bc}$	-	-	-	-	0	-

	Subject	J.M.B.	Cape S.W.A.	Natal	O.F.S.	Tvl.	E.A & Science
15.	The solution of triangles	x	x	0	x	x	x
16.	(a) Two dimentional problems	x	x	0	x	x	x
	(b) Simple examples of 3 dimensions	-	x	-	-	0	-
17.	Values for (a) Sin (A+B), Cos (A+B)	x	x	0	x	0	x
	(b) Sin (A-B), Cos (A-B)	-	x	0	-	0	-
	(c) Tan (A±B), Where A and B are both acute angles. (It is recommended that the formula for Sin (A+B) is derived with the aid of the formila for the area of the triangle)	-	-	0	-	-	-
18.	Measurement of arc	-	-	0	-	-	-
19.	Formulae for Sin 2A and Cos 2A	x	x	-	x	-	x
20.	Proof of the result $\text{Lim} \frac{\text{Sin } x}{x} = 1, x \longrightarrow 0$	0	-	-	-	-	-

<u>Analytical Geometry</u>

	Subject	J.M.B.	Cape S.W.A.	Natal	O.F.S.	Tvl.	E.A & Science
1.	The distance between two points with given co-ordinates	x	x	0	x	0	x
2.	The midpoint of the line joining two given points	x	x	0	x	0	x
3.	The distance of a point to a straight line	-	-	0	-	-	-
4.	The gradient of a straight line	x	x	0	x	0	x
5.	Conditions for straight lines to be						
	(i) parallel	x	x	0	x	0	x
	(ii) perpendicular to one another	x	x	0	x	0	x
	(iii) deduce from (i) that gradient m_1 = gradient m_2	-	-	-	-	0	-
	(iv) deduce from (ii) that $m_1 \times m_2 = -1$	-	-	-	-	0	-
6.	The equation of a straight line						
	(a) through a given point on the y-axis and with a given gradient	x	x	0	x	0	x
	(b) through any given point and with a given gradient	x	x	0	x	0	x
	(c) through two given points (it should be emphasised that the equation ax+by+c = 0 represents a straight line. The proof is not required)	x	x	0	x	0	x
7.	The angle between two straight lines	-	-	0	-	-	-
8.	The equation of a straight line which bisects the angle between two straight lines	-	-	0	-	-	-
9.	The locus of a point which moves so that the conditions y = mx+c are fulfilled	-	-	-	-	0	-

	Subjects	J.M.B.	Cape S.W.A.	Natal	O.F.S.	Tvl.	E. A & Science
10.	Elementary treatment of the circle with any centre - including tangency	-	-	0	-	-	-
11.	The gradient of a straight line is equal to tan x where x is a positive angle (measured anti clockwise) of rotation from the x axis to the position of the line, deduce that: $\tan x = \frac{y_2 - y_1}{x_2 - x_1}$	-	-	-	-	0	-
12.	Deduce also that $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \tan x = m(\text{gradient})$ $\therefore \quad m(x - x_1) = y - y_1$	-	-	-	-	0	-
NOTE:	Questions on analytical Geometry are limited to simple numerical problems which may serve inter alia to illustrate that both the perpendicular bisectors of the sides of the triangle and the altitudes of a triangle are concurrent in cases where the co-ordinates of the corners or the equations of the sides of the triangle are given	x	x	0	-	-	x
Differential calculus							
1.	Rate of change of a function and gradient of a curve	0	-	0	-	x	-
2.	Differentiation from first principles of functions of the type ax^n, of sums and differences of the products and the quotients of such functions	-	-	0	-	x	-
3.	Differentiation of a function	0	-	0	-	x	-
4.	Integration as the converse of differentiation of functions of the type $ax^n dx$ and sums and differences of these functions	0	-	0	-	-	-
5.	Simple application to inverse values, to the drawing of arcs, to change rate of change, to problems, to maxima and minima, to areas and volumes of revolution	0	-	0	-	-	-
6.	Differentiation of x, x^2, x^3 etc. $\frac{1}{x}$, $\frac{1}{x^2}$ etc. and of simple direct trigonometrical functions	0	-	-	-	-	-

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APPENDIX 8

SYLLABI FOR ADDITIONAL MATHEMATICS

		Joint Matriculation Board	Natal	
		Additional Mathematics	Additional Mathematics	Advanced Grade
(a)	ALGEBRA			
1.	Proof of the remainder theorem	x	-	x
2.	Elementary quadratic surds including simple examples of rationalization	x	x	-
3.	Indices and logarithms including formal proofs of elementary results	x	x [1)]	-
	(Proofs of theorems on indices will be required only for positive integral indices)	x	-	-
4.	Properties of the quadratic function, including its algebraic sign and turning value, together with simple properties of the roots of a quadratic equation	x	-	x
5.	Ratio and proportion	x	-	-
6.	Variation including simple cases of more than two variables	x	-	-
7.	Arithmetic and Geometric series (finite and infinite), with simple applications	x	x	-
8.	The meaning of homogeneity, symmetry and cyclic order, with simple applications of these concepts	x	-	x [2)]
(b)	GEOMETRY			
	Theorems on surfaces			
1.	In any triangle the square on the side opposite the acute angle is equal to the sum of the squares on the other two sides less twice the rectangle contained by one of these sides and a projection of the other on it	x	-	-
2.	In an obtuse angled triangle the square on the side opposite the obtuse angle is equal to the sum of the squares on the other two sides plus twice the rectangle made by one of these sides and the projection of the other side on it	x	-	-

[1)] Natal Education Department, Additional Mathematics, Theory of Logarithms including change of base with formal proof.

[2)] Polynomials and the degree of Polynomials. Homogeneous expressions in two variables.

	Joint Matriculation Board	Natal	
	Additional Mathematics	Additional Mathematics	Advanced Grade
Properties of the triangle			
3. The bisectors of the angles of a triangle are concurrent	x	-	x
4. The perpendicular bisectors of the sides of a triangle are concurrent	x	-	x
5. The medians of a triangle are concurrent	x	-	x
6. The altitudes of a triangle are concurrent	x	-	x
Proportionality and similar figures			
7. The interior and exterior bisectors of any angle of a triangle divide the opposite sides internally and externally respectively in the same proportion as the sides which include the angles, and conversely	x	-	-
Similar Triangles			
8. Whenever two triangles are equiangular, their corresponding sides are proportional, and conversely	x	-	x
9. Whenever in two triangles, an angle of one is equal to an angle of the other, and the sides which include these angles are proportional, those triangles are similar	x	-	-
10. The ratio of the areas of similar triangles is proportional to the ratio of the squares on the corresponding sides	x	-	-
11. Whenever two chords of a circle cut one another, inside or outside the circle, then the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other	x	-	x
12. The rectangle contained by the segments of a chord drawn from a point outside a circle is equal to the square on the tangent from that point to the circle	x	-	x
13. Whenever the straight lines AB and CD (produced if necessary) cut one another at O and the rectangles OA. OB and OC. OD are equal, then the points A, B, C and D lie on a circle	x	-	-
Solid Geometry			
14. Elementary concepts in solid geometry, e.g. angle between two surfaces, angle between line and plane surface, great and small circles of a sphere	x	-	-

		Joint Matriculation Board	Natal	
		Additional Mathematics	Additional Mathematics	Advanced Grade
(c)	TRIGONOMETRY			
1.	Concept of an angle as formed by the rotation of a straight line about one of its ends	x	-	-
2.	Circular measure, together with the length	x	x	-
	of arc and area of sector of circle in terms of the circular measure of the angle at the centre	x	-	-
3.	The trigonometrical function of any angle	x	x	-
4.	Formulae for the sine, cosine, tangent of the sum or difference of two angles, with simple applications. (Proof of these formulae will only be required for instances where all the angles are acute angles)	x	tangents	sines and cosines
5.	General solution of triangles, including proofs of standard formulae	x	-	-
6.	Simple equations and identities	x	x	-
7.	Graphs of trigonometrical functions for any angle with simple applications	x	-	x
8.	Simple problems in three dimensions, in which the necessary figures must be drawn by the candidate	x	x	-
9.	The proof of the following. $\lim_{\theta \to 0} \frac{\text{Sin } \theta}{\theta} = 1,$	x	x	-
(d)	DIFFERENTIAL CALCULUS AND ANALYTICAL GEOMETRY (Only right angled axes)			
1.	Rate of change of a function and gradient of a curve, with practical applications	x	x	-
2.	Differentiation of x, x^2, x^3, etc. $\frac{1}{x}$, $\frac{1}{x^2}$, etc. and of simple direct trigonometrical functions	x	x	-
3.	Differentiation of sums and differences of the product and the quotient of two functions and of a function of a function	x	x	-
	(No discussion of the general case $\frac{d}{dx}(x^m)$ is required)	x	-	-
4.	Notation for definite integrals	x	x	-

	Joint Matriculation Board	Natal	
	Additional Mathematics	Additional Mathematics	Advanced Grade
5. Application of these processes to problems on increasing and decreasing functions, maxima and minima, equations of tangents and normals to curves, including $y = f(x)$ where $f(x)$ is a polynomial in x, rate of increase, velocity and acceleration, areas, and volumes.	x	x	-
The notation for small increments with easy applications	x	x	-
6. The equation as representing the locus of a point	x	x	-
7. Distance between points, co-ordinates of the midpoint of a line joining two points	x	-	x
8. Equations of straight lines in the form: $y = mx + c$; $\frac{x}{a} + \frac{y}{b} = 1$; $ax + by + c = 0$	x	-	x
9. Equation of a straight line with a given slope through a given point; equation of the straight line joining two given points	x	-	x
10. Length of the perpendicular from a point to a line	x	x	-
11. Conditions that two straight lines should be parallel or perpendicular to one another	x	-	x
12. The angle between two straight lines	x	x	-
13. The equation to a circle and simple applications	x	x	-
14. Equation of the locus of a point satisfying given conditions	x	-	-
15. The equation of the (straight) lines which bisect the angles between two given (straight) lines	-	x	-
16. The co-ordinates of a point which divides the line joining two given points in a given ratio	-	x	-

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APPENDIX 9

ARITHMETIC TEXTBOOKS IN USE

		Transvaal	Natal	Orange Free State	Cape
1.	Barnard, Taute and Lamprecht: Everyday Arithmetic	-	-	-	x
2.	Behr, Steyn, Lange and Grobler: Arithmetic for Stds. 7 and 8*	x	-	-	x
3.	Behr: Arithmetic for all*	x	-	-	-
4.	Behr: Arithmetic is easy*	-	x	-	-
5.	Behr and Plekker: Arithmetic is fun	x	-	-	-
6.	Blair, Redgrave: Revised Perfected Arithmetic, Std. VI	-	x	-	x
7.	Blair and De Waal: New Junior Certificate Arithmetic	x	x	x	x
8.	Blair and De Waal: New Junior Certificate Arithmetic*	x	x	x	x
9.	Blair and Goodwin: New Junior Certificate Arithmetic	-	x	-	-
10.	Botha: Commercial Arithmetic	x	x	x	x
11.	Botha: Commercial Arithmetic*	x	x	x	x
12.	De Waal: New Junior Certificate Arithmetic	-	x	x	-
13.	De Waal and Winterbottom, Natal: Arithmetic for Junior Certificate	-	x	-	-
14.	De Waal and Winterbottom, Natal: Arithmetic for Junior Certificate*	-	x	-	-
15.	Durell: General Arithmetic for Schools	-	x	-	-
16.	Du Toit: Junior Arithmetic Book*	-	-	x	-
17.	Fletcher and Stone: Arithmetic for the Secondary school*	x	x	-	x
18.	Fletcher and Stone: Arithmetic for the Secondary school	x	x	-	x
19.	Fowler: Arithmetic for Junior Certificate*	-	x	-	x
20.	Grobler and Lehman: Arithmetic*	x	-	-	x
21.	Hugo, Barnard and De Wet: Arithmetic for Orange Free State schools*	-	-	x	-
22.	Hugo and De Wet: Arithmetic for all*	-	-	x	-
23.	Hugo and De Wet: Exercises in Arithmetic for Junior Certificate*	-	x	x	-
24.	Hutchinson and van Wyk: Commercial Arithmetic*	x	-	x	x
25.	Hutchinson and van Wyk: Arithmetic for Senior Pupils*	x	-	-	-
26.	Jacobs and Van Rensburg: Commercial Arithmetic for Stds. 8 to 10*	x	-	-	x
27.	Joubert and Gilloway: Commercial Arithmetic for High Schools	-	-	-	x
28.	Kock, I. W.: New Arithmetic	-	x	-	-
29.	Kock: Form 1 Arithmetic	-	-	x	x
30.	Kock: Form 1 Arithmetic*	-	-	x	x
31.	Lambert: Calculate correctly	-	x	x	-
32.	Levinsohn and De Waal: Arithmetic for today's Stds. 7 and 8*	-	x	-	x
33.	Longmans Revised Arithmetic for Natal schools*	-	x	-	-
34.	Longmans Revised Arithmetic for Natal schools	-	x	-	-
35.	McDougall: Arithmetic for Std. VI	-	x	-	-
36.	Mohr: Arithmetic for Std. 6*	-	-	-	x
37.	Mohr and Viljoen: Commercial Arithmetic*	-	-	x	x
38.	Nixon: Arithmetic for Forms II and III	x	-	-	x
39.	Nixon and De Waal: Form I Arithmetic	-	x	-	-
40.	Ogden and Van Vuuren: Graded Exercises in Arithmetic	x	-	-	x
41.	Olckers and Katzke: Arithmetic for Stds. 7 and 8*	x	-	-	x
42.	Olckers and Katzke: Arithmetic for all*	x	x	x	-
43.	Pendelbury and Beard: Arithmetic*	x	-	-	-
44.	Steyn and Goosen: Commercial Arithmetic - 1961*	x	-	-	-
45.	Stone, Lambert and Du Preez: Arithmetic for secondary schools, II and III*	x	x	-	x

* Book published in Afrikaans.

		Trans-vaal	Natal	Orange Free State	Cape
46.	Stone, Lambert and Du Preez: Arithmetic for the secondary school, II and III	x	x	-	x
47.	Walker: New course in Arithmetic	-	-	-	x
48.	Wiles: Graded Arithmetic	x	x	x	x

GENERAL MATHEMATICS

		Trans-vaal	Natal	Orange Free State	Cape
1.	Blair, Redgrave and Matz: General Mathematics*	x	-	-	x
2.	Barnard, Lambrechts, Taute: Everyday Mathematics*	x	-	-	x
3.	Behr, Lange, Steyn, Grobler: General Mathematics Std. 6 - 1960*	x	-	x	x
4.	Boshoff and Le Roux: General Mathematics - 1955*	x	-	-	x
5.	Boshoff: General Mathematics for the Junior secondary school*	x	-	-	-
6.	Corbett: General Mathematics	x	-	-	-
7.	Dreyer: General Mathematics Stds. 6, 7, 8 - 1961*	x	-	x	x
8.	Du Plooy: General Mathematics*	x	-	-	-
9.	Durell: Revision Course in General Mathematics	-	-	x	-
10.	Gonin and Slabber: General Mathematics	-	-	-	x
11.	Geldenhuys & Corbett: General Mathematics*	-	-	-	x
12.	Hugo, Barnard & De Wet: General Mathematics for Std. 6*	-	-	x	-
13.	Hugo, Barnard & De Wet: General Mathematics for Stds. 7, 8*	-	-	x	-
14.	Joubert, Gilloway & McLelland: Technical High School Mathematics*	x	x	-	x
15.	Lange: Mathematics is easy*	x	-	-	-
16.	Marquard and Faure: Mathematics for Std. 6*	-	-	x	-
17.	Marquard, Faure and Strauss: General Mathematics for Std. 7, 8*	-	-	x	-
18.	Moll, de Swart and Slabbert: Mathematics for Stds. 7, 8*	x	-	-	-
19.	Parkinson: General Mathematics	-	-	-	x
20.	Olckers & Katzke: General Mathematics*	x	-	-	x
21.	Potgieter and du Toit: Mathematics for Std. 6*	x	-	-	x
22.	Sagel and Sybert: Everyday Mathematics*	x	x	x	x
23.	Schmidt: General Mathematics*	-	-	-	x
24.	V. Dalsen: Mathematics for Stds. 7 and 8*	x	-	-	x
25.	Van Schouwenburg, et al.: Mathematics is easy*	x	-	-	-
26.	V. Zyl: General Mathematics*	-	-	-	x
27.	Wynn & Prinz: General Mathematics for Std. 8*	x	x	-	-
28.	Wilkinson and van Staden: Mathematics for Std. 6*	-	-	x	-

ANALYTICAL GEOMETRY

		Trans-vaal	Natal	Orange Free State	Cape
1.	Bolt: Co-ordinate Geometry	-	x	-	-
2.	Briggs & Bryant: Co-ordinate geometry	-	x	-	-
3.	Brown and Munro: Elementary Analytical Geometry	-	x	-	-
4.	Loney: The elements of Co-ordinate geometry	x	-	-	-
5.	Welman and Steyn: Geometry and Analytical Geometry - 1960*	x	x	-	x
6.	Watters: Deductive Geometry for Schools	-	x	-	-

TESTS

		Trans-vaal	Natal	Orange Free State	Cape
1.	Van Zyl: Diagnostic Tests and drill exercises*	-	-	x	x

* Book published in Afrikaans.

		Trans-vaal	Natal	Orange Free State	Cape
	ALGEBRA TEXTBOOKS				
1.	Abbott: Teach yourself Algebra	x	-	-	-
2.	Baker and Bourne: Algebra	x	x	x	-
3.	Beard: Algebra	-	x	-	-
4.	Beard: Easy exercises in Algebra for beginners	x	-	-	-
5.	Blackie's Algebra for beginners	-	x	-	-
6.	Bordherd: A first course in algebra	-	-	x	-
7.	Channon, McLeish, Smith: School algebra	-	x	x	-
8.	Channon and Smith: A school algebra and arithmetic	x	-	-	x
9.	De Kock and de Waal: Algebra for Junior Certificate	-	-	x	x
10.	De Kock and De Waal: Algebra for Junior Certificate*	-	-	x	x
11.	De Kock and De Waal: Algebra for Senior Certificate - 1955	-	x	x	-
12.	De Kock and De Waal: Algebra for Senior Certificate	-	x	x	-
13.	Dreyer, J.J.: Algebra for Junior Certificate	-	x	-	-
14.	Dreyer and Schmidt: Algebra for Senior Certificate - 1940*	x	x	x	x
15.	Durell: A New Algebra for Schools - 1930	-	x	-	-
16.	Du Toit: Exercises in Algebra	-	-	x	-
17.	Gonin, Archer and Slabber: Algebra for senior certificate	-	x	-	-
18.	Grobler and Dixon: Algebra	-	-	x	-
19.	Hall and Knight: Elementary Algebra for Schools - 1885	x	x	x	-
20.	Hall and Knight: Higher Algebra - 1885	-	-	x	-
21.	Hall, H.S.: School Algebra, Parts I and II	-	x	-	x
22.	Hofmeyr: Algebra for Transvaal, Junior Certificate	x	-	-	-
23.	Hofmeyr, Ogden and Marais: Algebra and Graphs - 1951*	x	x	-	x
24.	Hofmeyr, Ogden and Marais: Algebra and Graphs - 1951	-	x	-	-
25.	Humphries: Exercises in Algebra	-	x	-	-
26.	Lange: Algebra is easy*	x	-	-	-
27.	Marais, Garson and Ogden: Algebra and Graphs II - 1951*	x	-	-	-
28.	Marquard: Exercises in Algebra for Junior Certificate*	x	-	-	-
29.	Matz and Sagel: Algebra for Senior Certificate*	-	x	-	-
30.	Schmidt: Algebra for Senior Certificate	-	x	-	-
31.	Schmidt: Algebra for Senior Certificate*	-	x	-	-
32.	Schmidt, Wilkenson, Strauss and Van Staden: Algebra for Stds. 9 and 10	-	-	x	-
33.	Shaurus: Elementary Algebra	x	-	-	-
34.	Steyn, et al.: Algebra for Stds. 9 and 10 - 1960*	x	-	-	x
35.	Walker and Miller: A new Course in Algebra	-	-	-	x
36.	Walker: A School Algebra	x	-	-	x
37.	Welman and Steyn: Algebra and Graphs, Stds. 9 and 10 - 1960*	x	-	-	-
38.	Williams: Algebra	x	-	-	-
	TRIGONOMETRY				
1.	Bordhardt and Perrot: New Trigonometry for Schools	-	-	x	x
2.	Brink, Ogden and Garson: Trigonometry for the Higher secondary school - 1961*	x	x	-	x
3.	Carslaw: Plane Trigonometry	x	-	x	x

* Book published in Afrikaans.

		Transvaal	Natal	Orange Free State	Cape
4.	Dreyer and Schmidt: Trigonometry*	-	x	-	-
5.	Durell: Matriculation Trigonometry	-	x	-	-
6.	Garson, Ogden and Marais: Trigonometry	x	-	-	-
7.	George and Dockman: Matriculation Trigonometry*	-	-	x	-
8.	Hall and Knight: Trigonometry	-	-	-	x
9.	Loney: Plane Trigonometry	-	-	x	-
10.	Schmidt: Trigonometry for Stds. 9 and 10 - 1931*	x	x	x	x
11.	Smit: Trigonometry for Senior Certificate	-	x	-	-
12.	Welman and Steyn: Trigonometry for Stds. 9 and 10 - 1961*	x	-	-	x
13.	Schmidt: Trigonometry for Stds. 9 and 10 - 1931	x	x	x	x

GEOMETRY

		Transvaal	Natal	Orange Free State	Cape
1.	Baker and Bourne: A School Geometry	-	-	-	x
2.	Block: Problems in Geometry for Senior Certificate	x	-	-	-
3.	Borchardt: Geometry	-	x	x	-
4.	Clement and Durell: A new geometry of schools	-	x	-	-
5.	De Kock: Juta's Geometry	x	-	-	-
6.	De Kock and De Waal: Geometry for Junior Certificate	x	x	x	x
7.	De Kock and De Waal: Geometry for Senior Certificate	-	x	x	x
8.	De Kock and De Waal: Geometry for Junior Certificate*	x	x	x	x
9.	De Kock and De Waal: Geometry for Senior Certificate*	-	x	x	x
10.	Dixon, Grobler, Malherbe: Geometry for Junior Certificate	-	x	-	-
11.	Dreyer and Schmidt: Geometry for Stds. 8 to 10*	-	-	x	-
12.	Durell: Geometry for schools (Senior Certificate)	-	x	-	-
13.	Durell: A new Geometry for schools	x	-	x	x
14.	Du Toit: Geometry	-	-	-	x
15.	Garson and Malherbe: Geometry for Transvaal High Schools	x	x	-	x
16.	Garson and Malherbe: Geometry and Co-ordinate Geometry for Transvaal High schools - 1956*	x	x	-	x
17.	Godfrey and Siddons: Solid Geometry	-	-	x	-
18.	Gray and Smith: A new sequence Geometry - (1955) (33rd Edition)	x	-	-	-
19.	Hall and Stevens: Geometry for Junior Certificate	-	x	-	-
20.	Hislop: Approach to Geometry, Vol. II	-	-	-	x
21.	Marquard: New Geometry for Junior Certificate*	x	x	x	-
22.	Marquard: Geometry for Stds. 9 and 10*	-	x	x	-
23.	Palmer and Parr: Geometry for schools	-	x	-	-
24.	Schmidt: Geometry for Junior Certificate*	x	-	-	x
25.	Schmidt, B.F.: Geometry for Matriculation*	-	-	x	-
26.	Smit: Geometry for Senior Certificate	-	x	-	-
27.	Smit: Geometry for Senior Certificate*	-	x	-	-
28.	Stevens and Hall: A shorter school geometry	x	-	-	x
29.	Steyn: Geometry for Std. 8 - 1961	x	-	-	x
30.	Walker and Miller: A new course in Geometry	x	x	x	-
31.	Ward and Dick: Geometry for Junior Certificate*	-	x	x	x
32.	Ward and Dick: Geometry for Junior Certificate	-	x	x	x
33.	Lacey: Intermediate Geometry	x	-	-	-

GEOMETRY AND ALGEBRA

		Transvaal	Natal	Orange Free State	Cape
1.	Beresford and Risk: Algebra and Geometry for Std. VI	-	x	-	-

* Book published in Afrikaans.

		Transvaal	Natal	Orange Free State	Cape
2.	Bookhouse and Houldsworth: Pure Mathematics. A First Course	-	-	-	x
3.	Boshoff and Le Roux: Mathematics for Junior Certificate*	-	-	-	x
4.	De Kock and Van Zyl: New Junior Mathematics*	x	-	-	-
5.	Dreyer: Mathematics for Junior Certificate	-	x	-	-
6.	De Jager: Algebra and Geometry for Std. VI	-	-	-	x
7.	Du Toit: Mathematics for Stds. 7 and 8*	-	-	x	-
8.	Eksteen: Mathematics for National Technical Certificate*	x	-	x	x
9.	Gonin, Archer, Slabber: Graded Mathematics for Junior Certificate - 1960*	x	-	-	x
10.	Gonin, Archer and Slabber: Graded Mathematics for Senior Certificate - 1960*	-	x	-	x
11.	Joubert, Gilloway and McLellan: Technical High School Mathematics*	-	-	x	x
12.	Kock: Mathematics for II and III	-	-	-	x
13.	Kock and Mohr: Mathematics	x	-	-	x
14.	Marquard and Faure: Mathematics for Std. 7 and 8*	-	-	x	-
15.	Schmidt: Mathematics for Junior Certificate*	-	-	x	-
16.	Van Zyl and de Kock: New Junior Mathematics*	-	-	x	x
17.	Welman and Steyn: Mathematics for Std. 7 and 8*	x	-	-	x
18.	Ward and Dick: Geometry and Algebra	x	-	x	x
19.	Williams: Mathematics National Technical Certificate I*	x	-	x	x

ALGEBRA, TRIGONOMETRY AND GEOMETRY

		Transvaal	Natal	Orange Free State	Cape
1.	Boshoff and Le Roux: Mathematics for Senior Certificate*	-	x	-	x
2.	De Jager: Notes and examples in Algebra, Trigonometry and Graphs	-	-	-	x
3.	De Kock: Mathematics for National Technical Certificate I and II*	x	-	x	x
4.	Dick: Senior Certificate Mathematics	-	x	-	x
5.	Duddy: Mathematics for National Technical Certificate I and II*	x	x	x	x
6.	Gonin, Archer and Malherbe: Graded Mathematics for Senior Certificate*	x	-	-	-
7.	Hugo, Barnard and De Wet: Mathematics for 9 and 10	-	-	x	-
8.	Joubert and Gilloway: Mathematics for Junior High Schools*	-	x	-	-
9.	Marlow and De Wet: Junior Certificate Papers in Mathematics	-	x	-	-
10.	Matz and Sagel: Mathematics for Senior Certificate*	x	x	x	x
11.	Moll, De Swardt and Slabbert: Mathematics for Std. VII	-	x	-	-
12.	Moll, De Swardt and Slabbert: Mathematics for Std. VII*	-	-	x	-
13.	Moll and Slabbert: Mathematics for A.T.C.[1] I and II*	-	-	x	x
14.	Parr: Mathematics	x	-	-	x
15.	Pretorius: Senior school mathematics	x	x	-	x
16.	Schmidt: Mathematics for Senior Certificate*	x	-	x	x
17.	Van Zyl and De Kock: Senior Mathematics*	-	-	x	-
18.	Wilkinson, Van Staden and Strauss: Mathematics according to the Orange Free State syllabus for Stds. 9 and 10*	-	-	x	-
19.	Williams: The Kingsway Mathematics III*	x	x	-	x
20.	Williams: The Kingsway Mathematics III	x	x	-	x
21.	Young and Lockhart: Many examples	-	x	-	-

1) Advanced Technical Certificate
* Book published in Afrikaans.

APPENDIX 10

N.B.375
G.

REPUBLIEK VAN SUID-AFRIKA

REPUBLIC OF SOUTH AFRICA

DEPARTEMENT VAN ONDERWYS,
KUNS EN WETENSKAP

DEPARTMENT OF EDUCATION,
ARTS AND SCIENCE

'N NASIONALE OPNAME

A NATIONAL SURVEY

DIE ONDERRIG VAN WISKUNDE AAN
HOëRSKOLE

THE TEACHING OF MATHEMATICS IN
HIGH SCHOOLS

SKOOLHOOFDE

PRINCIPALS

Hierdie vraelys moet deur die hoofde van hoërskole en skole met hoër of middelbare afdelings voltooi word.

This questionnaire is to be completed by the principals of high schools and schools with secondary divisions.

VERTROULIK

CONFIDENTIAL

DEPARTEMENT VAN ONDERWYS, KUNS EN WETENSKAP.

NASIONALE BURO VIR OPVOEDKUNDIGE EN MAATSKAPLIKE NAVORSING.

'n Nasionale opname van sekere aspekte van die onderrig van WISKUNDE, ALGEMENE WISKUNDE en REKENE aan middelbare en hoërskole.

HIERDIE VRAELYS

word aan die hoofde van bogenoemde inrigtings gestuur.

LET WEL:-

1. Die inligting wat u in hierdie vraelys verstrek, word alleen vir navorsing gebruik, en sal streng vertroulik behandel word.
2. Lees asseblief die aanwysings noukeurig voordat u die vraelys voltooi:
   (a) Waar die moontlike antwoorde verstrek word, trek 'n kringetjie om die syfer by die regte antwoord.
   (b) Verontagsaam asseblief die getalle wat aan die regterkant van die bladsye tussen hakies verskyn.
3. Ons sal dit besonder op prys stel indien u ook 'n aanvullende memorandum oor u beskouings aangaande die onderwys van Wiskunde aan hoërskole sal indien.

ALGEMENE INLIGTING.

1. Naam van skool: ----------------

   ----

2. Adres van skool: ----------------

   ----

3. Skoolraad: ---------------------

   (Natal: Landdrosdistrik)

4. Die skool is 'n:

   (a) seunskool
   (b) meisieskool
   (c) skool vir beide seuns en meisies

DEPARTMENT OF EDUCATION, ARTS AND SCIENCE.

NATIONAL BUREAU OF EDUCATIONAL AND SOCIAL RESEARCH.

A national survey of certain aspects concerning the teaching of MATHEMATICS, GENERAL MATHEMATICS and ARITHMETIC in secondary and high schools.

THIS QUESTIONNAIRE

is being forwarded to the principals of the abovementioned schools.

NOTE:-

The information given in this questionnaire is strictly confidential and will be used for research purposes only.

Please read the instructions carefully before completing the questionnaire:

Where possible answers are given, draw a circle round the number opposite the appropriate answer.

Kindly ignore the numbers between brackets at the right side of each page.

It would be greatly appreciated if you would submit a supplementary memorandum with your views concerning the teaching of Mathematics at high schools.

GENERAL INFORMATION. 1 (1)

(2-5)

Name of school: ------------------

Address of school: ----------------

School Board: --------------------

(Magisterial district for Natal)

The school is a:

boys' school	1	(6)
girls' school	2	
co-educational school	3	

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5. Medium van onderrig van die skool:	Medium of instruction at the school:		(7)
(a) Afrikaans	Afrikaans	1	
(b) Engels	English	2	
(c) Parallelmedium	Parallel medium	3	
(d) Dubbelmedium, hoofsaaklik Afrikaans	Dual medium, primarily Afrikaans	4	
(e) Dubbelmedium, hoofsaaklik Engels	Dual medium, primarily English	5	
(f) Dubbelmedium, Afrikaans en Engels gelyk	Dual medium, Afrikaans and English equally	6	
(g) Ander: ---------------------	Other: --------------------------	7	
6. Die soort skool:	The type of school:		(8)
(a) Hoërskool	High school	1	
(b) Skool met laer en middelbare klasse	Combined primary and secondary school	2	
(c) Landbouhoërskool	Agricultural high school	3	
(d) Tegniese kollege	Technical college	4	
(e) Hoër tegniese skool	Technical high school	5	
(f) Hoër handelskool	Commercial high school	6	
(g) Hoër handel- en tegniese skool	Commercial and technical high school	7	
(h) Nywerheidskool	School of industries	8	
(i) Hoër huishoudskool	Housecraft high school	9	
7. Die skool is:	The school is:		(9)
(1) onder die beheer van	under the control of		
(a) Die Onderwysdepartement van	The Department of Education of		
(i) Kaap die Goeie Hoop	Cape of Good Hope	1	
(ii) Natal	Natal	2	
(iii) Oranje-Vrystaat	Orange Free State	3	
(iv) Transvaal	Transvaal	4	
(v) Suidwes-Afrika	South West Africa	5	
(b) Die Departement van Onderwys, Kuns en Wetenskap	The Department of Education, Arts and Science	6	
(2) ondersteun deur	Subsidized by		
(c) 'n Provinsie	A province	7	
(d) Die Departement van Onderwys, Kuns en Wetenskap	The Department of Education, Arts and Science	8	
(3) 'n Privaatskool	A private school	9	
8. Naam van hoof: -----------------	Name of Principal: ---------------		
--------------------------------	--------------------------------		
(indien u gewillig is om dit te verstrek)	(if you are willing to furnish this)		
9. Die hoogste akademiese kwalifikasie van die skoolhoof	The highest academic qualification of the principal		(10)
(a) Baccalaureus	Bachelor	1	
(b) Baccalaureus (Honneurs)	Bachelor (Honours)	2	
(c) Magister	Master	3	
(d) Doktor	Doctor	4	
(e) Ander: --------------------	Other: -------------------------	0	

10.	Professionele kwalifikasie van skoolhoof	Professional qualification of principal		(11)
	(a) Universiteitsdiploma: U.O.D., S.O.D., H.O.D. ens.	University diploma: U.E.D., S.E.D., H.E.D., etc.	1	
	(b) Diploma van 'n opleidings- of onderwyskollege	Diploma of a teachers' training college	2	
	(c) Diploma van 'n tegniese kollege	Diploma of a technical college	3	
	(d) Ander: ------------------	Other: ------------------------	4	
	(e) Geen	None	5	
11.	Plaas in die ruimte tussen die hakies die aantal kursusse wat u in die volgende vakke geslaag het. (Vir grade hoër as Baccalaureus en kursusse aan tegniese kolleges, kyk na die kodelys hieronder):	Write in the space between brackets the number of courses successfully completed by you in the following subjects. (For degrees beyond Bachelor's and courses at technical colleges see the code list below):		
	(a) Wiskunde	Mathematics		
	(i) Akademiese kursus	Academic course	( )	(12)
	(ii) Ingenieurskursus	Engineering course	( )	(13)
	(iii) Spesialiseringskursus vir onderwysers	Specializing course for teachers	( )	(14)
	(b) Toegepaste Wiskunde	Applied Mathematics		
	(i) Akademiese kursus	Academic course	( )	(15)
	(ii) Ingenieurskursus	Engineering course	( )	(16)
	(c) Wiskundige Statistiek/Statistiese Wiskunde	Mathematical Statistics/Statistical Mathematics	( )	(17)
	(d) Statistiese Metodes/Ekonomiese Statistiek	Statistical Methods/Economic Statistics	( )	(18)
	(e) Ander: -------------------	Other: ------------------------	( )	(18)

KODELYS.		CODE LIST.	
Plaas in die betrokke ruimte tussen die hakies die hoogste van die volgende kodegetalle wat op u van toepassing is:		Write in the space between brackets the highest of the following code numbers applicable to you:	
Honneursgraad	5	Honours degree	5
Magistergraad	6	Master's degree	6
Doktorsgraad	7	Doctor's degree	7
G.T.S. I	8	A.T.C. I	8
G.T.S. II en Nasionale Diploma	9	A.T.C. II and National Diploma	9

12.	Is u tevrede met die onderwys van Wiskunde in u skool?	Are you satisfied with the teaching of Mathematics in your school?		(19)
	Ja in al die klasse	Yes in all the classes	1	
	Net in sommige klasse	Only in some classes	2	
	Nee	No	0	
13.	(a) Aantal leerlinge in die skool	Number of pupils in the school	( )	(20-23)
	(b) Aantal leerlinge in standerd 10	Number of pupils in standard 10	( )	(24-26)
	(c) Aantal leerlinge in standerd 10 wat Wiskunde neem	Number of pupils in standard 10 taking Mathematics	( )	(27-29)

II. DIE PERSONEEL IN U SKOOL VIR DIE WISKUNDIGE VAKKE.

THE STAFF FOR THE MATHEMATICAL SUBJECTS IN YOUR SCHOOL.

LET WEL: Vrae 1 tot 3 het nie betrekking op die skoolhoof self nie. NOTE: The reply to questions 1 to 3 should not include the principal.	Rekene Arithmetic	Algemene Wiskunde General Mathematics	Wiskunde Mathematics
1(a) Totale aantal onderwysers vir vak benodig. 1(a) Total number of teachers required for subject.	(30)	(31)	(32)
(b) Aantal poste deur permanent aangestelde onderwysers gevul. (b) Number of posts filled by permanently appointed teachers.	Manlik (33) Male	Manlik (34) Male	Manlik (35) Male
	Vroulik (36) Female	Vroulik (37) Female	Vroulik (38) Female
(c) Aantal poste gevul deur permanente aflospersoneel, sowel as tydelike aangestelde onderwysers wat vir permanente aanstelling geskik is. (c) Number of posts filled by permanent relief staff and by temporarily appointed teachers fit for permanent appointment.	(39) Manlik Male	(40) Manlik Male	(41) Manlik Male
	(42) Vroulik Female	(43) Vroulik Female	(44 (44) Vroulik Female
(d) Aantal poste gevul deur onderwysers sonder onderwysdiploma of -sertifikaat (d) Number of posts occupied by teachers without teacher's diploma or certificate.	Manlik (45) Male	Manlik (46) Male	Manlik (47) Male
	(48) Vroulik Female	(49) Vroulik Female	(50) Vroulik Female
(e) Aantal poste gevul deur getroude dames (nie vas aangestel nie) en afgetrede onderwysers. (e) Number of posts occupies by married women (not permanently appointed) and retired teachers.	(51) Manlik Male	(52) Manlik Male	(53) Manlik Male
	(54) Vroulik Female	(55) Vroulik Female	(56) Vroulik Female
(f) Aantal onderwysers vir die vak nog in u skool benodig. (f) Number of teachers for the subject still required in your school.	(57)	(58)	(59)

2. Die aantal onderwysers in die skool

   The number of teachers in the school ( ) (60-1)

3. Die aantal onderwysers in die skool met minstens twee suksesvol voltooide graad-kursusse, G.T.S. II of Nasionale Diploma in Wiskunde

   The number of teachers in the school with at least two successfully completed degree courses, A. T. C. II or National Diploma in Mathematics ( ) (62-3)

4. Gee die hoof persoonlik onderrig in die volgende vakke?

   Does the principal personally teach the following subjects?

Wiskunde	Ja 1/Nee 0	Mathematics	Yes 1/No 0	(64)
Algemene Wiskunde	Ja 1/Nee 0	General Mathematics	Yes 1/No 0	(65)
Rekene	Ja 1/Nee 0	Arithmetic	Yes 1/No 0	(66)

5. (a) Skep die tekort aan onderwysers in die wiskundige vakke probleme in u skool?

   Does the shortage of teachers in the mathematical subjects create problems in your school? (67)

Ja	Yes	1
Nee	No	0

	(b) Indien u sodanige probleme ervaar, watter van die volgende metodes wend u aan om die probleme op te los?	If you experience such problems, how do you attempt to solve these problems?		
	(i) Die samevoeging van meer klasgroepe per onderwyser	More class groups are given to the same teacher at the same time		
	Ja	Yes	1	(68)
	Nee	No	0	
	(ii) Minder klasperiodes per week word aan die wiskundige vakke toegestaan	Fewer class periods per week are allotted for the mathematical subjects		
	Ja	Yes	1	(69)
	Nee	No	0	
	(iii) 'n Keusevak word teenoor die wiskundige vak gestel	A subject alternative to the mathematical subject is offered		
	Ja	Yes	1	(70)
	Nee	No	0	
	(iv) Die gebruikmaking van onderwysers sonder die nodige kwalifikasies in die wiskundige vakke	The employment of teachers without the necessary qualifications in the mathematical subjects		
	Ja	Yes	1	(71)
	Nee	No	0	
	(v) Watter ander metodes gebruik u om te probeer om probleme in verband met die tekort aan onderwysers op te los?	Which other methods do you employ to attempt to solve the problems caused by the shortage of teachers?		(72)
	------------------------------	--------------------------------		
	------------------------------	--------------------------------		
	------------------------------	--------------------------------		
6.	(a) Hoeveel onderwysers met minstens 2 voltooide graadkursusse in Wiskunde gee nie onderwys in die wiskundige vakke in u skool nie?	How many teachers with at least two completed degree courses in Mathematics do not teach a mathematical subject at your school?	( )	(66)
	(b) In hoeveel gevalle is dit weens:	How many teachers thus qualified do not teach a mathematical subject due to:		
	(i) onbevoegdheid	imcompetency	( )	(67)
	(ii) die gee van onderwys in natuurwetenskap	the teaching of science	( )	(68)
	(iii) administratiewe pligte	administrative duties	( )	(69)
	(iv) ander redes: -------------	other reasons: ----------------		
	-------------------------	------------------------------		(70)

III. KEUSEVAKKE. OPTIONAL SUBJECTS.

W
2 (1)
K/C 1 (2-18)

1. Dui met 'n kruisie in die betrokke kolom aan watter van die volgende vakke in die betrokke standerds aangebied word.

Indicate with a cross in the appropriate column which of the following subjects are taught in the standards shown.

Standerd Standard	Rekene Arithmetic	Wiskunde Mathematics	Algemene Wiskunde General Mathematics
6	(19)	(20)	(21)
7	(22)	(23)	(24)
8	(25)	(26)	(27)

2. Indien Rekene as 'n aparte vak in u skool doseer word, kan die leerlinge 'n keusevak in die plek daarvan neem?

If Arithmetic is taught as a separate subject in your school can a pupil take an optional subject instead? (28)

Ja — Yes 1
Nee — No 0

3. Indien 'n keusevak in die plek van Rekene geneem kan word, watter vak is dit vir:

If an optional subject can be taken instead of Arithmetic, which subject is this for:

(a) Seuns St. 6 ---------------- Boys Std. 6 -------------------- (29)
(b) Meisies St. 6 -------------- Girls Std. 6 -------------------- (30)
(c) Seuns St. 7 ---------------- Boys Std. 7 -------------------- (31)
(d) Meisies St. 7 -------------- Girls Std. 7 -------------------- (32)
(e) Seuns St. 8 ---------------- Boys Std. 8 -------------------- (33)
(f) Meisies St. 8 -------------- Girls Std. 8 -------------------- (34)

4. Kan leerlinge in u skool keusevakke in die plek van Wiskunde of Algemene Wiskunde neem?

Can pupils take optional subjects instead of Mathematics or General Mathematics in your school? (35)

Ja — Yes 1
Nee — No 0

5. Indien sodanige keusevakke geneem kan word watter is dit vir

If such optional subjects can be taken, which are these for

(a) St. 6 — Std. 6
Seuns: ------------------- Boys: ------------------------- (36-7)
Meisies: ----------------- Girls: ------------------------- (38-9)

(b) St. 7 — Std. 7
Seuns: ------------------- Boys: ------------------------- (40-1)
Meisies: ----------------- Girls: ------------------------- (42-3)

(c) St. 8 — Std. 8
Seuns: ------------------- Boys: ------------------------- (44-5)
Meisies: ----------------- Girls: ------------------------- (46-7)

(d) St. 9 Seuns: ------------------ ------------------	Std. 9 Boys: ------------------------ ------------------------	(48-9)
Meisies: ------------------ ------------------	Girls: ------------------------ ------------------------	(50-1)
(e) St. 10 Seuns: ------------------ ------------------	Std. 10 Boys: ------------------------ ------------------------	(52-3)
Meisies: ------------------ ------------------	Girls: ------------------------ ------------------------	(54-5)

IV. TOEGEPASTE WISKUNDE OF MEGANIKA. / APPLIED MATHEMATICS OR MECHANICS.

1. Is daar leerlinge wat Meganika as een van die skoolvakke in u skool neem?

   Are there pupils taking Mechanics as one of the subjects in your school? (56)

Ja	Yes	1
Nee	No	0

2. Is daar van u leerlinge wat Meganika na-uurs neem?

   Are some of your pupils taking Mechanics extramurally? (57)

Ja	Yes	1
Nee	No	0

3. Indien leerlinge Meganika neem, hoeveel is daar in:

   If there are pupils taking Mechanics, how many are there in:

St. 9	Std. 9	( ) (58-9)
St. 10	Std. 10	( ) (60-1)

4. Neem die leerlinge Meganika in die plek van Natuurkunde of Skei/Nat?

   Do the pupils take Mechanics instead of Physics or Physical Science? (62)

Ja	Yes	1
Nee	No	0

5. Neem die leerlinge Meganika as 'n sewende vak saam met Natuurkunde of Skei/Nat.?

   Do the pupils take Mechanics as a seventh subject together with Physical Science or Physics? (63)

Ja	Yes	1
Nee	No	0

6. Indien Meganika nie in u skool onderwys word nie, wat is die belangrikste redes?

   If Mechanics is not taught in your school, what are the main reasons? (64-5)

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N.B. 376
B.

REPUBLIEK VAN SUID-AFRIKA

DEPARTEMENT VAN ONDERWYS, KUNS EN WETENSKAP

Nasionale Buro vir Opvoedkundige en Maatskaplike Navorsing.

'N NASIONALE OPNAME

DIE ONDERRIG VAN WISKUNDE AAN HOËR-SKOLE.

Hierdie vraelys word gestuur aan:

(a) die volgende departemente van universiteite:

Wiskunde en Toegepaste Wiskunde,

Wiskundige Statistiek, Natuurkunde,

Landmeetkunde, Sterrekunde,

Departemente van Ingenieurswese;

(b) tegniese kolleges wat na-matrikulasie-kursusse verskaf;

vir voltooiing deur die departementshoofde na raadpleging met hul personeel.

VERTROULIK

REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF EDUCATION, ARTS AND SCIENCE

National Bureau of Educational and Social Research.

A NATIONAL SURVEY

THE TEACHING OF MATHEMATICS IN HIGH SCHOOLS.

This questionnaire is being distributed to:

(a) the following departments of universities:

Mathematics and Applied Mathematics,

Mathematical Statistics, Physics,

Land Surveying, Astronomy,

Departments of Engineering;

(b) technical colleges provinding courses at the post-matriculation level;

for the completion by the head of the department in consultation with his staff.

CONFIDENTIAL

DEPARTEMENT VAN ONDERWYS, KUNS EN WETENSKAP.

DEPARTMENT OF EDUCATION, ARTS AND SCIENCE.

Nasionale Buro vir Opvoedkundige en Maatskaplike Navorsing.

National Bureau of Educational and Social Research.

'n Nasionale opname van sekere aspekte van die onderrig van WISKUNDE, ALGEMENE WISKUNDE en REKENE aan middelbare en hoërskole.

A national survey of certain aspects concerning the teaching of MATHEMATICS, GENERAL MATHEMATICS and ARITHMETIC in secondary and high schools.

HIERDIE VRAELYS

THIS QUESTIONNAIRE

word aan inrigtings waar die wiskundige vakke na matrikulasie gedoseer word, gestuur.

is being forwarded to all institutions at which the mathematical subjects are taught at post-matriculation level.

<u>LET WEL</u>:-

<u>NOTE</u>:-

1. Die inligting wat u in hierdie vraelys verstrek, word alleen vir navorsing gebruik, en sal streng vertroulik behandel word.

The information given in this questionnaire is strictly confidential, and will be used for research purposes only.

2. Lees asseblief die aanwysings noukeurig voordat u die vraelys voltooi:

Please read the instructions carefully, before completing the questionnaire:

(a) Waar moontlike antwoorde verstrek word, trek 'n kringetjie om die syfer by die regte antwoord.

Where possible answers are given, draw a circle round the number opposite the appropriate answer.

(b) Verontagsaam asseblief die getalle wat aan die regterkant van die bladsye tussen hakies verskyn.

Kindly ignore the numbers between brackets at the right side of each page.

3. Ons sal dit besonder op prys stel indien u ook 'n aanvullende memorandum oor u beskouings aangaande die onderwys van Wiskunde aan hoërskole sal indien.

It would be greatly appreciated if you would submit a supplementary memorandum with your views concerning the teaching of Mathematics at high schools.

I. <u>ALGEMENE INLIGTING.</u> / <u>GENERAL INFORMATION</u> 3 (1)

1. Naam van inrigting: -------------

Name of institution: --------------

------------------------------- (2-3)

2. U departement / Your department

(a)	Wiskunde	Mathematics	1	(4)
(b)	Toegepaste Wiskunde	Applied Mathematics	2	
(c)	Wiskunde en Toegepaste Wiskunde	Mathematics and Applied Mathematics	3	
(d)	Wiskundige Statistiek	Mathematical Statistics	4	
(e)	Landmeetkunde	Land Surveying	5	
(f)	Sterrekunde	Astronomy	6	
(g)	Ingenieurswese: Departement: --------------- ---------------------------	Engineering: Department: --------------------- -------------------------------	7	

II. DIE HOëRSKOOLLEERGANG. / THE HIGH SCHOOL SYLLABUS.

1.	Bevat die bestaande leergange vir Wiskunde op skool voldoende stof ter voorbereiding van die studente vir u eerstejaarskursus?	Do the present syllabi for school Mathematics contain sufficient subject-matter for the preparation of students for your first year course?		(5)
	Ja	Yes	1	
	Nee	No	0	
	Die vak word nie aan eerstejaarstudente gebied nie	This subject is not presented to first year students	2	
2.	Beskou u dit as nodig dat toekomstige studente in u vak ook Addisionele Wiskunde (of Hoër Wiskunde) op skool moet neem?	Do you consider it essential that students-to-be in your subject should also take Additional Mathematics (or Higher Mathematics)?		(6)
	Ja	Yes	1	
	Nee	No	0	
3.	Word die stof doeltreffend genoeg aangebied sodat die leerling voldoende kennis neem van grondliggende begrippe en metodes vir verdere studie in die Wiskunde?	Are the contents presented in such a way that the pupils becomes sufficiently aware of the basic concepts and methods for further study in Mathematics?		(7)
	Ja	Yes	1	
	Nee	No	0	
4.	Is u ten gunste van 'n metode van onderwys wat meer op die grondbeginsels gebaseer is om op die wyse 'n meer verenigde behandeling van die verskillende temas te verseker?	Are you in favour of a method of teaching which leans more heavily on the underlying principles and so presents a more unified treatment of the various topics?		(8)
	Ja	Yes	1	
	Nee	No	0	

5. Watter van die volgende vakgebiede lewer stof wat met vrug in die Wiskundeleergang van die hoërskool gevolg kan word?

Which of the following fields contain subject matter that can be profitably included in the high school Mathematics syllabus?

Plaas 'n kruisie langs die betrokke onderwerp:

Place a cross next to the subject:

(a)	Wiskundige logika	Mathematical logic	( )	(9)
(b)	Versamelingsleer	Set theory	( )	(10)
(c)	Topologie	Topology	( )	(11)
(d)	Getalleteorie	Number theory	( )	(12)
(e)	Teorie van reële funksies	Theory of real functions	( )	(13)
(f)	Teorie van komplekse funksies	Theory of complex functions	( )	(14)
(g)	Abstrakte Algebra	Abstract Algebra	( )	(15)
(h)	Lineêre Algebra	Linear Algebra	( )	(16)
(i)	Meetkunde (Nie-Euclidies)	Geometry (Non-Euclidean)	( )	(17)
(j)	Analitiese Meetkunde	Analytical Geometry	( )	(18)
(k)	Numerieke Analise	Numerical Analysis	( )	(19)
(l)	Differensiaalvergelykings	Differential equations	( )	(20)
(m)	Informasieteorie	Information theory	( )	(21)
(n)	Spelteorie	Game theory	( )	(22)
(o)	Lineêre programmering	Linear programming	( )	(23)
(p)	Geskiedenis van die Wiskunde	History of Mathematics	( )	(24)
(q)	Teorie van Wiskundige Statistiek	Theory of Mathematical Statistics	( )	(25)
(r)	Kansrekene	Probability	( )	(26)
(s)	Statistiese metodes	Statistical methods	( )	(27)

(t)	Ekonometrie	Econometry	( )	(28)
(u)	Klassieke Meganika	Classical Mechanics	( )	(29)
(v)	Relatiwiteitsteorie	Relativity theory	( )	(30)
(w)	Kwantummeganika	Quantum Mechanics	( )	(31)
(x)	Statistiese Meganika	Statistical Mechanics	( )	(32)
(y)	---------------------------	--------------------------------	( )	(33)
(z)	---------------------------	--------------------------------	( )	(34)

6. Gee u in u kursus enige spesiale aandag aan die behoeftes van toekomstige Wiskunde-onderwysers of dosente in u bepaalde rigting, bv. deur aan te toon hoe die besondere feit of betrekking in die hoërskool-Wiskunde aangebied word?

Do you pay any attention in your course to the needs of future teachers in Mathematics or lecturers in your field e.g. by demonstrating how a fact or relation will be presented in the high school Mathematics? (35)

Ja	Yes	1
Nee	No	0

7. Moet die volgende onderwerpe in die leergang van die hoërskool voorkom?

Should the following subjects be included in the syllabus of the high school?

(a) Elementêre Versamelingsleer (Element, versameling, vereniging, snyding)

Elementary sets (Elements, set, union, intersection) (36)

Ja	Yes	1
Nee	No	0

(b) Analitiese Meetkunde — Analytical Geometry (37)

Ja	Yes	1
Nee	No	0

(c) Elementêre Statistiek — Elementary Statistics (38)

Ja	Yes	1
Nee	No	0

(d) Ongelykhede — Inequalities (39)

Ja	Yes	1
Nee	No	0

(e) Elementêre teorie van Kansrekene — Elementary theory of Probability (40)

Ja	Yes	1
Nee	No	0

(f) Rekenkundige en meetkundige rye — Arithmetical and geometrical series (41)

Ja	Yes	1
Nee	No	0

(g) Die goniometriese verhoudings van die som en die verskil van twee hoeke

The trigonometrical relations of the sum and the difference of two angles (42)

Ja	Yes	1
Nee	No	0

8. Onderwerpe wat uit die hoërskoolleergang vir Wiskunde weggelaat kan word

Subjects that may be deleted from the Mathematics syllabus of the high school (44)

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III. DIE DOELSTELLINGS EN OPVOEDKUNDIGE WAARDE VAN DIE ONDERWYS VAN WISKUNDE (INSLUITENDE REKENE) AAN DIE HOëR-SKOOL.

THE AIMS AND EDUCATIONAL VALUE OF THE TEACHING OF MATHEMATICS (INCLUDING ARITHMETIC) AT THE HIGH SCHOOL.

1. Doelstellings.

Aangesien die doelstellings en waarde van Wiskunde-onderwys by verskillende tipes van leerlinge mag verskil, kan hulle in drie groepe verdeel word:

A. Leerlinge wat later 'n universiteitskursus kan volg.

B. Leerlinge wat die skooleindsertifikaat sonder matriek of matrikulasievrystelling kan verkry.

C. Leerlinge wat die skool sal verlaat sodra hulle oud genoeg is.

Dui die vyf belangrikste doelstellings in die Wiskunde-onderrig vir Groep A, Groep B en Groep C aan deur kruisies in die betrokke kolomme te maak. (U maak dus in elke kolom VYF kruisies).

Gee u eie mening.

Aims.

As the aims and value of the teaching of Mathematics may be different for the various types of pupils, they are divided into three groups:

A. Pupils capable of attending university later on.

B. Pupils capable of obtaining a senior or school leaving certificate without matriculation requirements.

C. Pupils who will leave school as soon as as they have fulfilled the minimum age requirements.

By drawing crosses in the appropriate columns, indicate the five principal aims in the teaching of Mathematics for Group A, Group B and Group C. (There should be FIVE crosses in each column).

Give your own opinion.

		Groep/ Group (1) A	Groep/ Group (2) B	Groep/ Group (3) C	
(i) Uitbreiding en afronding van die laerskoolwerk.	Extension and completion of the primary school work.				(45)
(ii) Wiskundige begripsvorming.	Formation of mathematical concepts.				(46)
(iii) Om die leerling in staat te stel om sy weg deur die wêreld te vind.	To enable the pupil to make his way in the world.				(47)
(iv) Dissiplinering.	Discipline.				(48)
(v) Om 'n akkurate rekenvermoë aan te kweek.	To train pupils to calculate correctly.				(49)
(vi) Om basiese opleiding vir 'n toekomstige loopbaan te verskaf.	To provide a basic training for a future career.				(50)
(vii) Om waardevolle feitekennis in verband met die alledaagse lewe oor te dra.	To teach factual information of value in everyday life.				(51)
(viii) Om die intellektuele genot te ervaar wanneer die wetmatige in getal en ruimte openbaar word.	To experience the intellectual enjoyment of discovering the laws of number and space.				(52)
(ix) Om heldere denke en logiese afleidingsvermoë te bevorder.	To promote habits of clear thinking and logical deduction.				(53)
(x) Om in 'n eksamen te slaag.	To pass an examination				(54)
(xi) Om die aantal persone met opleiding in Wiskunde te vermeerder.	To increase the number of persons with training in Mathematics.				(55)

		Groep/ Group			
		(1) A	(2) B	(3) C	
(xii) Om persoonlikheid en karaktervorming te bevorder.	To promote the development of personality and character.				(56)
(xiii) Om by leerlinge 'n liefde en belangstelling vir Wiskunde te kweek.	To inspire a love and interest in Mathematics.				(57)
(xiv) ---------------------------	----------------------				(58)

IV. DIE DOELTREFFENDHEID VAN DIE WISKUNDE-ONDERWYS AAN HOëRSKOLE. / THE EFFECTIVENESS OF THE HIGH SCHOOL TEACHING IN MATHEMATICS.

A. Klassifiseer asseblief onder die volgende hoofde oorsake wat moontlik tot die druiping van eerstejaarstudente bydra:

Kindly classify under the following headings reasons which possibly contribute to the failure of first year students:

1. Kom dikwels onder matrikulante voor. — Prevalent among matriculants.
2. Kan soms 'n oorsaak wees. — Only sometimes a contributing factor.
3. Kom nie beduidend onder matrikulante voor nie. — Not of consequence amongst matriculants.

			Kom dikwels voor Prevalent	Kom soms voor Only sometimes	Onbeduidend Not of consequence	
a.	Onvoldoende wiskundige begripsvermoë.	Inadequate mathematical conceptual ability.	1	2	3	(59)
b.	Onvoldoende kennis van elementêre algebraïese bewerkings.	Inadequate knowledge of elementary algebraic operations.	1	2	3	(60)
c.	Feitekennis sonder die nodige insig.	Factual knowledge without insight.	1	2	3	(61)
d.	Gebrek aan die nodige wiskundige benaderingswyse.	Inadequate conception of the mathematical approach.	1	2	3	(62)
e.	Onbevredigende houding teenoor die vak.	Resentment towards subject in general.	1	2	3	(63)
f.	Onnoukeurigheid in berekenings.	Negligence in calculations.	1	2	3	(64)
g.	Ontoereikende wiskundige vaardigheid.	Inadequate mathematical ability.	1	2	3	(65)
h.	Onbevredigende deduktiewe denke.	Inadequate deductive reasoning.	1	2	3	(66)
i.	Onbevredigende induktiewe denke.	Inadequate inductive reasoning.	1	2	3	(67)
j.	Onvermoë om onafhanklike literatuurstudie te doen.	Inability to obtain necessary data or literature on a subject.	1	2	3	(68)
k.	Onvermoë om selfstandig te studeer.	Inability to study independently.	1	2	3	(69)
l.	Onvoldoende besteding van tyd aan Wiskunde in die hoërskool.	Insufficient allocation of time to Mathematics at high school.	1	2	3	(70)
m.	------------------------------ ------------------------------	----------------- -----------------	1	2	3	(71)

B. Beskou u dit as nodig dat 'n leerling wat universiteit toe gaan, maar nie van plan is om Wiskunde te neem nie, dieselfde Wiskunde-eksamen in matriek moet aflê as dié leerling wat wel van plan is om met Wiskunde aan te gaan?

Do you consider it necessary for a pupil who does not plan to take Mathematics at university to write the same matric papers in Mathematics as a pupil who is planning to take further courses in Mathematics? (72)

Ja	Yes	1
Nee	No	0

No. .... (73-5)

APPENDIX 12

N.B. 377
W.

REPUBLIEK VAN SUID-AFRIKA

REPUBLIC OF SOUTH AFRICA

DEPARTEMENT VAN ONDERWYS,
KUNS EN WETENSKAP

Nasionale Buro vir Opvoedkundige en Maatskaplike Navorsing.

DEPARTMENT OF EDUCATION,
ARTS AND SCIENCE

National Bureau of Educational and Social Research.

'N NASIONALE OPNAME

A NATIONAL SURVEY

DIE ONDERRIG VAN WISKUNDE AAN HOëR-SKOLE.

THE TEACHING OF MATHEMATICS IN HIGH SCHOOLS.

ONDERWYSERS

TEACHERS

Hierdie vraelys moet deur almal wat onderwys in Wiskunde, Algemene Wiskunde en Rekene gee, voltooi word.

This questionnaire is to be completed by all who teach Mathematics, General Mathematics and Arithmetic.

VERTROULIK

CONFIDENTIAL

DEPARTEMENT VAN ONDERWYS, KUNS EN WETENSKAP

Nasionale Buro vir Opvoedkundige en Maatskaplike Navorsing

'n Nasionale opname van sekere aspekte van die onderrig van WISKUNDE, ALGEMENE WISKUNDE en REKENE aan middelbare en hoërskole.

HIERDIE VRAELYS

word aan alle persone wat aan bogenoemde inrigtings bostaande vakke doseer, gestuur.

LET WEL:-

1. Die inligting wat u in hierdie vraelys verstrek, word alleen vir navorsing gebruik, en sal streng vertroulik behandel word.

2. Lees asseblief die aanwysings noukeurig, voordat u die vraelys voltooi:

   (a) Waar die moontlike antwoorde verstrek word, trek 'n kringetjie om die syfer by die regte antwoord.

   (b) Verontagsaam asseblief die getalle wat aan die regterkant van die bladsye tussen hakies verskyn.

   (c) Verstrek asseblief die gegewens aangaande uself.

3. Ons sal dit besonder op prys stel indien u ook 'n aanvullende memorandum oor u beskouings aangaande die onderwys van Wiskunde aan hoërskole sal indien.

AFDELING I.

ALGEMENE INLIGTING.

1. Naam van skool: -----------------

----

2. Adres van skool: ----------------

DEPARTMENT OF EDUCATION, ARTS AND SCIENCE

National Bureau of Educational and Social Research

A national survey of certain aspects concerning the teaching of MATHEMATICS, GENERAL MATHEMATICS and ARITHMETIC in secondary and high schools.

THIS QUESTIONNAIRE

is being forwarded to all persons teaching the abovementioned subjects at the secondary level.

NOTE:-

1. The information given in this questionnaire is strictly confidential and will be used for research purposes only.

2. Please read the instructions carefully before completing the questionnaire:

   (a) Where possible answers are given draw a circle round the number opposite the appropriate answer.

   (b) Kindly ignore the numbers between brackets at the right side of each page.

   (c) Kindly complete the questionnaire in respect of yourself.

3. It would be greatly appreciated if you would submit a supplementary memorandum with your views concerning the teaching of Mathematics at high schools.

SECTION I.

W
4 (1)
(77-80)

GENERAL INFORMATION.

Name of school: ------------------

Address of school: ---------------

3.	Skoolraad: --------------------- -------------------------------- (Natal: Landdrosdistrik).	School Board: -------------------- --------------------------------- (Magisterial district for Natal).		
4.	Die skool is 'n:	The school is a:		(2)
	(a) seunskool	boys' school	1	
	(b) meisieskool	girls' school	2	
	(c) skool vir beide seuns en meisies	co-educational school	3	
5.	Medium van onderrig van die skool:	Medium of instruction at school:		(3)
	(a) Afrikaans	Afrikaans	1	
	(b) Engels	English	2	
	(c) Parallelmedium	Parallel medium	3	
	(d) Dubbelmedium hoofsaaklik Afrikaans	Dual medium, primarily Afrikaans	4	
	(e) Dubbelmedium hoofsaaklik Engels	Dual medium, primarily English	5	
	(f) Dubbelmedium Afrikaans en Engels gelyk	Dual medium, Afrikaans and English equally	6	
	(g) Ander	Other	7	
6.	Die soort skool:	The type of school:		(4)
	(a) Hoërskool	High school	1	
	(b) Skool met laer en middelbare klasse	Combined primary and secondary school	2	
	(c) Landbouhoërskool	Agricultural high school	3	
	(d) Tegniese kollege	Technical college	4	
	(e) Hoër tegniese skool	Technical high school	5	
	(f) Hoër handelskool	Commercial high school	6	
	(g) Hoër handel- en tegniese skool	Commercial and technical high school	7	
	(h) Nywerheidskool	School of industries	8	
	(i) Hoër huishoudskool	Housecraft high school	9	
7.	Die skool is:	The school is:		(5)
	(1) onder die beheer van	under the control of		
	(a) Die Onderwysdepartement van -	The Department of Education of -		
	(i) Kaap die Goeie Hoop	Cape of Good Hope	1	
	(ii) Natal	Natal	2	
	(iii) Oranje-Vrystaat	Orange Free State	3	
	(iv) Transvaal	Transvaal	4	
	(v) Suidwes-Afrika	South West Africa	5	
	(b) Die Departement van Onderwys, Kuns en Wetenskap	The Department of Education, Arts and Science	6	
	(2) ondersteun deur	subsidised by		
	(c) 'n Provinsie	A province	7	
	(d) Die Departement van Onderwys, Kuns en Wetenskap	The Department of Education, Arts and Science	8	
	(3) 'n Privaatskool	A Private school	9	
8.	Naam van onderwyser: ----------- ------------------------------	Name of teacher: ---------------- ------------------------------		

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9. Ouderdom van onderwyser:	Age of teacher:		(6)
19 jaar en jonger	19 years and younger	1	
20 tot 24 jaar	20 to 24 years	2	
25 tot 29 jaar	25 to 29 years	3	
30 tot 39 jaar	30 to 39 years	4	
40 tot 49 jaar	40 to 49 years	5	
50 tot 59 jaar	50 to 59 years	6	
60 tot 64 jaar	60 to 64 years	7	
65 jaar en ouer	65 years and older	8	
10. U onderwyservaring in Wiskunde, Rekene en Meganika in voltooide jare:	Your experience in teaching Mathematics, Arithmetic and Mechanics in completed years:		(7)
0 - 2 jaar	0 - 2 years	1	
3 - 5 jaar	3 - 5 years	2	
6 - 10 jaar	6 - 10 years	3	
11 - 19 jaar	11 - 19 years	4	
20 - 29 jaar	20 - 29 years	5	
30 - 39 jaar	30 - 39 years	6	
40 jaar en langer	40 years and longer	7	
11. Pos wat u beklee:	Position occupied:		(8)
(a) Hoof	Principal	1	
(b) Onderhoof	Vice-principal	2	
(c) Departementshoof/Senior onderwyser	Head of department/Senior teacher	3	
(d) Lektor/dosent	Lecturer	4	
(e) Assistent-onderwyser	Assistant teacher	5	
12. U kwalifikasies:	Your qualifications:		(9)
Grade: -----------------------	Degrees: ----------------------		
Professioneel (diplomas, sertifikate):	Professional (diplomas, certificates):		(10)
------------------------------	------------------------------		
13. Plaas tussen die hakies die aantal kursusse wat u in die volgende vakke geslaag het: (Vir grade hoër as Baccalaureus en kursusse in tegniese kolleges kyk na die kodelys hieronder)	Write between the brackets the number of courses successfully completed by you in the following subjects: (For degrees beyond Bachelor's and courses at technical colleges see the code list below)		
(a) Wiskunde:	Mathematics:		
(i) Akademiese kursus	Academic course	( )	(11)
(ii) Ingenieurskursus	Engineering course	( )	(12)
(iii) Spesialiseringskursus vir onderwysers	Specializing course for teachers	( )	(13)
(b) Toegepaste Wiskunde:	Applied Mathematics:		
(i) Akademiese kursus	Academic course	( )	(14)
(ii) Ingenieurskursus	Engineering course	( )	(15)
(c) Wiskundige Statistiek/Statistiese Wiskunde	Mathematical Statistics/Statistical Mathematics	( )	(16)
(d) Statistiese Metodes/Ekonomiese Statistiek	Statistical Methods/Economic Statistics	( )	(17)

(e) Ander: --------------------	Other: ------------------------	

KODELYS.		CODE LIST.	
Plaas tussen die hakies die hoogste van die volgende kodegetalle wat op u van toepassing is.		Write between the brackets the highest of the following code numbers applicable to you.	
Honneursgraad	5	Honours degree	5
Magistersgraad	6	Master's degree	6
Doktorsgraad	7	Doctor's degree	7
G.T.S. I	8	A.T.C. I	8
G.T.S. II en Nasionale Diploma	9	A.T.C. II and National Diploma	9

14. U hoogste graad in Opvoedkunde behaal met 'n verhandeling of proefskrif oor 'n wiskundige onderwerp: | Your highest degree in Education obtained with a thesis on a mathematical subject: (18)

B.Ed.	B.Ed.	1	
M.Ed.	M.Ed.	2	
Doktorsgraad	Doctor's degree	3	

15. Die titel van die verhandeling of proefskrif: | The title of the thesis:

B.Ed. ------------------------ | B.Ed. ------------------------

------------------------ | ------------------------

M.Ed. ------------------------ | M.Ed. ------------------------

------------------------ | ------------------------

Doktorsgraad: ---------------- | Doctor's degree: ----------------

------------------------ | ------------------------

16. Aard van u aanstelling: | Nature of your appointment: (19)

(a) Permanent aangestel:	Permanent:	
Manlik	Male	1
Vroulik	Female	2
(b) Tydelike aangestel, maar geskik vir permanente aanstelling: Manlik	Temporary, but eligible for permanent appointment: Male	3
Vroulik	Female	4
(c) Afgetrede onderwyser	Retired teacher: Male	5
Afgetrede onderwyseres	Female	6
(d) Getroude vrou (nie geskik vir permanente aanstelling nie)	Married woman (not eligible for permanent appointment)	7
(e) Sonder erkende onderwysdiploma/sertifikaat:	Without recognized teacher's diploma/certificate:	
Manlik	Male	8
Vroulik	Female	9

17. Aan watter onderwyskollege of universiteit het u u professionele opleiding as onderwyser/es ontvang?

At which college of education or university did you receive your professional training as a teacher? (20-1)

-------------------------------- --------------------------------

-------------------------------- --------------------------------

18. Het u opleiding gehad in die metodiek van die middelbare onderwys van die wiskundige vakke?

Did you follow a course in the methods of teaching mathematical subjects at the secondary level?

(a) Wiskunde	Ja Nee	Mathematics	Yes No	1 0	(22)
(b) Rekenkunde vir die hoërskool	Ja Nee	High school arithmetic	Yes No	1 0	(23)
(c) Meganika	Ja Nee	Mechanics	Yes No	1 0	(24)

19. Was die opleiding wat u in die metodiek van die wiskundige vakke gehad het, voldoende?

Was the training which you had in the methods of teaching the mathematical subjects sufficient? (25)

Ja		Yes	1
Nee		No	0

20. Indien u dit as onvoldoende ag, verstrek asseblief 'n paar van u redes

If you deem it insufficient, state a few of your reasons (26-7)

----

21. (a) Is u die skrywer of medeskrywer van 'n handboek vir Wiskunde en/of Rekene?

Are you the author or co-author of a textbook in Mathematics and/or Arithmetic? (28)

Ja		Yes	1
Nee		No	0

(b) Indien wel, verskaf asseblief die titel.

If your answer is "Yes", kindly furnish the title.

----

22. Skrywe hieronder die name van die belangrikste tydksrifte wat u in verband met die onderwys van Wiskunde en Rekene gereeld lees:

List hereunder the most important periodicals on the teaching of Mathematics and Arithmetic, which you read regularly:

--- (29-33)

23. Bestaan daar 'n behoefte aan 'n Suid-Afrikaanse wiskundige tydskrif vir onderwysers en dosente?	Is there a need for a South African mathematical periodical for teachers and lecturers?		(34)
Ja	Yes	1	
Nee	No	0	
24. Bestaan daar 'n behoefte aan 'n Suid-Afrikaanse wiskundige tydskrif vir leerlinge en studente?	Is there a need for a South African mathematical periodical for pupils and students?		(35)
Ja	Yes	1	
Nee	No	0	
25. Bestaan daar 'n behoefte aan 'n reeks kort verhandelings geskrywe deur vooraanstaande wiskundiges om die aandag te vestig op Wiskunde geskik vir hoërskoolleerlinge en sodoende die belangstelling van begaafde leerlinge te prikkel, en om Wiskunde as 'n bevredigende en betekenisvolle menslike aktiwiteit voor te stel?	Is there a need for a series of short papers, written by well-known mathematicians, with the object of drawing attention to fields of study in Mathematics suitable for high school pupils, in order to obtain and stimulate the interest of gifted pupils, and to present Mathematics as a satisfying and useful human activity?		(36)
Ja	Yes	1	
Nee	No	0	
26. Onder watter van die volgende voorwaardes sal u bereid wees om vakansiekursusse in Wiskunde-onderwys by te woon?	Under which of the following conditions would you attend vacation courses in the teaching of Mathematics?		(37)
(i) Onvoorwaardelik	Unconditionally	8	
(ii) By die ontvangs van departementele erkenning	On the receipt of departmental recognition	1	
(iii) Indien soveel informele kontak as moontlik met die dosente verseker word	If the maximum of informal contact with the lecturers is ensured	2	
(iv) Teen vergoeding van reis- en verblyfkoste	On payment of subsistence and travelling expenses	3	
27. Is u 'n lid van:	Are you a member of:		
(a) Die Suid-Afrikaanse Wiskunde-Vereniging?	The South African Mathematical Association?		(38)
Ja	Yes	1	
Nee	No	0	
(b) Die Suid-Afrikaanse Statistiese Vereniging?	The South African Statistical Association?		(39)
Ja	Yes	1	
Nee	No	0	

(c) 'n Onderwysersvereniging?	A teacher's association?			(40)
Ja		Yes	1	
Nee		No	0	
(d) 'n Ander sodanige vereniging?	Another such society?			(41)
Ja		Yes	1	
Nee		No	0	
28. Woon u die Wiskundevakvergaderings van u onderwysersvereniging by?	Do you attend the Mathematics group meetings of your teachers' association?			(42)
Ja		Yes	1	
Nee		No	0	

AFDELING II. / SECTION II.

DIE ONDERWYS. / THE TEACHING.

1. Indien u net die Rekenegedeelte van Algemene Wiskunde doseer, voltooi dan die betrokke gedeeltes ten opsigte van Rekene.

   If you teach the Arithmetic part of General Mathematics only, kindly complete the sections for Arithmetic.

2. Verstrek asseblief die volgende gegewens teenoor elke vak waarin u in die betrokke standerds onderrig gee:

   Kindly furnish the following information opposite each subject you teach in the different standards:

   (a) die aantal klasse in die standerd waarvoor u onderrig gee;

   the number of classes in each standard to which you teach the subject concerned;

   (b) die aantal periodes per week wat bestee word aan die onderrig van die vak in die betrokke standerd;

   the number of periods per week allocated to the tuition of the subject in the standard concerned;

   (c) die aantal leerlinge in u grootste klas;

   the number of pupils in the largest class;

   (d) die gemiddelde aantal leerling in u klasse per standerd per vak.

   the average number of pupils per standard per subject in your classes.

TABEL/TABLE I

Die klasse waarvoor u onderwys gee/The classes you teach personally.

Vak en standerd Subject and standard	Aantal klasse in standerd. Number of classes in standard.	Aantal periodes per week per klas. Number of periods per week per class.	Aantal leerlinge in die grootste klas. Number of pupils in largest class.	Gemiddelde aantal leerlinge per klas. Average number of pupils per class.
1. Wiskunde/ Mathematics				
St(d) 6	(2)	(3)	(4-5)	(6-7)
St(d) 7	(8)	(9)	(10-1)	(12-3)
St(d) 8	(14)	(15)	(16-7)	(18-9)
St(d) 9	(20)	(21)	(22-3)	(24-5)
St(d) 10	(26)	(27)	(28-9)	(30-1)
2. Algemene Wiskunde/General Mathematics				
St(d) 6	(32)	(33)	(34-5)	(36-7)
St(d) 7	(38)	(39)	(40-1)	(42-3)
St(d) 8	(44)	(45)	(46-7)	(48-9)
3. Rekene/Arithmetic				
St(d) 6	(50)	(51)	(52-3)	(54-5)
St(d) 7	(56)	(57)	(58-9)	(60-1)
St(d) 8	(62)	(63)	(64-5)	(66-7)

K.4:2-10: (68-76)

AFDELING III.

DIE DOELSTELLINGS EN OPVOEDKUNDIGE WAARDE VAN DIE ONDERWYS VAN WISKUNDE (INSLUITENDE REKENE).

1. Doelstellings.

Aangesien die doelstellings en waarde van Wiskunde-onderwys kan verskil van leerling tot leerling, word tussen drie groepe onderskei:

A. Leerlinge wat later 'n universiteitskursus kan volg.

B. Leerlinge wat die skooleindsertifikaat sonder matriek of matrikulasievrystelling kan verkry.

C. Leerlinge wat die skool sal verlaat sodra hulle oud genoeg is.

SECTION III.

THE AIMS AND EDUCATIONAL VALUE OF THE TEACHING OF MATHEMATICS (INCLUDING ARITHMETIC).

Aims.

As the aims and value of the teaching of Mathematics may vary in respect of different types of pupils, they are divided into three groups:

Pupils capable of attending university later.

Pupils capable of obtaining a senior or school leaving certificate without matriculation requirements.

Pupils who will leave school as soon as they fulfil the minimum age requirements.

Deur kruisies in die betrokke kolomme te maak, dui die vyf belangrikste doelstellings vir Groep A, Groep B en Groep C aan. (U maak dus in elke kolom VYF kruisies).

By drawing crosses in the appropriate columns, indicate the five principal aims for Group A, Group B and Group C. (There should be FIVE crosses in each column). W

Gee u eie mening.

Give your own opinion. K(c) 4

			Groep/ Group			
			(1) A	(2) B	(3) C	
(i)	Uitbreiding en afronding van die laerskoolwerk	Extension and completion of the primary school work				(43)
(ii)	Wiskundige begripsvorming	Formation of mathematical concepts				(44)
(iii)	Om die leerling in staat te stel om sy weg deur die wêreld te vind	To enable the pupil to make his way in the world				(45)
(iv)	Dissiplinering	Discipline				(46)
(v)	Om 'n akkurate rekenvermoë aan te kweek	To train pupils to calculate correctly				(47)
(vi)	Om basiese opleiding vir 'n toekomstige loopbaan te verskaf	To provide a basic training for a future career				(48)
(vii)	Om waardevolle feitekennis in verband met die alledaagse lewe tuis te bring	To teach factual information of value in everyday life				(49)
(viii)	Om die intellektuele genot te ervaar wanneer die wetmatige in getal en ruimte openbaar word	To experience the intellectual enjoyment of discovering the laws of number and space				(50)
(ix)	Om die vermoë om helder te dink en logiese afleidings te maak te bevorder	To promote the ability to think clearly and to make logical deductions				(51)
(x)	Om in 'n eksamen te slaag	To pass an examination				(52)
(xi)	Om die aantal persone met opleiding in Wiskunde te verhoog	To increase the number of persons with training in Mathematics				(53)
(xii)	Om persoonlikheid- en karaktervorming te bevorder	To promote the building up of personality and character				(54)
(xiii)	Om by leerlinge 'n liefde en belangstelling in Wiskunde te kweek	To inspire a love and interest in Mathematics				(55)
(xiv)	---------------------------- ----------------------------	----------------------- -----------------------				(56)

2\. (a) Het u departement (eksaminerende liggaam vir privaatskool) 'n voorgeskrewe leerplan vir Algemene Wiskunde?

Has your Department (Examining body in the case of a private school) a prescribed syllabus in General Mathematics? (57)

Ja	Yes	1	
Nee	No	0	

(b) Indien „Ja", meen u dat u departement se leerplan vir Algemene Wiskunde aan die doelstellings wat in dieselfde leerplan uiteengesit is, beantwoord?

If "Yes", do you think that the content of the syllabus for the subject General Mathematics furthers the aims as set out in the same syllabus? (58)

Hoofsaaklik, „Ja"	Chiefly "Yes"	1
Hoofsaaklik, „Nee"	Chiefly "No"	0

3. (a)	In watter wiskundige vak gee u hoofsaaklik onderrig?	Which mathematical subject do you mostly teach?		(59)
	Rekene	Arithmetic	1	
	Algemene Wiskunde	General Mathematics	2	
	Wiskunde, sts. 6, 7 en 8	Mathematics, stds. 6, 7 and 8	3	
	Wiskunde, sts. 9 en 10	Mathematics, stds. 9 and 10	4	
(b)	Beskou u die doelstellings van u departemente se leerplan vir die wiskundige vak wat u hoofsaaklik onderrig, as bevredigend?	Do you consider the aims of the departmental syllabus for the mathematical subject, which you mostly teach, satisfactory?		(60)
	Ja	Yes	1	
	Nee	No	0	
(c)	Vind u dit moontlik om hierdie doelstellings in die praktyk uit te voer?	Do you find it possible to carry out these aims in practice?		(61)
	Ja	Yes	1	
	Nee	No	0	
(d)	Indien u antwoord op vraag 3(c) „Nee" is, wat beskou u as die vernaamste rede?	If your answer to question 3(c) is "No", what do you regard as the most important reason?		(62)
	(i) Die inhoud van die leerplan	The content of the syllabus	1	
	(ii) Gebrek aan tyd	The lack of time	2	

AFDELING IV. / SECTION IV.

DIE WISKUNDE-LEERPLAN. / THE MATHEMATICS SYLLABUS.

1.	Is daar 'n sentrale begrip in die Wiskunde van die hoërskool?	Is there a central concept in the Mathematics of the high school?		(63)
(a)	Volgens u mening,	In your opinion,		
	Ja	Yes	1	
	Nee	No	0	
(b)	Volgens die amptelike leerplan,	According to the official syllabus,		(64)
	Ja	Yes	1	
	Nee	No	0	
2. (a)	Wat is volgens u mening die sentrale begrip van hoërskool-Wiskunde?	In your opinion what is the central concept of high school Mathematics?		(65)
	-------------------------------	-------------------------------		
	-------------------------------	-------------------------------		
(b)	Wat is volgens die amptelike leerplan die sentrale begrip van hoërskool-Wiskunde?	According to the official syllabus what is the central concept of high school Mathematics?		(66)
	-------------------------------	-------------------------------		
	-------------------------------	-------------------------------		

3. Wat is volgens u mening en volgens die amptelike leerplan die sentrale begrip in die onderwys van Rekene, Algebra, Meetkunde en Driehoeks-meting?

Dui asseblief met kruisies in Tabel 2 aan.

In your opinion and according to the official syllabus, which is the central concept in the teaching of Arithmetic, Algebra, Geometry and Trigonometry?

Indicate with crosses in Table 2.

TABEL/TABLE 2.

Sentrale Begrip / The Central Concept.

Begrip/Concept	U OORDEEL YOUR OPINION				AMPTELIKE LEERGANG OFFICIAL SYLLABUS			
	Rekene Arith-metic	Algebra Algebra	Meet-kunde Geo-metry	Drie-hoeks-meting Trigo-nome-try	Rekene Arith-metic	Algebra Algebra	Meet-kunde Geo-metry	Drie-hoeks-meting Trigo-nome-try
	(67)	(68)	(69)	(70)	(71)	(72)	(73)	(74)
Vergelyking/ Equation 1								
Funksie/ Function 2								
Verhouding/ Ratio 3								
Versameling/ Set 4								
Gelykheid/ Equality 5								
Betrekking/ Relation 6								
Kongruensie/ Congruence 7								
Grootheid/ Magnitude 8								
Hoeveelheid/ Multitude 9								
Geen sentrale begrip/No central concept 0								
Ander/Other ----------- X								
----------- Y								

R6(1) K/C 4 (2-26)

4. Meen u dat die kloof tussen Wiskunde op hoërskool en dié in die begin van die eerste jaar op universiteit te wyd is?

Do you think that the gap between high school Mathematics and that at the beginning of the first year at university is too wide? (27)

Ja	Yes	1
Nee	No	0
Kan die vraag nie beantwoord nie	Unable to answer	2

5. Indien die kloof tussen skool en universiteit te wyd is, is dit omdat	If the gap between school and university is too wide, is it because		
(a) die standaard van hoërskool-Wiskunde te laag is?	the standard of high school Mathematics is too low?		(28)
Ja	Yes	1	
Nee	No	0	
(b) die universiteits-Wiskunde in die begin	the university Mathematics starts off at		
(i) teen 'n te hoë tempo aangebied word?	too rapid a pace?		(29)
Ja	Yes	1	
Nee	No	0	
(ii) 'n te hoë standaard handhaaf?	too high a standard?		(30)
Ja	Yes	1	
Nee	No	0	
(c) daar verskille voorkom ten opsigte van die wiskundige taal wat gebruik word?	of variations in the usage of mathematical language?		(31)
Ja	Yes	1	
Nee	No	0	
(d) te veel verskille voorkom wat die Wiskunde-leerstof betref?	too many variations occur in respect of subject matter in Mathematics?		(32)
Ja	Yes	1	
Nee	No	0	
(e) daar te veel verskil in leermetode is?	there is too much difference in the method of studying?		(33)
Ja	Yes	1	
Nee	No	0	
6. Gebruik u dikwels die drilmetode in die onderwys van Wiskunde?	Do you use the drill method extensively in the teaching of Mathematics?		(34)
Ja	Yes	1	
Nee	No	0	
7. Meen u dat die soort vrae wat gewoonlik in die skooleindeksamen voorkom, die gebruik van die drilmetode in die onderwys van Wiskunde in die hand werk?	Do you think that the type of question generally set in the final school examination encourages the use of the drill method in the teaching of Mathematics?		(35)
Ja	Yes	1	
Nee	No	0	
Indien u hoofsaaklik <u>Rekene</u> onderrig, beant beantwoord asseblief <u>onderstaande</u> vrae 8 en 9 ten opsigte van Rekene.	If you teach mainly <u>Arithmetic</u>, kindly answer the <u>following</u> questions 8 and 9 with reference to Arithmetic.		

8. (a)	Word die vermeerderende toepassing van Wiskunde op die gebied van die natuurwetenskap en tegniek behoorlik in die leerplan in ag geneem?	Does the school Mathematics syllabus pay sufficient attention to the increasing application of Mathematics in scientific and technical fields?		(36)
	Ja	Yes	1	
	Nee	No	0	
(b)	Word die jongste voortgang van die Wiskunde self genoegsaam in die leerplan weerspieël?	Does the syllabus satisfactorily reflect recent advances made in Mathematics itself?		(37)
	Ja	Yes	1	
	Nee	No	0	
(c)	Ontvang die wiskundige grondbegrippe genoegsame aandag in die leerplan?	Do the basic mathematical concepts receive enough attention in the syllabus?		(38)
	Ja	Yes	1	
	Nee	No	0	
(d)	Bevat die leerplan stof wat gebruik kan word om 'n wiskundige struktuur uit te beeld?	Does the syllabus contain subject matter that can be utilized to present a mathematical structure?		(39)
	Ja	Yes	1	
	Nee	No	0	
9.	In watter standerd moet die volgende onderwerpe die eerste keer onderwys word?	In which standard should the following subjects be taught for the first time?	St(d)	
(a)	Positiewe en negatiewe getalle (slegs syfergetalle) soos onderskei van natuurlike getalle	Positive and negative numbers (numerals only) as opposed to natural numbers	5 6 7 8 9	(40)
(b)	Positiewe en negatiewe getalle (syfergetalle en letters)	Positive en negative numbers (letters and numerals)	5 6 7 8	(41)
(c)	Driehoeksmeting	Trigonometry	5 6 7 8	(42)
(d)	Grafieke	Graphs	5 6 7 8	(43)
(e)	Praktiese Meetkunde	Practical Geometry	5 6 7	(44)
(f)	Meetkunde as 'n logiese struktuur	Geometry as a logical structure	6 7 8 9	(45)

10. Moet die volgende onderwerpe in die leerplan van die hoërskool voorkom?	Should the following subjects be included in the syllabus of the high school?		
(a) Elementêre versamelingsleer (Element, versameling, vereniging, snyding)	Elementary sets (Elements, set, union, intersection)		(46)
Ja	Yes	1	
Nee	No	0	
(b) Analitiese Meetkunde	Analytical Geometry		(47)
Ja	Yes	1	
Nee	No	0	
(c) Elementêre Statistiese Wiskunde	Elementary Statistical Mathematics		(48)
Ja	Yes	1	
Nee	No	0	
(d) Ongelykhede	Inequalities		(49)
Ja	Yes	1	
Nee	No	0	
(e) Elementêre teorie van Kansrekene	Elementary theory of probability		(50)
Ja	Yes	1	
Nee	No	0	
(f) Rekenkundige en meetkundige reekse	Arithmetical and geometrical series		(51)
Ja	Yes	1	
Nee	No	0	
(g) Die goniometriese verhoudings van die som en die verskil van twee hoeke	The trigonometrical relations of the sum and the difference of two angles		(52)
Ja	Yes	1	
Nee	No	0	
11. Reken u dat daar leemtes in die bestaande Wiskundeleerplanne van die hoërskool is?	Do you think that there are defects in the existing high school Mathematics syllabuses?		(53)
Ja	Yes	1	
Nee	No	0	
12. Onderwerpe wat uit die hoërskoolleerplan vir Wiskunde weggelaat kan word	Topics that may be deleted from the Mathematics syllabus of the high school		
------------------------------	------------------------------		
------------------------------	------------------------------		(54-5)
13. Onderwerpe nie in vraag 10 genoem nie, wat in die hoërskoolleerplan vir Wiskunde gevoeg kan word	Topics not mentioned in question 10 which could be included in the Mathematics syllabus of the high school		
------------------------------	------------------------------		
------------------------------	------------------------------		(56-7)

AFDELING V.

SECTION V.

HUISWERK.

HOMEWORK.

1. Hoeveel keer per week gee u Rekene-huiswerk aan die volgende standerds? (Trek 'n sirkel om die gepaste getal)

How many times per week do you give homework in Arithmetic to the following standards? (Encircle the appropriate number).

(a) St. 6 / Std. 6

0 1 2 3 4 5 (58)

(b) St. 7 / Std. 7

0 1 2 3 4 5 (59)

(c) St. 8 / Std. 8

0 1 2 3 4 5 (60)

2. Hoeveel keer per week gee u Wiskunde-huiswerk aan die volgende standerds?

How many times per week do you give homework in Mathematics to the following standards?

(a) St. 6 / Std. 6

0 1 2 3 4 5 (61)

(b) St. 7 / Std. 7

0 1 2 3 4 5 (62)

(c) St. 8 / Std. 8

0 1 2 3 4 5 (63)

(d) St. 9 / Std. 9

0 1 2 3 4 5 (64)

(e) St. 10 / Std. 10

0 1 2 3 4 5 (65)

3. Moet die skooldag verleng word, sodat die „huiswerk" in die skool onder toesig gedoen kan word?

Should the school-day be lengthened so that the "homework" could be done at school under supervision? (66)

Ja / Yes 1
Nee / No 0

4. Gemiddeld hoeveel uur per skoolweek bestee u aan Rekene-nasienwerk, uitgesonderd nasienwerk in die klas?

On an average how many hours per school week do you devote to correcting work in Arithmetic, excluding marking in class? ( ) (67-8)

5. Gemiddeld hoeveel uur per week bestee u aan Wiskunde-nasienwerk, uitgesonderd nasienwerk in die klas?

On an average how many hours per school week do you devote to correcting work in Mathematics, excluding marking in class? ( ) (69-0)

AFDELING VI. | SECTION VI.

(B)

HANDBOEKE EN ANDER HULPMIDDELS. | **TEXTBOOKS AND OTHER TEACHING AIDS.** (1)

7

K/C4

A. HANDBOEKE. | A. TEXTBOOKS (2-22)

1, Watter handboeke gebruik u die meeste vir Rekene? | Which textbooks do you use most often for Arithmetic?

St(d) 6 -- (23)

St(d) 7 --

St(d) 8 -- (24-5)

2. Watter handboeke gebruik u die meeste vir Algebra? | Which textbooks do you use most often for Algebra?

St(d) 6 -- (26)

St(d) 7 --

St(d) 8 -- (27-8)

St(d) 9 --

St(d) 10 -- (29-0)

3. Watter handboeke gebruik u die meeste vir Meetkunde? | Which textbooks do you use most often for Geometry?

St(d) 6 -- (31)

St(d) 7 --

St(d) 8 -- (32-3)

St(d) 9 --

St(d) 10 -- (34-5)

4. Watter handboeke gebruik u die meeste vir Driehoeksmeting? | Which textbooks do you use most often for Trigonometry?

St(d) 7 --

St(d) 8 -- (36-7)

St(d) 9 --

St(d) 10 -- (38-9)

5. Watter ander handboeke gebruik u vir Rekenkunde? | Which other textbooks do you use for Arithmetic?

St(d) 6 -- (40)

St(d) 7 --

St(d) 8 -- (41-2)

6. Watter ander handboeke gebruik u vir Algebra? | Which other textbooks do you use for Algebra?

St(d) 6 -- (43)

St(d) 7 --

St(d) 8 -- (44-5)

St(d) 9 --

St(d) 10 --- (46-7)

7. Watter ander handboeke gebruik u vir Meetkunde?

Which other textbooks do you use for Geometry?

St(d) 6 -- (48)

St(d) 7 --

St(d) 8 -- (49-0)

St(d) 9 --

St(d) 10 --- (51-2)

8. Watter ander handboeke gebruik u vir Driehoeksmeting?

Which other textbooks do you use for Trigonometry?

St(d) 6 -- (53)

St(d) 7 --

St(d) 8 -- (54-5)

St(d) 9 --

St(d) 10 --- (56-7)

9. Meen u dat u genoeg handboeke in u eie taal tot u beskikking het om die vak behoorlik te onderrig en om u eie agtergrond uit te brei?

Are you satisfied that enough textbooks written in your own language are available to enable you to teach the subject properly and to give yourself more background?

Afrikaans		Afrikaans			(58)
	Ja		Yes	1	
	Nee		No	0	
Engels		English			(59)
	Ja		Yes	1	
	Nee		No	0	

B. HULPMIDDELS.

B. TEACHING AIDS.

1. Skrywe asseblief 'n lys van die belangrikste hulpmiddels waarvan u in die onderwys van Wiskunde/Rekene gebruik maak.

Kindly list the most important teaching aids employed by you in the teaching of Mathematics/Arithmetic.

(a) -- (60)

(b) -- (61)

(c) -- (62)

(d) -- (63)

(-- (64)

(f) -- (65)

(g) -- (66)

2.	Maak u gebruik van 'n meetmiddel waarmee die wiskundige en rekenvermoë van leerlinge objektief bepaal kan word?	Do you use an objective measure of the mathematical/arithmetical ability of pupils?		(67)
	Ja	Yes	1	
	Nee	No	0	
3.	Het u behoefte aan wetenskaplikopgestelde toetse wat leerlinge se wiskundige gebreke op die volgende gebiede sal blootlê?	Do you need scientifically compiled tests to diagnose mathematical disabilities of pupils in the following fields?		
	(a) Basiese kennis	Basic knowledge		(68)
	Ja	Yes	1	
	Nee	No	0	
	(b) Begripsvorming	Formation of concepts		(69)
	Ja	Yes	1	
	Nee	No	0	
	(c) Vaardigheid	Ability		(70)
	Ja	Yes	1	
	Nee	No	0	
	(d) Houding teenoor vak	Attitude to subject		(71)
	Ja	Yes	1	
	Nee	No	0	
4.	Watter maatreëls tref u om te verseker dat die leerlinge korrek werk?	What steps do you take to ensure that the pupils do their work correctly?		(72)
	------------------------------	------------------------------		
	------------------------------	------------------------------		

Bruin
Brown
8 (1)
K/C
4 (2-23)

AFDELING VII. / SECTION VII.

BESPREKING VAN HANDBOEKE. / DISCUSSION OF TEXTBOOKS.

'n Algemene Wiskunde-, 'n Algebra-, Meetkunde- en 'n Driehoeksmetinghandboek waarvan u in u skoolwerk gebruik maak, word hier bespreek. Bespreek asseblief net een handboek in elk van hierdie vakgebiede.

A General Mathematics, an Algebra, a Geometry and a Trigonometry textbook actually used by you for teaching, are being discussed here. Kindly select only one textbook in each of these subjects for discussion.

A. ALGEMENE WISKUNDE / GENERAL MATHEMATICS

1.	Naam van boek: ----------------	Name of book: ------------------	
	------------------------------	------------------------------	
2.	Skrywer(s): ------------------	Author(s): ----------------------	(24-5)
	------------------------------	------------------------------	

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	Afrikaans	English		Code	Col.
3.	Vir standerds: ----------------	For standards: ------------	6	1	
			7, 8	2	(26)
	-------------------------------	------------------------	9,10	3	
4.	Dek die inhoud die leerplan van u Departement?	Do the contents cover the syllabus of your department?			(27)
	Ja		Yes	1	
	Nee		No	0	
5.	Hoe is die boek ingedeel?	How is the book divided?			(28)
	(a) Rekene, Algebra en Meetkunde apart.	Arithmetic, Algebra and Geometry separately		1	
	(b) Volgens temas en begrippe wat Rekene, Algebra en Meetkunde betrek.	According to themes and concepts involving Arithmetic, Algebra and Geometry.		2	
	(c) ---------------------------	-------------------------------		3	
6.	Is die antwoorde agter in die boek?	Are the answers at the back of the book?			(29)
	Ja		Yes	1	
	Nee		No	0	
7.	Is die feite, definisies en stellings akkuraat?	Are the facts, definitions and theorems accurate?			(30)
	Ja		Yes	1	
	Nee		No	0	
8.	Is die vrae gegradeer volgens moeilikheid?	Are the questions arranged in order of difficulty?			(31)
	Ja		Yes	1	
	Nee		No	0	
9.	Is die taal duidelik en wiskundig korrek?	Is the language clear and mathematically correct?			(32)
	Ja		Yes	1	
	Nee		No	0	
10.	Word die ontwikkeling van die wiskundige grondbegrippe genoegsaam beklemtoon in die boek?	Is the development of the fundamental mathematical concepts sufficiently emphasised by the book?			(33)
	Ja		Yes	1	
	Nee		No	0	
11.	Is die boek meer modern in sy benadering as sy voorgangers?	Is this book more modern in its approach than earlier publications?			(34)
	Ja		Yes	1	
	Nee		No	0	
12.	Wat is u persoonlike mening oor die boek?	What is your personal opinion of the book?			(35)
	Baie goed		Very good	1	
	Goed		Good	2	
	Middelmatig		Fair	3	
	Swak		Poor	4	
	Baie swak		Very poor	5	

B. ALGEBRA. — ALGEBRA.

1.	Naam van boek: ----------------	Name of book: -----------------	(36-7)
	-------------------------------	-------------------------------	

2.	Skrywer(s): --------------------	Author(s): --------------------		
	------------------------------	------------------------------		
3.	Vir standerds: -----------------	For standards: ------------ 6	1	(38)
		7, 8	2	
		9, 10	3	
4.	Dek die inhoud die leergang van u departement?	Do the contents cover the syllabus of your department?		(39)
	Ja	Yes	1	
	Nee	No	0	
5.	Hoe is die boek ingedeel?	How is the book divided?		(40)
	(a) Algebra en Grafieke apart	Algebra and Graphs separately	1	
	(b) Grafieke word as 'n inherente deel van Algebra behandel	Graphs are treated as an integrated part of Algebra	2	
	(c) --------------------------	------------------------------	3	
6.	Is die antwoorde agter in die boek?	Are the answers at the back of the book?		(41)
	Ja	Yes	1	
	Nee	No	0	
7.	Wat is die sentrale begrip van die boek?	What is the central concept of the book?		(42)
	(a) Versamelings	Sets	1	
	(b) Vergelykings	Equations	2	
	(c) Funksie	Functionality	3	
	(d) Verandering	Variation	4	
	(e) Verhouding	Ratio	5	
	(f) Betrekking/Relasie	Relation	6	
	(g) --------------------------	------------------------------	7	
8.	Is die taal duidelik, taal- en wiskundig korrek?	Is the language clear, grammatically and mathematically correct		(43)
	Duidelik	Clear	1	
	Taalkundig korrek	Grammatically correct	2	
	Wiskundig korrek	Mathematically correct	3	
	Foute in die aanbieding van wiskundige feite en taalfoute kom voor	Mistakes in the language and in the presentation of mathematical facts	0	
9.	Ontwikkel die boek 'n duidelike wiskundige struktuur?	Does the book develop a clear mathematical structure?		(44)
	Ja	Yes	1	
	Nee	No	0	
10.	Is hierdie boek meer modern in sy benadering as sy voorgangers?	Is this book more modern in its approach than earlier publications?		(45)
	Ja	Yes	1	
	Nee	No	0	
11.	Wat is u persoonlike mening oor die boek?	What is your personal opinion of the book?		(46)
	Baie goed	Very good	1	
	Goed	Good	2	
	Middelmatig	Fair	3	
	Swak	Poor	4	
	Baie swak	Very poor	5	

C. **MEEKUNDE.**	**GEOMETRY.**			
1. Naam van boek: ------------------	Name of book: ------------------			(47-8)
--------------------------------	--------------------------------			
2. Skrywer(s): --------------------	Author(s): ----------------------			
--------------------------------	--------------------------------			
3. Vir standerds: -----------------	For standards: -----------	6	1	(49)
		7, 8	2	
		9, 10	3	
4. Dek die inhoud die leerplan van u departement?	Do the contents cover the syllabus of your department?			(50)
Ja		Yes	1	
Nee		No	0	
5. Is die bewoording van die aksiomas, definisies en stellings taalkundig sowel as wiskundig korrek?	Are the axioms, definitions and theorems grammatically and mathematically correctly stated?			(51)
(a) Wiskundig korrek	Mathematically correct		1	
(b) Taalkundig korrek	Grammatically correct		2	
(c) Taalkundige foute sowel as foute in die aanbieding van wiskundige feite kom voor	Mistakes appear in the language and in the presentation of mathematical facts		0	
6. Is die tekeninge duidelik en akkuraat?	Are the diagrams well-drawn and accurate?			(52)
Ja		Yes	1	
Nee		No	0	
7. Kan die boek as meer modern in sy benadering as sy voorgangers beskou word?	Is this book more modern in its approach than earlier publications?			(53)
Ja		Yes	1	
Nee		No	0	
8. Wat is u persoonlike mening oor die boek?	What is your personal rating of this book?			(54)
Baie goed		Very good	1	
Goed		Good	2	
Middelmatig		Fair	3	
Swak		Poor	4	
Baie swak		Very poor	5	
D. **DRIEHOEKSMETING.**	**TRIGONOMETRY.**			
1. Naam van boek: -----------------	Name of book: ------------------			(55-6)
--------------------------------	--------------------------------			
2. Skrywer(s): -------------------	Author(s): ----------------------			
--------------------------------	--------------------------------			
3. Vir standerds: ----------------	For standards: -----------	6	1	(57)
		7, 8	2	
		9, 10	3	

4. Dek die inhoud die leerplan van u departement?	Do the contents cover the syllabus of your department?		(58)
Ja	Yes	1	
Nee	No	0	
5. Is die bewoording taalkundig sowel as wiskundig korrek?	Are the contents grammatically and mathematically correctly stated?		(59)
(a) Wiskundig korrek	Mathematically correct	1	
(b) Taalkundig korrek	Grammatically correct	2	
(c) Taalfoute, sowel as foute in die aanbieding van wiskundige feite, kom voor	Mistakes appear in the language and in the presentation of mathematical facts	0	
6. Is die tekeninge duidelik en akkuraat?	Are the diagrams well-drawn and accurate?		(60)
Ja	Yes	1	
Nee	No	0	
7. Kan die boek as meer modern in sy benadering as sy voorgangers beskou word?	Is this book more modern in its approach than previous publications?		(61)
Ja	Yes	1	
Nee	No	0	
E. <u>EKSPERIMENTELE HANDBOEKE.</u>	<u>EXPERIMENTAL TEXTBOOKS.</u>		
<u>LET WEL:-</u>	<u>NOTE:-</u>		
Met „eksperimentele handboeke" word handboeke bedoel wat in voorlopige vorm aan skole beskikbaar gestel word om op proef gestel te word. Die kommentaar van die onderwysers word verkry, en 'n nuwe stel word aan die hand daarvan opgestel. Hierdie proses word voortgesit totdat 'n bevredigende resultaat verkry is.	"Experimental textbooks" are textbooks made available to schools in preliminary form to be tried out. The remarks of the teachers are welcomed and a new set of books are compiled incorporating suggestions that seem feasible. This process is continued until a satisfactory result is obtained.		
1. Meen u dat daar geleentheid is vir 'n ingrypende vernuwing van	Do you think that there is room for a complete overhaul of		
(a) ons Wiskunde-onderwys	the teaching of Mathematics		(62)
Ja	Yes	1	
Nee	No	0	
(b) ons Rekene-onderwys?	teaching of Arithmetic		(63)
Ja	Yes	1	
Nee	No	0	
2. Meen u dat 'n stel eksperimentele handboeke 'n belangrike rol kan speel in die vernuwing van die onderwys van Wiskunde en Rekene?	Do you think that a series of experimental textbooks could play an important part in a new approach in the teaching of Mathematics and Arithmetic?		(64)
Ja	Yes	1	
Nee	No	0	
3. Indien 'n sodanige stel eksperimentele handboeke geskryf moet word, deur wie moet dit gedoen word?	If such a series of experimental textbooks were to be written by whom should it be done?		(65)
(a) 'n Groep onderwysers	A group of teachers	1	

	(b) 'n Groep universiteitsdosente	A group of university professors and lecturers	2	
	(c) 'n Onafhanklike skrywer	An independent author	3	
	(d) 'n Komitee van onderwysers en universiteitsdosente	A committee of teachers and university professors and lecturers	4	
	(e) ---------------------------	------------------------------	5	
4.	Moet hierdie eksperimentele handboeke dien as:	Should these experimental textbooks serve as:		(66)
	(a) 'n model vir toekomstige handboekskrywers?	a model for future authors of school textbooks?		(66)
	Ja	Yes	1	
	Nee	No	0	
	(b) 'n hulpmiddel vir die opleiding van onderwysers?	an aid in the training of teachers?		(67)
	Ja	Yes	1	
	Nee	No	0	
	(c) 'n voorbeeld van hoe die wiskundige begrippe in hoërskole ontwikkel behoort te word?	an example of how the mathematical concept ought to be developed in high schools?		(68)
	Ja	Yes	1	
	Nee	No	0	
	(d) 'n voorbeeld van die uitbouing van 'n wiskundige struktuur?	an example of the development of a mathematical structure?		(69)
	Ja	Yes	1	
	Nee	No	0	
	(e) voorlopige skoolhandboeke totdat soortgelyke boeke in die handel verkrygbaar is?	preliminary textbooks until such books are obtainable commercially?		(70)
	Ja	Yes	1	
	Nee	No	0	
5.	Waar moet die antwoorde op die vrae wat in die eksperimentele handboeke voorkom, verskyn?	Where should the answers to questions in the experimental textbooks appear?		(71)
	(a) Agterin dieselfde boek	At the back of the same book	1	
	(b) In 'n aparte antwoordboekie net vir onderwysers	In a separate booklet for teachers only	2	
	(c) In 'n handleiding vir die onderwysers waarin die metodiek uiteengesit word	In a teachers' manual containing the teaching methods.	3	
	(d) ---------------------------	------------------------------	4	
6.	Onder die beskerming van watter liggaam moet die eksperimentele handboeke gepubliseer word?	Under whose aegis should these experimental textbooks be written?		(72)
	(a) Die Nasionale Raad vir Sosiale Navorsing	The National Council for Social Research	1	
	(b) Die Suid-Afrikaanse Wiskundevereniging	The South African Mathematical Association	2	
	(c) 'n Komitee wat verskillende instansies met belang by Wiskundeonderwys verteenwoordig.	A joint committee of various organisations interested in the teaching of Mathematics	3	

APPENDIX 13

N.B. 378
R.

REPUBLIEK VAN SUID-AFRIKA

REPUBLIC OF SOUTH AFRICA

DEPARTEMENT VAN ONDERWYS, KUNS EN WETENSKAP

DEPARTMENT OF EDUCATION, ARTS AND SCIENCE

Nasionale Buro vir Opvoedkundige en Maatskaplike Navorsing.

National Bureau of Educational and Social Research.

'N NASIONALE OPNAME

A NATIONAL SURVEY

DIE ONDERRIG VAN WISKUNDE AAN HOëR-SKOLE.

THE TEACHING OF MATHEMATICS IN HIGH SCHOOLS.

Hierdie vraelys moet deur almal wat die Metodiek van Wiskunde-onderwys doseer, voltooi word.

This questionnaire is to be completed by all who teach the Methods of Teaching Mathematics.

VERTROULIK

CONFIDENTIAL

DEPARTEMENT VAN ONDERWYS, KUNS EN WETENSKAP

DEPARTMENT OF EDUCATION, ARTS AND SCIENCE

Nasionale Buro vir Opvoedkundige en Maatskaplike Navorsing

National Bureau of Educational and Social Research

'n Nasionale opname van sekere aspekte van die onderrig van WISKUNDE, ALGEMENE WISKUNDE en REKENE aan middelbare en hoërskole.

A national survey of certain aspects concerning the teaching of MATHEMATICS, GENERAL MATHEMATICS and ARITHMETIC in secondary and high schools.

HIERDIE VRAELYS

THIS QUESTIONNAIRE

word aan alle dosente in die sekondêre Metodiek van Wiskunde en Rekene aan onderwyskolleges, tegniese kolleges (onderwysersopleidingsdepartemente) en universiteite gestuur. Dit sluit universiteitsdosente in wat hierdie metodiek aan onderwyskolleges doseer.

is being forwarded to all persons teaching the methods of secondary school Mathematics and Arithmetic in any of the above subjects at teacher training colleges, technical colleges (teacher training departments) and universities, including university staff lecturing at teacher training colleges in these methods.

LET WEL:-

NOTE:-

1. Die inligting wat u in hierdie vraelys verstrek, word alleen vir navorsingsdoeleindes gebruik, en sal streng vertroulik behandel word.

The information given in this questionnaire is strictly confidential and will be used for research purposes only.

2. Lees asseblief die aanwysings noukeurig voordat u die vraelys voltooi:

Please read the instructions carefully before completing the questionnaire:

(a) Waar die moontlike antwoorde verstrek word, trek 'n kringetjie om die syfer by die regte antwoord.

Where the possible answers are given, draw a circle round the number opposite the appropriate answer.

(b) Verontagsaam asseblief die getalle wat aan die regterkant van die bladsye tussen hakies verskyn.

Kindly ignore the numbers between brackets at the right side of each page. K/c

9 (1)

(c) Die vrae oor Wiskunde het ook op Rekene betrekking.

The questions on Mathematics include Arithmetic.

1. ALGEMENE INLIGTING.

GENERAL INFORMATION.

1. Naam van kollege: --------------

Name of college: ---------------- (2,3)
----

2. Naam van universiteit: ---------

Name of university: -------------
----

3. Adres: -----------------------

Address: ---------------------

	Afrikaans	English		
4.	Naam van dosent: --------------	Name of professor or lecturer: ----		
	------------------------------	------------------------------		
5.	U eintlike departement:	Your principal department:		(4)
	(a) Wiskunde	Mathematics	1	
	(b) Toegepaste Wiskunde	Applied Mathematics	2	
	(c) Wiskunde en Toegepaste Wiskunde	Mathematics and Applied Mathematics	3	
	(d) Wiskundige Statistiek	Mathematical Statistics	4	
	(e) Ander (spesifiseer): --------	Other (specify): ----------------		
	--------------------------	------------------------------	5	
	(f) Departement in Opvoedkunde (spesifiseer): -------------	Department in Education (specify): ------------------------------	6	

	Afrikaans	English		
II.	<u>DIE PROFESSIONELE KURSUS VIR TOEKOMSTIGE ONDERWYSERS IN WISKUNDE/REKENE.</u>	<u>THE PROFESSIONAL COURSE FOR FUTURE TEACHERS OF MATHEMATICS/ARITHMETIC.</u>		
1.	In watter van die volgende kursusse gee u klas in die Metodiek van Wiskunde en/of Hoërskoolrekene?	In which of the following courses do you lecture the Methods of Mathematics and/or High School Arithmetic?		
(A)	Driejarige professionele kursus met spesialisering in die Onderwys van Wiskunde	Three-year professional course with specialisation in the Teaching of Mathematics		(5)
	Ja	Yes	1	
	Nee	No	0	
(B)	Driejarige gekombineerde kursus bestaande uit 'n eerstejaarse universiteitskursus vir die Baccalaureusgraad en 'n tweejarige algemene professionele opleiding.	Three-year combined course consisting of a first year university course for the Bachelor's degree and a two-year general professional training.		(6)
	Ja	Yes	1	
	Nee	No	0	
(C)	Driejarige gekombineerde kursus bestaande uit die eerste- en tweedejaar kursus vir die Baccalaureusgraad en 'n eenjarige algemene professionele opleiding.	Three-year combined course consisting of a first and second year course for the Bachelor's degree and a general professional training of one year.		(7)
	Ja	Yes	1	
	Nee	No	0	
(D)	'n Vierjarige gekombineerde graad- en diplomakursus.	A four-year combined degree and professional course.		(8)
	Ja	Yes	1	
	Nee	No	0	
(E)	Ander (spesifiseer): -----------	Other (specify): ----------------		
	------------------------------	------------------------------		
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	------------------------------	------------------------------		

2. Volg die studente aan wie u die Metodiek van die Onderwys van Wiskunde en/of Rekene (Hoërskool) doseer die volgende kursusse?

Do the students to whom you lecture on the Methods of Teaching Mathematics and/or High School Arithmetic take the following courses? (9)

Kursus A	Ja	Course A	Yes	1	
	Nee		No	0	
Kursus B	Ja	Course B	Yes	1	(10)
	Nee		No	0	
Kursus C	Ja	Course C	Yes	1	(11)
	Nee		No	0	
Kursus D	Ja	Course D	Yes	1	(12)
	Nee		No	0	
Kursus E	Ja	Course E	Yes	1	(13)
	Nee		No	0	

3. Indien die Metodiek-kursusse wat u doseer aan die studente wat Kursusse A, B, C, D of E volg, in inhoud van mekaar verskil, moet u asseblief van meer as een van hierdie vraelyste gebruik maak. Hierdie vraelys word voltooi ten opsigte van die studente wat Kursus(se) -------------- volg.

If the Methods courses taught by you to the students following Courses A, B, C, D and E differ in content, kindly use more than one of these questionnaires. This questionnaire is being completed in respect of students following Course(s) -------------------------- (14-18)

4. Word die volgende onderwerpe as deel van die opleiding van Wiskundeonderwysers aangebied?

Indien die onderwerp nie aangebied word nie, plaas 'n 0 in die spasie tussen die hakies, Word dit wel aangebied, plaas asseblief die volle getal klas-ure wat gedurende die aspirantonderwyser se opleiding aan die onderwerp bestee word, in die spasie tussen die hakies.

Are the following topics included as part of the course for aspirant teachers in Mathematics?

If the topic is not included, put an 0 in the space between brackets. If it is dealt with, kindly indicate in the space between brackets the number of lecture hours allotted to this topic during the training of the future teachers.

(a)	Klaskamerbeheer en organisasie.	Classroom management and organisation.	( )	(19-0)
(b)	Beplanning van werkseenhede gebaseer op die Wiskundeleerplan.	Planning of units of work based on the Mathematics syllabus.	( )	(21-2)
(c)	Tegniek van Wiskunde-onderwys.	Technique of teaching Mathematics.	( )	(23-4)
(d)	Die gebruik van die biblioteek en ander bronne.	The use of the library and other sources.	( )	(25-6)
(e)	Die opstelling van Wiskundetoetse.	Construction of Mathematics tests.	( )	(27-8)
(f)	Die jongste ontwikkelings op wiskundige gebied.	The most recent developments in the field of Mathematics.	( )	(29-0)
(g)	Die beroeps- en vakorganisasies en die voordele van lidmaatskap.	The various mathematical and professional organisations and the value of membership.	( )	(31-32)
(h)	Die gebruik van die rekenliniaal.	The use of the sliderule.	( )	(33-4)
(i)	Die Wiskunde-handboek op skool.	The school Mathematics textbook.	( )	(35-6)
(j)	Die geskiedenis van Wiskunde.	The history of Mathematics.	( )	(37-8)
(k)	Die wiskundige begrippe en die leerlinge se begripsvermoë.	The mathematical concepts and the ability of the pupils to grasp the concepts.	( )	(39-0)

(l)	Die wysgerige grondslae van Wiskunde.	The philosophical bases of Mathematics.	( )	(41-2)
(m)	Die wiskundige struktuur.	The mathematical structure.	( )	(43-4)
(n)	Die wiskundige bewys.	The mathematical proof.	( )	(45-6)
(o)	Deduksie en induksie in die Wiskunde.	Deduction and induction in Mathematics.	( )	(47-8)
(p)	Analise en sintese in Wiskunde.	Analysis and synthesis in Mathematics.	( )	(49-0)
(q)	Die aksiomatiese metode.	The axiomatic method.	( )	(51-2)
(r)	Die grondbegrippe: Lengte, tyd en massa.	The basic concepts: Length, time and mass.	( )	(53-4)
(s)	Die afgeleide begrippe, bv. die energiebegrip.	The derived concepts, e.g. the concept of energy.	( )	(55-6)

5. Proefonderwys: — Practice Teaching:

	Hoe vind die proefonderwys plaas?	How is student teaching practice organised?		(57)
(a)	Gewone skoolonderwys onder die leiding van 'n ervare onderwyser.	Ordinary teaching routine in the schools, under the direction of an expert teacher.	1	
(b)	Onderwys onder die leiding van opleidingsonderwysers aan oefenskole.	Teaching practice under the guidance of training personnel in training schools.	2	
(c)	Die gebruik van 'n eksperimentele klaskamer in die kollege of universiteit.	Use of an experimental classroom at the college or university.	3	
(d)	---------------------------	--------------------------------	4	
	---------------------------	--------------------------------		

III. SPESIALE KURSUS. — SPECIAL COURSE.

1.	Indien u inrigting nie 'n spesiale professionele kursus om persone in staat te stel om Wiskunde-onderwysers aan hoërskole te word, bied nie sal 'n sodanige kursus binnekort ingestel word?	If your institution does not have a special professional course to train people to teach Mathematics in high schools, will a course be started soon?		(58)
	Ja	Yes	1	
	Nee	No	0	
2.	Bestaan sodanige kursus reeds?	Has such a course already been introduced?		
	Ja	Yes	3	
	Nee	No	4	
3.	Indien wel, hoeveel studente sal na verwagting 'n sodanige kursus	If this is already the case, how many students are expected to finish the course at		
	(a) vanjaar voltooi?	(a) the end of this year	( )	(59-0)
	(b) die einde van volgende jaar?	(b) the end of next year?	( )	(61-2)
	(c) die einde van 1964?	(c) the end of 1964?	( )	(63-4)
4.	Wat is die lengte van die kursus?	What is the duration of the course?		(65)
	1 jaar	1 year	1	
	2 jaar	2 years	2	
	3 jaar	3 years	3	

5.	Is die kursus 'n voltydse kursus?	Is this a full-time course?			(66)
	Ja		Yes	1	
	Nee		No	0	
6.	Is die kursus bedoel vir die opleiding van onderwysers in diens?	Is this an in-service course for teachers?			(67)
	Ja		Yes	1	
	Nee		No	0	
7.	Hoeveel uur word jaarliks in die kursus aan die Metodiek van Wiskunde-onderwys bestee?	How many hours are spent on the Methods of Teaching Mathematics in this course annually?		( )	(68-9)
8.	Was u self 'n Wiskunde-onderwyser?	Were you a Mathematics teacher yourself?			(70)
	Ja		Yes	1	
	Nee		No	0	
9.	(a) U akademiese kwalifikasies.	Your academic qualifications.			(71)
	--------------------------------	--------------------------------			
	(b) U professionele kwalifikasies.	Your professional qualifications.			(72)
	--------------------------------	--------------------------------			

No. ------------ (73-5)

ONS SAL DIT BESONDER OP PRYS STEL OM 'N AANVULLENDE MEMORANDUM OMTRENT U SIENSWYSE OOR DIE OPLEIDING VAN WISKUNDE-ONDERWYSERS TE ONTVANG.

A MEMORANDUM ON YOUR VIEW OF FURTHER ASPECTS RE THE TRAINING OF MATHEMATICS TEACHERS WILL BE MUCH APPRECIATED.

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