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the instruction of mathematics at secondary school level in a number of countries in western europe

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PREFACE

A comprehensive research programme concerning the instruction of Mathematics and Arithmetic at primary and secondary schools was launched by the HSRC in order to carry into effect a request by the South African Mathematical Association.

This report is the second in a series and it deals mainly with data collected by the author during a visit of two months to a number of countries in Western Europe. The aim of the overseas visit was to obtain a picture of the way in which innovation in the instruction of Mathematics was undertaken in overseas countries. The necessity of the participation of teachers, the practical evaluation of amendments in syllabuses, and full-time staff for planning and innovation were discussed in the report.

The attention of all interested parties is drawn to the fact that although the researcher is responsible for the factual information, the content of the report and the recommendations contained in it, were approved by the ad hoc committee for research on the instruction of Mathematics in South African schools.

I would like to express my sincere gratitude towards members of the ad hoc committee who sacrificed their time in order to attend the meetings and to evaluate the report.

It is hoped that this report will contribute towards purposeful planning and implementation of innovation in the instruction of Mathematics in the Republic of South Africa.

PRESIDENT

C O N T E N T S

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O P S O M M I N G

Hierdie verslag spruit uit 'n oorsese besoek aan sekondêre onder=wysinrigtings, universiteite, navorsingsinstitute en onderwysde=partemente in Brittanje, België, Nederland en Wes-Duitsland. Die doel met die besoek was om 'n beeld te kry van die wyse waarop vernuwing in Wiskundeonderrig in oorsese lande aangepak word. Dit gaan om sowel die organisatoriese as die inhoudelike aspekte van vernuwing.

In Nederland is daar van 1961 af vernuwingsprogramme in Wiskunde=onderrig onderneem, maar teen 1972 het 'n wesenlike klemverskui=wing ten opsigte van die benadering tot vernuwing plaasgevind. Tans word met die beplanning en implementering van vernuwing in Wiskundeonderrig van die standpunt uitgegaan dat dit in die eerste plek gaan om die opvoedingswaarde van Wiskundeonderrig te verbreed en te verdiep, en nie om Wiskunde as vak te bevorder nie. In 1971 is die beplanning en implementering van vernuwing van Wiskundeonderrig op alle skoolvlakke in Nederland op 'n gesen=traliseerde basis georganiseer met die totstandkoming van die <u>Instituut voor Ontwikkeling van het Wiskunde Onderwijs</u> (IOWO). Uit ondervinding sedertdien het IOWO gevind dat die volgende sake van deurslaggewende aard is by die beplanning en implemen=tering van vernuwing in Wiskundeonderrig:

(1) Vernuwing ten opsigte van sillabusinhoud alleenlik is van beperkte waarde, terwyl vernuwing ten opsigte van onderrig=metodiek noodsaaklik is.

(2) Dit is gebiedend noodsaaklik dat gedetailleerde metodolo=giese advies in die vorm van eksperimenteel beproefde hand=boeke en hulpmiddels beskikbaar gestel word.

(3) Sillabuswysigings moet ook deur konsultasie met onderwysers voorafgegaan word.

In Wes-Duitsland is daar tot die besef gekom dat gedetailleerde inligting en leiding aan onderwysers gegee moet word met die implementering van nuwe sillabusse, en tans word "curricularer Lehrplan" aan onderwysers beskikbaar gestel. Met 'n kurriku=lêre sillabus word dan bedoel 'n sillabus in die gewone sin van die woord tesame met uitvoerige interpretasie daarvan en metodo=logiese riglyne. Daar is verder bevind dat dit noodsaaklik is dat onderwysers self aktief deelneem aan die ontwikkeling van nuwe kurrikula. Ten einde leiding te gee met betrekking tot vernuwing in vakonderrig op skool is besluit op die oprigting van nasionale institute vir navorsing oor die vakdidaktiek van Wiskunde en Wetenskap.

Vernuwing in Wiskundeonderrig in Engeland en Wallis is in sekere opsigte maklik implementeerbaar vanweë die outonomie van skole. Daar is dan ook die afgelope dekade heelwat nuwe Wiskundesilla= busse geïmplementeer, soos onder andere die School Mathematics Project, St Dunstan's Project en die Midlands Mathematics Expe= riment. Daar is tot die gevolgtrekking geraak dat die moderni= sering van die Wiskundeleerstof as sodanig van relatief min be= tekenis is, tensy die toepaslikheid en betekenis van Wiskunde beklemtoon word ten einde die leerlinge te motiveer. Soortge= lyke bevindinge het in België aan die lig gekom. Van vroeg af (1961) is 'n sentrum opgerig om die vernuwing in Wiskundeonderrig te inisieer en te koördineer, naamlik die Belgiese Sentrum vir die Metodiek van Wiskundeonderwys (BCMW).

Oor die algemeen kan daar gepraat word van 'n kontra-vernuwing ten opsigte van Wiskundeonderrig in Wes-Europa, omdat die moderni= sering van die Wiskundeleerstof as sodanig plek gemaak het vir die besef dat die <u>onderrig</u> van die vak eerder vernuwe moet word. Hierdie vernuwing kan egter alleenlik geskied indien die Wiskunde= sillabusse die onderwysers die geleentheid bied om weg te breek van die pedagogies sinlose gebruik om Wiskunde as geïsoleerde vak te onderrig. Wiskundeonderrig kan slegs effektief tot die op= voeding van kinders bydra indien die samehang tussen Wiskunde en die totale leefwerklikheid op skool ter sprake kom. Die aanbe= velings wat in die lig hiervan gedoen word, is kortliks soos volg:

(1) Wiskundesillabusse moet op grond van navorsing voortdurend hersien word.

(2) Sekere skole moet periodiek deur die Gemeenskaplike Matri= kulasieraad (GMR) vrygestel word van die st. 10-eindeksamens ten einde sillabuswysigings aan hierdie skole vooraf eers effektief te evalueer.

(3) Sillabuswysigings moet slegs geïmplementeer word nadat dit op eksperimentele basis in die praktyk geëvalueer is.

(4) Voltydse navorsers moet aangestel word vir die evaluering van sillabuswysigings en Wiskundeonderwysers moet aktief daarby betrek word.

S U M M A R Y

The findings in this report are the result of a visit paid to schools, universities, research bureaux and education departments in Britain, Belgium, the Netherlands and West Germany. The aim of the visit was to find out in what way innovation in the teach= ing of Mathematics was planned and put into practice in overseas countries. The way in which innovation projects are organized and the content matter introduced in the "new" syllabuses are therefore the subject of this report.

Innovation projects as regards Mathematics teaching have been undertaken in the Netherlands since 1961. By 1972 a marked shift of emphasis as regards the approach to this type of project had occurred. The aim at present with the planning and imple= mentation of innovation with regard to the teaching of Mathema= tics, is the broadening and deepening of the educational (forma= tive) value of Mathematics and not the promotion of the subject as such. In 1971 the planning and organization of innovation projects in Mathematics for all school levels was placed under the control of a single institution, the Instituut voor Ontwik= keling van het Wiskunde Onderwijs (IOWO). Research at this institute resulted in the following findings:

(1) Innovation with respect to sillabus content alone is of little value. Innovation as regards the teaching methods must accompany new content matter.

(2) It is most necessary that detailed advice on method should be made available in the form of experimentally evaluated textbooks and teaching aids.

(3) Changes in the sillabus content should only be made after consultation with teachers.

In West Germany experience has also shown that detailed infor= mation and guide-lines must be supplied to teachers on the in= troduction of new syllabuses. At present teachers are supplied with a "curricularer Lehrplan", which is a syllabus in the usual form but accompanied by a detailed interpretation of the content matter and suggestions as to teaching it. A further finding has been that it is important to draw teachers into active par= ticipation in any innovation project as regards curricula. In order to lay down guide-lines pertaining to the renewal of sub= ject teaching in schools, national institutes for research into the subject didactics of Mathematics and Science have been es= tablished.

Innovation in the teaching of Mathematics in England and Wales is relatively easier because of their decentralised education system. During the past decade numerous projects have been initiated, e.g. the School Mathematics Project, St Dunstan's Project and the Midlands Mathematics Experiment. Evaluation of these projects once again showed that the inclusion of "new" topics in a syllabus is of relatively little importance unless the applications and meanings these topics have in everyday life can be put across to pupils in order to motivate them. Similar conclusions were arrived at in Belgium. As early as 1961 a centre was established in Belgium to initiate and co-ordinate innovation projects with respect to Mathematics teaching. This institute is known as the Belgian Centre for the Methodology of Mathematics (BCMW).

In Western Europe it is at present probably more accurate to speak of contra-innovation with regard to Mathematics teaching, since the emphasis has shifted from "new" content matter to "new" teaching methods. This innovation with respect to teaching methods, however, is only possible if teachers are allowed to break away from the unpedagogical practice of teaching Mathematics as an isolated subject. With a view to the true education of children through the teaching of Mathematics, the connections between actual life situations and Mathematics must be experienced by the pupils. In view of the above, the following recommendations are made:

(1) Mathematics syllabuses must be continually renewed on the strength of research.

(2) The Joint Matriculation Board (JMB) must periodically exempt some schools from the Std 10 examinations so that new syllabuses may be evaluated beforehand in these schools.

(3) New syllabuses must be implemented in the schools only after they have been experimentally evaluated.

(4) Research workers must be appointed in a full-time capacity in order to evaluate new syllabuses, and Mathematics teachers must participate actively in such a programme.

CHAPTER 1

INTRODUCTORY REMARKS

1.1 REASON FOR THIS RESEARCH

At the request of The Mathematical Association of South Africa, the HSRC launched a comprehensive research programme concerning the instruction of Mathematics at South African schools, with a view to innovation. The request of the Mathematical Asso= ciation reads that the research will cover, inter alia, the following:

"The study of the functioning and results of school mathematics projects in other countries with special reference to:

(1) Methods of examinations;

(2) methods of providing new textbooks;

(3) the training and retraining of teachers;

(4) principles and norms by which existing subject-matter was eliminated and new matter introduced" (letter dated 20.4.1970).

This request was complied with in part, by means of a two month visit by a research officer of the HSRC to countries in Western Europe. This report deals mainly with the data collected during this visit.

1.2 MANNER OF COLLECTING DATA

The data were collected by means of visits to schools, universi= ties, research institutes and education departments. Interviews were also conducted with Mathematics teachers, principals of schools, inspectors, administrative education officers, research= ers and university lecturers, and research was undertaken in libraries. The following countries were visited: Britain, Belgium, the Netherlands, West Germany and Sweden. With the exception of Sweden, the different countries are reported on separately in this publication. At the beginning of the study tour the Second International Conference on Mathematical Educa= tion (ICME) was attended in Exeter, England, and on this occasion background data were collected and personal contacts established, which were later followed up during the visits to the various countries.

1.3 ORIENTATIONAL REMARKS

1.3.1 Aims of this research

In accordance with the contents of the request of the Mathema= tical Association, the primary aim of this research was to obtain a picture of the way in which innovation in Mathematics instruction is tackled in overseas countries, as well as a pic= ture of what innovation has comprised so far and what is planned for the future. The issue is therefore the organizational as well as contextual aspects of innovation.

1.3.2 Differences between the school systems of various countries

Innovation of subject instruction, by its very nature, cannot be planned and implemented in the same manner in the different school systems. In order to evaluate particular organizational forms for innovation with a view to recommendations concerning the innovation of Mathematics instruction in South Africa, it is necessary to evaluate such organizational forms every time against the background of the school system in which it is implemented. For every country covered by this report, a pic= ture of the school system is provided in outline.

1.4 MANNER OF REPORTING

Concerning the organizational aspects of innovation, a report is given in Chapters 2 to 5 on the ways in which innovation occurs in the different countries. Chapter 6 reports on recent tendencies in respect of contextual and didactical innovation. A few synoptical remarks are made in Chapter 7 concerning the organization of innovation, and a few recommendations in Chapter 7.

CHAPTER 2

INNOVATION OF MATHEMATICS INSTRUCTION IN THE NETHERLANDS

2.1 THE DUTCH SCHOOL SYSTEM

The Netherlands has a centralised school system in which the Minister of Education and Science is responsible for legislation concerning education and the implementation of such laws. Syllabuses and curricula are laid down at a national level, although private schools have a limited degree of freedom in this connection.

In addition to nursery school education and special education, distinction is made in the Netherlands between primary education ("basisonderwijs") and secondary education ("voortgezet onderwijs"). Primary education continues for 6 years (age 6-12). With regard to secondary education, distinction is made between:

(a) Preparatory scientific education (PSE),
(b) general continued education (GCE), and
(c) vocational education.

Preparatory scientific education continues for 6 years, is presented at the school types "gymnasia", "athenea" and "lycea", and leads to a diploma which entails university admission. At the "gymnasia" pupils specialise from their fourth year of study either in the classics (Section A), or in Mathematics and Natural Sciences (Section B). At the "athenea" pupils specialise from the fourth year of study in either Economics and Social Sciences (Section A), or Mathematics and Natural Sciences (Section B). The appellation "lyceum" is used for a school which comprises a "gymnasium" as well as an "atheneum".

With regard to general post-primary education, a distinction is made between lower general post-primary education (LAVO) which takes 2 years, intermediate general post-primary education (MAVO) which takes 3 or 4 years, and higher general post-primary education (HAVO) which takes 5 years. Normally the different kinds of post-primary education are presented at different schools.

With regard to vocational education, there is a variety of school types, which will not be discussed here.

2.2 INNOVATION IN THE INSTRUCTION OF MATHEMATICS SINCE 1960

In 1961 the Commissie Modernisering Leerplan Wiskunde (CMLW) was brought into being in the Netherlands by the Minister of Educa= tion. The tasks assigned to this Commission (of which the members served in a part-time capacity), were the following -

(a) the composition of a new Mathematics syllabus for the gymnasium,

(b) the retraining of Mathematics teachers, and

(c) the initiation of "school experiments".

From 1963 to 1966 all Mathematics teachers (secondary schools) attended in-service training courses. During the period 1964-1968 "experimental" syllabuses were implemented at various gymnasia. These schools were exempted from the normal final examinations and the pupils sat for special examinations based on the "experimental" syllabuses. New Mathematics syllabuses for secondary schools were introduced in 1968. These new syllabuses were composed in accordance with the results of the implementation of "experimental" syllabuses during the 1964-1968 period.

In 1968 two subcommissions of the CMLW were formed, viz -

(a) a subcommission for Mathematics instruction in vocational training, assigned to draw up new syllabuses and to be responsible for the retraining of teachers, and

(b) a subcommission for Mathematics instruction in the primary school, with the task of modernising Mathematics instruc= tion in the primary school and in the training of primary school teachers and to be responsible for the retraining of serving teachers. The activities of this subcommission are organised in the form of a so-called "project" with the name WISKOBAS.

The planning and implementation of innovation of Mathematics instruction at all school levels in the Netherlands were orga= nised on a centralised basis in 1971 upon the establishment of the Instituut voor Ontwikkeling van het Wiskunde Onderwijs (IOWO) at the state university of Utrecht.

The IOWO is the first official institute for the development of a curriculum in the Netherlands (similar institutes for other school subjects are planned to be erected in the foreseeable future). For administrative purposes IOWO forms part of the University of Utrecht. However, IOWO is responsible to the CMLW (see above).

2.3 DUTCH FINDINGS IN RESPECT OF INNOVATION IN THE INSTRUCTION OF MATHEMATICS

There is a fundamental difference between the present (1972) Dutch approach to the instruction of Mathematics, and the approach in practice in 1961 at the start of the attempt at innovation.

The attempt at innovation undertaken since 1961, was largely motivated by the findings of the international conference on the instruction of Mathematics, held in 1959 in Royamont, France, on the initiative of the Organization for European Economic Co-operation. In accordance with the approach at this conference (14), emphasis in the Dutch innovation attempt fell strongly on innovation of the mathematical-contextual aspects of Mathematics instruction. It was concerned more specifically with "modernization", in a mathematical sense, of Mathematics syllabuses (15).

By 1972 a fundamental shift in emphasis had taken place in respect of the approach to innovation, as is evident from the following quotation from a recent publication of the IOWO (8):
"...... it became increasingly clear:
- that rather than mathematics our concern should be the pair <u>mathematics-education.</u>" In the planning and implementation of innovation, the view is held at present that the first concern is to widen and deepen the educational value of Mathematics instruction, and not to promote Mathematics as a subject. This point of view is strongly propounded by Prof. H. Freudenthal, the present director of the IOWO, in his recent book <u>Mathematics is an educational task</u> (5).

2.4 THE ORGANIZATION OF INNOVATION

The innovation of Mathematics instruction which took place in the Netherlands between 1961 and 1970, in outline, was organised as follows:

(a) Experimental implementation of amended syllabuses in a number of schools.

(b) Mathematical retraining of teachers.

(c) Design of new syllabuses by a syllabus committee.

(d) Implementation of new syllabuses.

The present (1972) method of organization in respect of inno= vation is as follows:

(a) Basic curriculum design, namely selection of contents and definition of methodological ways of approach.

(b) Composition of draft syllabuses in a thematological manner.

(c) Making available textbooks and aids to children, teachers and persons who are responsible for the training of teach= ers.

(d) Revision and refinement of syllabuses, textbooks and aids on the strength of experimental implementation.

(e) Implementation of new courses.

The alterations in the organization of innovation indicate that, *inter alia*, the following matters were realised:

(a) That innovation in respect of syllabus content is only of limited value, and that innovation in respect of in= struction methodology is essential.

(b) That the implementation of an amended syllabus may progress effectively only if detailed methodological advice is made available in the form of experimentally tested textbooks and aids.

(c) That, for the sake of effective innovation, syllabus alte= rations should not only be preceded by experimental imple= mentation of draft syllabuses in a number of schools, but also by consultation with the teaching corps during reorien= tation courses.

2.5 THE INSTITUTE FOR THE DEVELOPMENT OF THE TEACHING OF MATHEMATICS (IOWO)

2.5.1 The organizational form of the IOWO

At present (1972) the IOWO has a full-time staff of approximately 35, of whom 20 are professional and 15 clerical officers. The wider organizational form is as follows:

(a) A department of general affairs, with sections for:

- (i) Co-ordination.
- (ii) Curriculum development (general).

(iii) Training of full-time and part-time staff.

(b) A department of primary education, with sections for:

(i) Curriculum development for primary schools.
(ii) Curriculum development for the training of primary school teachers.
(iii) Further training (in-service training) of primary school teachers.
(iv) Information.
(v) Television.

(c) A department of secondary education, with sections for:

(i) The gymnasiums (VWO).
(ii) The general secondary schools (AVO).
(iii) Vocational schools.
(iv) Higher vocational schools.
(v) Special subjects.

2.5.2 The activities of the IOWO

a. The department of general affairs

This department is at present (1972) occupied with mainly two subjects, namely -

(a) the planning of methods of work for curriculum development and the implementation of new curricula, and

(b) the training of full-time and part-time staff.

b. The department of primary education

This department is at present occupied with the following tasks:

(a) Curriculum development for the primary school (see Para= graph 2.5.1 (b)(i)).

(b) Curriculum development for the training and in-service training of teachers.

(c) Publication of a journal (Wiskobas Bulletin).

(d) Preparation of television broadcasts.

(e) Organization of and guidance to work groups of persons who are responsible for teacher training.

c. <u>The department of secondary education</u>

(a) Organization of in-service training courses for teachers.

(b) Liaison with the new institutions for the training of secondary teachers.

(c) Curriculum development for vocational schools and higher vocational schools, and in special mathematical subjects namely Computer Science, Probability Theory, Statistics and Applied Mathematics.

2.6 ORGANIZATION OF CURRICULUM DEVELOPMENT

The method of work of the IOWO with regard to curriculum development covers the following four aspects:

a. <u>The design and composition of "experimental" study material (textbook and other aids) and guides for teachers for particular subject-matter fields</u>

This material is compiled in the light of anticipated innovation potentialities with regard to subject-matter as well as method of instruction. This work is done by full-time members of staff of the institute. Normally it occurs on this basis that one person collects draft material, which is then evaluated by a team of his colleagues. On the strength of this evaluation, alterations are made where necessary. For the WISKOBAS project, for example, such experimental "curriculum units" were compiled for the themes co-ordinates, graphs, sets-relations logic, geometry, open sentences, number systems and probability.

b. <u>The evaluation and refinement of "experimental" material in a "design school"</u>

The "experimental" material is implemented in a school which has been allocated to IOWO for this purpose, and is evaluated and refined by the IOWO staff in co-operation with the staff of the "design school". For the WISKOBAS project, for example, a primary school in Arnhem is used as design school, whereas a secondary school in The Hague is used for Computer Science.

The staff of the design school are trained by IOWO staff members on the strength of weekly lectures and discussions with regard to experimental curriculum units. At the same time these discussions serve as an opportunity for the teachers to inform IOWO staff members about problems experienced in the implementation of the experimental material, as well as recommendations for alterations and refinement. The staff of the "design school" receive additional remuneration for the time they devote to these discussions.

IOWO staff members regularly attend classes in which the experimental material is used and give some lessons themselves. There is thus close co-operation between IOWO and the staff of "design schools". In so far as there are aspects of the existing syllabus which are not covered by the experimental curriculum units, the teaching of Mathematics proceeds in the normal manner. In the case of the WISKOBAS project the "experimental" instruction comprises only about 1/5 of the total time allocated to the teaching of Mathematics.

During this phase of curriculum development, a team of at least four IOWO staff members are set apart to give advice to the teachers of the design school.

This second phase eventually culminates in the study material and teacher guides being revised in the light of the experience gained in them at the design school.

c. <u>Training of teachers, and implementation of "experimental" material in a number of schools</u>

Upon completion of the second phase, teachers are invited to receive voluntary training with regard to the "new" curriculum units. In October 1971, for example, a start was made with such training courses in the WISKOBAS programme. The latter courses cover two years and twenty two-hour sessions, and are presented at teacher training colleges by teams of lecturers, which each consists of a mathematician and a didactician (IOWO staff). The teachers' guides which were compiled during the first and second phases are used as study material for these courses. The idea is that teachers who attend these courses, implement the new curriculum units as enrichment at the schools where they are employed, while the courses are still in progress. The main aim of these courses is to re-orientate teachers with a view to future innovation throughout the country.

d. <u>Integration of curriculum units with existing syllabuses</u>

After completing the second phase a work group is isolated at the IOWO to integrate the new subject-matter and method of presentation with existing syllabuses and methods of presentation, thus forming an "integrated" curriculum which may serve as a basis for innovation throughout the country. It is intended that such an integrated curriculum for the primary school will be published in 1975 and that the composition and implementation throughout the country of syllabuses based on it, will take from 1975 to 1990.

2.7 DEVELOPMENT OF A CURRICULUM FOR COMPUTER SCIENCE

2.7.1 Introductory remarks

In 1968 the CMLW initiated the introduction of Computer Science as a subject at the secondary schools in the Netherlands. A distinction was made between Computer Science as a subject at schools for general secondary education (AVO) and as a subject in vocational and higher vocational schools.

The point of view was held that since a variety of subjects are considered for inclusion in the curriculum for general secondary education, clarity should be obtained on the aims of the instruction of Computer Science, before the contents and method of such instruction can be planned. The formulation of aims for the teaching of Computer Science in the Netherlands, is discussed in the following subparagraph.

Instruction in Computer Science in the Netherlands is at present still in an "experimental" stage, and only approximately 3 per cent of all secondary school pupils receive such instruction at present.

2.7.2 Methods with regard to the introduction of Computer Science as a subject

Since 1967 special training courses in Computer Science have been offered for officiating teachers. Initially lecturing in these courses was undertaken by university professors, but this task has since been taken over by a team of lecturers trained at the IOWO.

Since 1968 "Computerkunde" has been offered on an experimental basis as an optional subject at certain schools by teachers who attended some of the above-mentioned training courses. Since that time these teachers have received assistance in the form of -

(a) a teachers' guide compiled by the IOWO,
(b) a journal published by the IOWO, and
(c) regular meetings of the teachers and IOWO staff.

The textbooks in use at present are regarded as experimental material and are altered from time to time on the strength of experience acquired in using them.

2.7.3 Motivation for the inclusion of Computer Science in the curriculum for general secondary education

The view is held that electronic computers at present fill an unavoidable place in Western culture and have a decided effect on various facets of society. Particularly in the vocational dispensation, almost all adults are involved with a computer in some respect or another. On the strength of this it is averred that all adults in future should have an idea of the uses and performance capacity (quantitatively and qualitatively) of computers. Viewed in this light, instruction in Computer Science may be of educational value.

When a computer is used for some task or another, it is necessary for the user, to a particular degree, exactly to define the data and interdependence which are mentioned. The issue is therefore that knowledge should be made "operational" by explication and insight. Since a computer can only carry out one or another routine of processing (for example classification, arrangement and arithmetical processing), the user should be able to observe (identify) and describe the processing routine (algorithm, process) which is required for a particular task. He should also be able to distinguish between various levels of complexity with regard to a problem, as well as between various aspects of the problem and its solution. In brief, the user must be able to organise the execution of a task. In the light of the forementioned, the instruction of Computer Science may contribute to the forming of a child with regard to the "operationalization" of knowledge, the recognition and description of routines (algorithms) and organization.

The foregoing is at present accepted as aims of Computer Science instruction at secondary schools in the Netherlands and at the same time as motivation for such instruction to all secondary school pupils.

2.7.4 Syllabus composition for Computer Science

In view of the forementioned aims, the point of view is held that subject matter such as the binary number system, time sharing, data storage and composition of programming languages, need not be mentioned at school. On the other hand, it is stressed that the pupil should acquire insight into the principles of computer usage, by solving simple problems with the aid of a computer. The primary issue is therefore "algorithmitics" as subject-matter.

It is considered essential that pupils solve problems themselves with the aid of a computer, so that they may view the computer in the correct perspective, namely as a mere instrument. An attempt should nevertheless be made to restrict programming technique to a minimum, otherwise the subject instruction could easily deteriorate into mere training as programmers. Appli= cations are also stressed in order to reveal the significance of computers in culture.

In view of the foregoing arguments, the present "experimental" syllabus covers the following:

(a) Flow charts, allocation of values to variables, allocation of storage space to variables, input and output of data, statements, branching, loops, variables with subtitles, files.

(b) Principles of important applications: personnel adminis= tration, banking, reservations, simulations, games.

The above-mentioned syllabus is completed in a year and three periods (40 minutes) per week.

CHAPTER 3

INNOVATION OF MATHEMATICS INSTRUCTION IN WEST GERMANY

3.1 THE WEST GERMAN EDUCATION SYSTEM

3.1.1 Educational administration

West Germany is a federal republic consisting of 11 states or "Länder", each with a local government responsible, inter alia, for education at school level. School education in every state is managed by the department of cultural affairs. Co-ordination of cultural matters at national level, including school education, is accomplished by regular (once a month) meetings of the ministers of cultural affairs of the various states. The activities of these meetings are organised by means of a permanent secretariat in Bonn. The meeting of ministers of cultural affairs is called the Ständigen Konferenz der Kultusminister der Länder (Permanent Conference of ministers of cultural affairs). The abbreviation SKK will henceforth be used for this body.

The SKK has no legislative power, but its decisions are normally in the form of recommendations to the local governments. The local governments of the Länder may implement such recommendations as legislation. Recently, for example, the SKK made recommendations concerning a differentiated school system, university entrance examinations and Mathematics syllabuses, which were implemented in all the West German states.

In addition to the SKK there is another body, the Deutsche Bildungsrat (German education council) which has a co-ordinating function with regard to education. This body consists of one representative from each state and three representatives nominated by the federal government. Normally the members of the Bildungsrat (Education Council) are senior education officials, who retain their normal function. The task of the Bildungsrat is to make recommendations to the SKK with regard to educational matters.

In every state educational financing is handled by the local government. Syllabuses and curricula are normally prescribed by the departments of cultural affairs, and the work of schools is controlled by inspectors.

In respect of examination and issuing of certificates, every school is autonomous, except in as far as the form and contents of the Hochschulreifeprüfung are prescribed by the local governments.

3.1.2 Types of schools

In West Germany there are five basic types of schools, namely the following:

(a) The Gründschule, a lower primary school which makes provi= sion for the first four classes (called Primarstufe), i.e. pupils of 6 to 9 years of age.

(b) The Hauptschule, which makes provision for Classes 5 to 9 (10-14 year old pupils). Pupils who attend the Hauptschule normally go to vocational or trade schools later on, on a full-time or part-time basis. After the 6th or 7th Class, pupils may change over to one of the other school types.

(c) The Realschule, which also makes provision for Classes 5 to 9, but which leads to further technical, commercial or academic training (the Hauptschule normally does not lead to this).

(d) The gymnasium. At present there are two types of gymnasiums, namely the so-called Normalform which makes provision for Classes 5 to 13, and the so-called Aufbauform which makes provision only for Classes 8 to 13. In Classes 11 to 13 five types of gymnasium instruction are distinguished, namely -

(i) classical languages,
(ii) modern languages,
(iii) the mathematical-natural sciences field,
(iv) the commercial field, and
(v) various highly specialised fields.

(e) Specialised subject schools, for example technical and commercial schools. Pupils go to these schools after completion of Class 10 at a gymnasium or Realschule.

(f) Trade and vocational schools, which make provision for the further training of pupils who have completed Class 9 at the Hauptschule.

In addition to the above-mentioned types of schools there are at present (1972) a number of Gesamtschulen (comprehensive schools) in which a gymnasium, a Realschule and a Hauptschule exist as one school. There is uncertainty whether the comprehensive schools will gradually replace the other types of schools.

3.1.3 Examining

Promotion from one class (standard, form) to the next is auto= matic. No children are held back. For the Realschule and gymnasium there are entrance qualifications, which are not the same everywhere. In most cases entrance depends on a report of the staff of the school which the pupil has attended, and/or a written or oral entrance examination, and/or a "trial period" during which the pupil attends the school where he/she wants to go to. However, the entrance qualification is not absolute, in this sense that a child's parents have the final decision on the type of school which the child will attend. Normally parents are guided by the recommendations of the schools.

There are two kinds of final examinations, viz the university entrance examination (Hochschulreifeprüfung) which is entered for after Class 13 of the gymnasium or at the end of a period of attendance at certain subject schools, and the final examina= tions of subject, trade and vocational schools.

To be allowed to enter for the university entrance examination, pupils have to take the following compulsory subjects in Classes 12 and 13: German, Mathematics, Religious Instruction, Community Science or History, Geography and Social Studies, Physical Educa= tion, a Music subject and one additional optional subject.

In addition to the above-mentioned, two more subjects are com= pulsory in every type of gymnasium, for example Latin and Greek in a gymnasium for classical languages; and Physics and a modern language in a mathematics-general sience gymnasium. These two subjects together with German and Mathematics are known as compulsory core subjects.

The university entrance examination consists of written, oral and practical parts. The written examination covers the four compulsory core subjects. The oral examination covers these four subjects, as well as Community Science (Normalform Gymna= sium) or History, Geography and Social Studies (Aufbauform Gymnasium), and one optional subject. The practical examina= tion covers Physical Education, Music and Art, and Natural Sciences. The university entrance examination is taken down by an examination committee. Every gymnasium has its own examination committee, under the chairmanship of a person nomi= nated by the local education authorities for each school. The principal (Director) of a school may be nominated for this task. The form and general contents (syllabus) for the examination are prescribed by the local education authority, at present in accordance with recommendations of the SKK. However, the exami=

nation committee as such is responsible for the detailed con= tents of the examination. In outline, it is therefore an internal examination on the strength of broad prescriptions.

3.1.4 Differentiation with regard to the teaching of Mathe= matics

Mathematics is a compulsory subject for all pupils at all levels of school in West Germany. In Classes 1 to 4 (Grundschule) there is no differentiation. In Classes 5 and 6 children attend different school types (gymnasium, Realschule and Hauptschule), but they take the same Mathematics syllabus. From Class 7 onwards (12-13 years old pupils) there is differentiation in subject matter in this sense that pupils at the Hauptschule follow a limited syllabus in comparison with pupils at the other school types (see Appendix 2). From Class 9 onwards pupils of the Realschule follow a less comprehensive syllabus than gymnasium pupils. In Classes 11 to 13 pupils of the various other types of gymnasiums follow less comprehensive syllabuses than pupils at the mathematics-natural sciences gymnasium.

3.2 INNOVATION IN THE INSTRUCTION OF MATHEMATICS SINCE 1958

In 1958 the SKK made recommendations to the 11 states in respect of innovation in the instruction of Mathematics. These recom= mendations were mainly of a contextual nature and were directed at the introduction of modern mathematical concepts such as sets, mapping, relation, group, ring, body, space, etc. at school level.

Between 1958 to 1968 the above-mentioned recommendations were implemented to some degree or other by the various states.

On 3 October 1968 the SKK decided to recommend a "modernised" Mathematics syllabus for all school levels to the governments of the various states. The SKK published a core syllabus for all school levels and it is expected that all the states will eventually base their syllabuses on this core syllabus (see Appendix 2).

At present (1972) innovation activities in West Germany are almost completely limited to implementation by the various states of the syllabuses which were recommended by the SKK in 1968, and the planning and establishing of new organizations to cope with innovation in future (see Paragraph 5).

Experience in the implementation of new Mathematics syllabuses in the various West German states since 1958 revealed that it is insufficient simply to publish a new syllabus as a "catalogue of subject-matter". It was realised that it is necessary to provide detailed guidance to teachers with regard to what is intended with the instruction of various learning contents, to which degree of difficulty particular learning contents should be treated, and which methods of presentation may be followed. In order to meet with this demand, the concept curricularer Lehrplan (curricular syllabus) came into use. Curricular syllabus implies a syllabus in the ordinary sense of the word, together with detailed interpretation of it and methodological directives.

3.3 PLANNING FOR FUTURE INNOVATION

In order to initiate innovation of the instruction of various subjects on a co-ordinated basis in West Germany, the Bildungs= rat nominated a subcommittee for curriculum development. According to Prof. Bauersfeld, the present chairman of this committee, experience over the past 10-15 years in West Germany showed that prescribing new syllabuses, even if it should be accompanied by the making available of new textbooks and orien= tation courses for teachers, is not an effective way of innova= tion. According to him it is essential that teachers should actively participate in the "development" of new curricula, in order to guarantee that they will effectively implement such curricula.

The forementioned subcommittee of the Bildungsrat recently made a recommendation in this connection to the SKK.

In outline the recommendation amounts to teacher centres being established to promote curriculum development on a decentra= lised basis. The idea is that each such centre will serve from 2 000 to 5 000 teachers (all subjects and school levels) of the schools in a particular area. According to what is planned the staff of these centres will consist of full-time specialists (psychologists, didacticians) as well as capable serving teachers who will be attached to these centres on a part-time basis. These centres will initiate curriculum development projects, and the idea is that full-time serving teachers will actively participate in and be responsible for aspects of curriculum development, for example the making avai= lable of study material.

Another recent development with regard to innovation in subject instruction at school (Mathematics and Natural Science), is the establishment of national institutes for subject didactics. The original initiative for the establishment of such insti= tutes came from the Volkswagen foundation which voted the necessary funds. An institute for Physics didactics was established in 1970, at the University of Kiel and a similar institute for Mathematics didactics is being established at the new University of Bieleveld.

For the first 5 years of its existence this institute will be financed by the Volkswagen foundation. It is estimated that the expenditure for this period will amount to approximately 9-10 million German Mark ($2\frac{1}{4}$ - $2\frac{1}{2}$ million Rand). It is planned that the governments of the German states will eventually take over the financial responsibility for this Institute. The initial planning and organization of the Institute is undertaken by a so-called Foundation Council. Leading Mathematics didac= ticians of the various states serve on this Council, as well as the chairman of the National Bildungsrat's subcommittee for curriculum development.

For administrative purposes the Institute will be integrated with the new University of Bieleveld. The staff members of the Institute will be appointed jointly in the departments of Mathematics and Education, but will have no lecturing assign= ments in respect of undergraduates.

The primary functions of the Institute and its staff will be the following:

(a) Guidance and advice in curriculum development projects.

(b) Specialised training of Mathematics didacticians.

(c) Documentation and provision of relevant information con= cerning research and innovation with regard to the teaching of mathematics.

It is intended that the staff of the Institute will participate actively in curriculum development projects throughout West Germany. This participation will amount especially to advising, distribution of information, organization of the exchange of information between various projects, and co-ordination with a view to making provision for all requirements in curriculum development.

On the whole, the function of the Institute will not be to <u>initiate</u> curriculum development projects, but rather to make a constructive and co-ordinating contribution towards projects initiated by other organizations. This approach is essential on account of the decentralised education system and the differences between the school systems of the various German states.

A second task of the Institute will be to train specialised Mathematics didacticians with a view to staffing the teacher centres previously referred to. In the training of these persons the emphasis will fall on the ability to interpret the results of subject didactical research in the light of the practical teaching situation and to transfer these interpretations to serving teachers.

CHAPTER 4

INNOVATION OF MATHEMATICS INSTRUCTION IN ENGLAND AND WALES

4.1 SCHOOL SYSTEM

4.1.1 Administration

Control over the school system in England and Wales is decentra=lised to a large degree, and is divided among the central govern=ment, local education authorities and various voluntary (private) organizations. Co-ordination between the various controlling bodies is accomplished by discussions and agreements, as well as by Her Majesty's inspectors, who act, in fact, as liaison officers between the central government and the local education authorities.

Political responsibility for school education in England is borne by the Secretary of State (minister) for Education and Science, who is the head of the Department of Education and Science. Political responsibility for school education in Wales is borne by the secretary of state for Wales and is administered by the Welsh Education Office in Cardiff.

The above-mentioned departments do not prescribe syllabuses, curricula and textbooks, do not conduct examinations and do not control the appointment of teachers. However, the depart=ments do control matters such as school buildings, recognition of teaching qualifications, pensions and support research.

In the 163 counties and municipal districts (county boroughs) of England and Wales, local management is the responsibility of chosen councils. These councils nominate education commit=tees which officiate as local education authorities. These authorities plan and control education in the various areas by determining which schools are to be attended by which children, appointing and paying teachers, building schools (subject to the approval of the particular secretary of state), making pro=vision for equipment and study material. The local authorities appoint inspectors to serve as liaison officers with the schools.

4.1.2 Syllabus composition

Primary education in England normally takes 5 years (pupil age 5-11 years). With regard to secondary education there is a variety of school types. Since syllabuses and curricula are not prescribed to schools by the authorities, but headmasters (principals) are autonomous in this connection, a description of the various school types is not relevant with regard to this report. Inspectors of the local education authorities as well as Her Majesty's inspectors may be consulted by the headmasters in connection with the composition of curricula and syllabuses. External examination (see below) inevitably affects the compo= sition of syllabuses.

4.1.3 Examining

At present all pupils (except those who receive special educa= tion) have to attend school up to the age of 15. (Compulsory education is soon to be raised to 16 years of age.) There is no compulsory national school leaving examination. Pupils may sit for examinations with a view to obtaining the General Cer= tificate of Education (GCE) or the Certificate of Secondary Education (CSE). The GCE examinations are conducted by nine independent examining bodies, most of which are associated with one or more universities. In most subjects these examinations may be written at two levels, viz ordinary level (Ordinary or O level) and advanced level (Advanced or A level). The O level examinations are normally written after completing a 5-year study at a secondary school. The A level examinations are normally written after another two years of study in the sixth form. Universities normally evaluate applications for entrance on the strength of the results of A level GCE examinations. There are no compulsory subjects for O level or A level examina= tions and candidates may offer any number of subjects. Candi= dates need not have passed a subject on the O level in order to be allowed to sit for the A level examination. A candidate who fails in an A level examination, may be awarded a pass mark at O level in the particular subject. Pass marks in A level exami= nations are awarded in the form of symbols (A, B, C, D and E). Candidates for A level examinations may enter for so-called "special" examinations in the same subjects, based on the same learning contents but more difficult than the A level examina= tions. School attendance is not a prerequisite for examina= tion entrance. Candidates under the age of 16 are normally not allowed to enter for these examinations.

The various examining bodies publish syllabuses for the subjects in which they conduct examinations, with the aim of indicating on which subject contents the examinations are based. For a particular subject an examining body may conduct more than one different examination, based on various syllabuses. The Uni= versity of Cambridge Local Examinations Syndicate, for example, conducts two different examinations in Mathematics at the O level, whereas the University of London University Entrance and School Examinations Council conducts three different examinations at the O level and five at the A level in Mathematics. Headmas= ters are autonomous with regard to the choice of examining bodies and examinations for the pupils of the school, and the choice is normally left to heads of subjects (heads of depart= ments).

GCE certificates are normally issued by the various examining bodies on behalf of the Secretary of State for education and science and examination papers must be approved by this Secre= tary of State. In this connection the Schools Council (see further on) acts for the Secretary of State. This body also has a co-ordinating and advising function with regard to the activities of the various examining bodies, including that for the GCE examinations (see further on). The Schools Council is concerned particularly with maintaining comparable examination standards.

CSE examinations have been conducted since 1965. The examina= tions are conducted in various ways (see further on), but are controlled in all instances by 14 examining bodies which are made up on a regional basis and consist exclusively of serving teachers. Each such body draws up syllabuses and conducts examinations. However, schools may also compile their own syllabuses (subject to the approval of an examining body) and conduct their own examinations (moderated or checked by the examining body). CSE examinations may be entered for by any pupil who has completed five years secondary education, and pass marks are awarded on the basis of a scale of symbols. The highest symbol is recognised as a GCE O level pass mark.

4.1.4 The Schools Council

The Schools Council is an autonomous body within the English ministry of education and its functions are (*inter alia*) to co-ordinate, and where necessary to initiate and finance educational research, to make recommendations to local education authorities and to exert control over examining and issuing of certificates.

4.2 INNOVATION IN THE INSTRUCTION OF MATHEMATICS IN ENGLAND AND WALES

On account of the autonomy of schools with regard to syllabuses and curricula, as well as the variety and partial autonomy of examining bodies with regard to examining, innovation in subject instruction in certain respects is more easily implemented than in countries with stricter centralised control over syllabuses, curricula and examinations. The situation in England and Wales regarding secondary education is such that an examining body may offer an examination based on a "new" syllabus (with the approval of the Schools Council) and schools are free to implement such a syllabus or not. The implication of this is that even if a small number of teachers would be willing and able to implement an innovation arrangement, they may do so, other than in countries with central control over syllabuses and examinations. During the past decade in England and Wales, a variety of new Mathema= tics syllabuses were implemented at secondary school level (and corresponding final examinations), such as the SMP (School Mathematics Project), St Dunstan's Project, Midlands Mathematics Experiment, Mathematics for the Majority Project, to mention a few. Further innovation projects are being initiated. If some person (Mathematics teacher or university lecturer) is of the opinion that he has the necessary insight for the establish= ment of a meaningful syllabus and final examination, he only needs to convince the Schools Council and one or more of the examining bodies with regard to his views, in order to imple= ment such a syllabus in one or more schools. For the purposes of this report two projects were studied in detail and they are discussed in the following paragraphs.

4.3 THE NUFFIELD MATHEMATICS PROJECT

4.3.1 General

The aim of this project was to bring about innovation in the instruction of Mathematics at primary school level, particularly with regard to instruction methodology. This was initiated in 1964 and was financed by the Nuffield Foundation, a private benevolent organization. The project is controlled jointly by the Nuffield Foundation and the Schools Council.

4.3.2 Method of work

The method of work of the project was to compile "guides" for primary school teachers, with the aim of bringing alternative (new) methodological ways of approach to the attention of teachers in the expectation that they will implement them and will themselves investigate further methodological innovation possibilities.

The project employed persons (mostly teachers) full-time for a number of years in order to write the teachers' guides. After the first drafts of the guides had been completed, they were made available in 1965 to primary school teachers in fourteen so-called "pilot areas" in England, Wales and Scotland. Seventy seven so-called "second phase areas" were added in 1966.

The teachers who were involved in the project, implemented the recommendations in the guides in their schools and on the strength of their experience, made recommendations to the authors of the guides which were amended on the strength of such recommendations. Final copies have been published since 1967.

The use of Nuffield guides has been considerably extended ever since and practically every local education authority at present has a local organiser. There are altogether approximately 200 teacher centres (see following paragraph) where Nuffield material is used as a basis for discussions.

In conjunction with the Institut des Sciences de L'Education in Geneva, Switzerland, Mathematics tests have been developed since 1966 with a view to the evaluation of learning readiness among children.

The activities of the project were gradually extended to the lower standards of the secondary school and since 1972 a series of modules, consisting of teachers' guides and school textbooks in the form of job cards, has been published for this school level. These modules are composed with a view to individua=lised, semi-programmed instruction.

4.3.3 The methodological approach on which the Nuffield project is based

The methodological approach which served as a point of departure for the Nuffield guides, is based to a large degree on the results of the research of Piaget. According to Piaget's views, a child's conceptual ability develops according to a certain pat=tern and the effect of instruction will be optimal if it occurs in accordance with the development of the conceptual ability of children, and if its instruction makes provision that children are enabled to discover concepts at the right stage of such development.

In the Nuffield approach the emphasis is placed on the fact that children must learn by doing and, as far as possible, by dis=covering. The task of the teacher is thus to create an "environ=ment", *inter alia* by means of a wide variety of concrete material, amongst which the child may explore, and to intervene when neces=sary by drawing the child's attention to relevant mathematical aspects of his experiences. The point of view is held that such an approach may lead to improved concept formation, that less <u>drilling</u> will be necessary, and that more time will be available for the child to obtain a broad perspective on Mathematics.

4.4 THE ESTABLISHMENT OF CENTRES FOR MATHEMATICS TEACHERS

The activities of the Nuffield project, SMP and other projects indicated that, with a view to effective innovation, it is essential that primary school teachers as well as Mathematics teachers at secondary schools should receive intensive re-orientation with regard to the subject contextual as well as methodological aspects of their work. It was realised that with this in view it is not only essential that teachers should communicate with one another, but also that opportunities should be provided for the re-training of teachers.

With a view to this, so-called teachers' centres have been established since 1965, where teachers regularly gather for discussions and where leading teachers as well as lecturers at training colleges and universities, provide guidance. At present approximately 130 local education authorities have such teachers' centres and especially primary school teachers and teachers of the lower classes of secondary schools (CSE and GCE O level) are involved in the activities of these centres.

4.5 THE SIXTH FORM MATHEMATICS PROJECT

4.5.1 <u>General</u>

This project is directed towards the making available of study material (as well as methodological directives for the teacher), rather than towards the development of a new syllabus. Separate textbooks of a semi-programmed nature are produced for the various themes which appear in syllabuses for the Sixth Form, with the intention that textbooks may be useful, irrespective of the composition of the particular syllabus in use at a school. The project is also directed towards solution of the problem that, owing to divergent variety of O level syllabuses which are being used at present, pupils in Sixth Form classes often have divergent mathematical backgrounds.

4.5.2 <u>Didactical assupmtion of a point of view</u>

In this project the point of view is assumed that Mathematics is fundamentally a study of "hypothetical states of things" (Pierce) and the variation "possibilities" comprised in it. Ormell, initiator and director of the project, writes the following (1972): "Such 'possibilities' are, in a wide sense of the term, candidate innovations in ordinary, scientific, technical, financial, commercial and social situations" (16, p. 127). The distinction between pure and applied Mathematics is cancelled to a large degree by this typification of Mathematics. In the light of this typification of Mathematics a stand is made against stressing the formal structure of Mathematics by which the majority of innovation projects were characterised in the sixties. One of the collaborators on the project, Knowles, puts it in the following way (1971): "When mathematics is studied in a completely formalist manner, it is the structure of the mathematics that holds a position of overriding importance .... of course, it is necessary to recognise, and make use of, the structural 'apparatus' of mathematics; but the chief difficulty in solving a real-life problem frequently lies in relating this structure to the realities of the situation. A completely abstract approach therefore fails to recognise the essentially dual nature of mathematical problem-solving" (10, p. 50). Ormell writes as follows in this connection: "...... although the aesthetic-formalist approach may foster sensibility, the sensibility it fosters tends to be of the 'ivory tower' variety. This approach may please (some) pure research mathematicians in the universities, but it is unlikely to satisfy those who have a larger conception of the place of mathematics in education, or who hope to employ graduates usefully on practical tasks" (16, p. 126), and further: "The value of an approach which lays stress on relevance is that it connects mathematics with other studies and with the main stream of human preoccupations and concerns ... It has become clear that the unique contribution which mathematics makes to society operates not via its relevance to the actual, but via its relevance to the not yet and would be actual: via the light it sheds on the hidden and semi-hidden implications of candidate innovations, hypotheses, proposals, programmes and projects of all kinds. The 'relevance' of mathematics is not to a crass utilitarianism, but to the possibilities of new patterns of explanation in science and new patterns of organization in social and technical affairs. ..... if mathematics is presented without a substantial effort being made to establish relevance, its connection with human experience in general must appear (to all except the expert) to be far more tenuous than those of history, literature, etc. Those teachers who have swung to approaches which treat mathematics as an autonomous, aesthetic exercise seem to have consciously sacrificed relevance in favour of a kind of mystique. They also seem unaware of the extent to which the viability of their approach is sustained by the attitude of the lay public, who believe by and large that mathematics is an extraordinarily important subject. The importance clearly stems from the relevance. If mathematics really were a self-sufficient, aesthetic exercise, it would be no more 'important' than elaborate embroi= dery or organ music. The issue perhaps reduces in the end to that of mystification versus demystification. Do we try to build up a sense of mystification about mathematics, or do we try to clear up the obscurity surrounding the importance of mathematics and its contribution to science and social progress?" (16, p. 129). (Compare in this connection the subject didactical assumption of a point of view of Lighthill (11), the present president of ICME, and of Freudenthal (5), the present director of the IOWO Institute in the Netherlands.) With the above-mentioned arguments the meaningfulness of the presentation of "modern Mathematics" at school level is not rejected, but a stand is taken against the absolutising over-emphasis thereof. Ormell gives the following reasons (inter alia) why "purely abstract" Mathematics should be offered at school level (16, p. 129):

(a) To develop the ability to construct mathematical models, for example by the mastery of a mathematical theory (such as the concept of function and its consequences) within which a variety of mathematical models may be constructed.

(b) To foster understanding of mathematical theories (at a less abstract level) and in this way to make the theories more useful for the learner.

(c) To promote economy and clarity in the use of mathematical concepts.

In this project the point of view is maintained that emphasis of the applicability and significance of Mathematics is essen= tial in the instruction in order to guarantee adequate motiva= tion of the pupils. Ormell writes in this connection: "The most natural way to create motivation is to let the subject speak for itself: to show what it does, or can do" (16, p. 127). The above-mentioned subject-didactical arguments are implemented in the study material of the project by introducing a new mathematical theme every time with an exemplary concrete problematical situation for which the particular theme may provide a functional model. The material is semi-programmed in this sense, that the assignments of ("exercises" for) the pupil are of such a nature that they may lead him to independent design of the mathematical model required by the problem situa= tion. As an example a part from the study material on limits is quoted:

"The design for a high-rise block of flats includes 20 flats on each floor, except the ground floor, which will be used for parking cars. A special building method is being considered:

each floor is to be built on ground level, the building being raised on the completion of each floor by powerful hydraulic jacks. It is estimated that each floor will cost £80 000 to build. The foundations will cost £25 000. The cost of in= stalling the jacks will be £16 000. Other expenses will total £19 000. Each time a floor is raised through 1 floor level the energy used by the jacks will cost £100. What will be the average cost per flat for building and raising a block of the kind described of (a) 2 storeys, (b) 3 storeys, (c) n storeys? Does the average cost per flat approach a limit as n increases? If so, what is the limiting cost per flat, and how many storeys must the builder build to get within 5% of this? Illustrate your answer with a graph showing the way the cost per flat changes as n increases.

Could the average cost per flat ever be as much as £8 000 in a block of this sort? If so, and if n is so large that 3 000/(n-1) is negligible, estimate how many storeys the building would have" (10, p. 50).

Since the point of view is maintained in this project that algorithmic ability as well as insight into the formal structure of Mathematics, are only indirect aims in Mathematics instruc= tion, and that the issue is actually that the pupil should be mathematically familiar with problem situations, similar (but more complex) questions as the one above are set in examinations.

4.5.3 Organization of the project

The project is financed by the Schools Council and administered by the University of Reading. The project was initiated in January 1969 and for the first two years introductory research was undertaken by one full-time officer. Since January 1971 this person has been assisted by four full-time assistants who are serving teachers seconded to this project for four years.

The above-mentioned team produces study material in the form of textbooks on various themes. The first series of textbooks deals with the themes exponents, generalization (nth term, "ge= nerating" functions, recursion, extrapolation and curve fitting), binomial theorem, limits, linear models, quadratic models, polynomial models, geometrical series, logarithmic and exponen= tial functions, biological models (particularly for biological growth).

Schools were invited to use the material (the Schools Council approved a separate A level examination for schools which used the material) and during 1972 altogether 126 schools made use of the offer. The officers of the project consistently evaluate the study material, _inter alia_ by discussions with the teachers who use the material. Large-scale alterations are made in the material on the strength of this evaluation. In addition to the semi-programmed "work-books", booklets with background data on the problems are written for the pupils and guides for the teachers.

CHAPTER 5

INNOVATION OF MATHEMATICS INSTRUCTION IN BELGIUM

5.1 SCHOOL SYSTEM

5.1.1 Administration

Regarding the manner of administration there are two kinds of schools in Belgium, namely Government schools (Rykskole) and Catholic schools. However there are separate departments (with separate ministers) of education for the French-speaking and Flemish-speaking sections of the population, so that there are actually four kinds of schools, namely:

(a) French government schools.
(b) French Catholic schools.
(c) Flemish government schools.
(d) Flemish Catholic schools.

(There are also a small number of so-called "congregational" and "provincial" schools which are controlled in a somewhat different way from the other schools.)

The differences with regard to control between the Government schools and Catholic schools basically amount to the Government schools being completely controlled by the two departments of education, whereas the church has a large degree of authority over the Catholic schools. One implication of the various ways of control, is that each of the forementioned four kinds of schools has different syllabuses for subject instruction. All syllabuses have to be approved by one or the other of the two ministers of education. There are thus four different Mathematics syllabuses in use in Belgian schools.

5.1.2 The school system

Primary education continues for 6 years and makes provision for pupils from 6 to 12 years. Secondary education continues for another 6 years, divided into lower and higher sections of three years each. In the lower secondary section all pupils follow the same Mathematics course (4 periods of 50 minutes each per week). In the higher secondary section Mathematics is still compulsory, although pupils may follow one of three courses, namely:

(a) A Mathematics course of seven periods per week, followed mainly by pupils who wish to study in Mathematics or Engineering after leaving school.

(b) A Mathematics course of five periods per week, followed mainly by pupils who wish to study in the biological scien= ces, including medicine, after leaving school.

(c) A Mathematics course of three periods per week.

In addition to the ordinary secondary schools, there are also the so-called normal schools where nursery school and primary school teachers are trained from the age of 12, that is, after completion of their primary education. Normal schools do not use the same syllabuses as ordinary secondary schools.

5.1.3 Examining

There is no external examination in Belgium. Each school con= ducts its own final examination. Question papers and marked examination papers are controlled by the ministries of education on a test sample basis. A comparable standard is maintained at various schools mainly because the popularity of schools is affected, inter alia, by the post-school academic achievements of ex-pupils.

5.2 INNOVATION IN THE INSTRUCTION OF MATHEMATICS IN BELGIUM

Innovation in the instruction of Mathematics in Belgium was ini= tiated by the introduction of an experimental "modernised" Mathe= matics syllabus at four Belgian normal schools for nursery school teachers in 1958 and 1959. This syllabus, used for 12 to 16 year old girls, included inter alia, elementary set theory, Venn diagrams, relations and elementary topology. In July 1959 a study day for Mathematics teachers was organised in Aarlen, on which occasion Prof. George Papy convinced those present of the necessity and practicability of modernising Mathematics instruc= tion. Vermandel (22) in 1966 drew attention to the fact that the further progress of innovation in Belgium would indicate that it is essential not only to prescribe new subject-matter for schools, but also to give detailed methodological instruc= tions to teachers. He also drew attention to the fact that numerous visits by Mathematics teachers to the experimental classes at the forementioned normal schools contributed towards promoting willingness among Belgian Mathematics teachers to accept innovation.

Between 1958 and 1961 a large number of re-orientation courses for Mathematics teachers were organised by the ministry for national education.

In 1961 on the strength of the insistence of Papy, an experimental syllabus for the first class of the secondary school was approved by the ministry and implemented in twenty trial classes throughout Belgium. In order to provide assistance to the teachers who would be responsible for this experimental instruction, the Belgian Centre for the Methodology of Mathematics Instruction (BCMW) was also established in 1961. This organization is discussed in more detail in the following paragraph. Since 1961-1962 15 work-groups for modern Mathematics have been established by the BCMW throughout Belgium, with the aim of re-orientating Mathematics teachers with regard to modern Mathematics.

In the years 1962-1964 the above-mentioned experiment was extended to the second, third and fourth classes of the secondary school. New Mathematics syllabuses for the first three classes of the secondary school were compiled in 1963-1964 by a commission consisting of five university professors, three Mathematics inspectors, four principals of schools (mathematicians) and twelve Mathematics teachers, selected from those teachers who had already implemented the experimental syllabuses and who could therefore assess the new subject-matter from the classroom situation.

Between 1964 and 1968 experiments were continued up to the highest class of the secondary school, and in 1968 new Mathematics syllabuses were approved for the secondary school for Catholic as well as Government schools and they have subsequently been implemented. Since 1964 inspectors of Catholic as well as Government schools have instructed some of the teachers who took part in the experimental instruction programme to start compiling textbooks for the intended new syllabuses on the strength of their practical experience in the experimental syllabuses. Such textbooks have been available since 1968.

It is noticeable that in Belgium the method of work with regard to innovation was based, from the beginning, on the fact that serving teachers, by means of "experimental" instruction, actively take part in the planning of innovation. The Belgian method of work is further characterised by the fact that provision is made at an early stage for an organization with full-time staff and the specific task to provide assistance and information to the teachers who do the pioneering (the BCMW). Vermandel (1966) has the following to say in this connection (22, p. 244):

"To accomplish a definite reformation, however, it is not sufficient to have good motives, but particular, necessary con= ditions have to be met. A first and most important condition is undoubtedly the voluntary decision on the part of the teach= ers to familiarise themselves, mathematically as well as didac= tically, with the new ideas. A second condition which is closely bound up with the first, is the provision of thorough scientific and pedagogical information. A third condition is that the responsible authorities at appropriate times should support the initiative and take up a stand.

The fact that the general modernising of Mathematics instruction in Belgium is well on its way to becoming a success, is due to the fact that the foregoing conditions have been amply satisfied" (translation).

Since 1967 innovation in Mathematics instruction at primary school level has been approached in a similar manner as for secondary education. At present (1972) experimental instruction is proceeding at a number of primary and nursery schools.

5.3 THE BELGIAN CENTRE FOR THE METHODOLOGY OF THE TEACHING OF MATHEMATICS (BCMW)

The BCMW was established in 1961 and the following tasks were entrusted to this organization:

(a) Identification and description of which Mathematics subject contents form the basis of Mathematics.

(b) Research concerning psycho-affective factors in the learning of Mathematics.

(c) Grounding of Mathematics didactics.

(d) Compiling Mathematics tests.

(e) Composition of Mathematics textbooks for pupils and teachers.

(f) Research concerning aids such as films in the instruction of Mathematics.

(g) Interpretation of existing Mathematics syllabuses in the light of contemporary science and techniques.

(h) Formulation of draft syllabuses.

(i) Organization and co-ordination of experiments with regard to the instruction of modern Mathematics.

The BCMW employs full-time researchers, namely Mathematics teachers seconded to the centre for one or more years. The BCMW also has part-time staff, mostly university lecturers in Mathematics, and full-time clerks.

With the passing of time the re-orientation of Mathematics teachers was also entrusted to the BCMW and monthly meetings of Mathematics teachers are held for this purpose. The BCMW also trains leaders for the work groups of Mathematics teachers. Before the implementation of new syllabuses in secondary schools throughout the country in 1968, numerous visits were arranged for teachers to schools where experimental syllabuses were being implemented.

The BCMW is financed by the ministries of education and science and a controlling council is nominated for the centre by the ministries. In addition to the controlling council, which is mainly concerned with financing and staffing of the centre, there is also a BCMW syllabus commission on which Mathematics teachers, principals, inspectors, university mathematicians, technologists and natural scientists serve. This commission provides guidance to the BCMW with regard to the composition of draft syllabuses.

CHAPTER 6

RECENT TRENDS REGARDING INNOVATION OF THE INSTRUCTION OF MATHE= MATICS

6.1 INTRODUCTORY REMARKS

As indicated in a previous report (7), innovation of Mathematics instruction in West European countries was ushered in by a series of international congresses which have been held since 1959. In 1973 Steiner (20) draws attention to the fact that this innova= tion was mainly motivated by the realization that the West is lagging in the technological field after the "Sputnik shock" of 1957, and the realization (in West European countries) that adequate provision of technologically trained manpower is a pre= requisite for material prosperity.

The initial fundamental motive for innovation of Mathematics instruction was modernization of Mathematics syllabuses in accordance with contemporary progress in the mathematical field (14, 15). This progress particularly entailed that the formal-conventional characteristics of mathematical systems were iden= tified and described on the basis of abstract concepts such as sets, relation, group, metrics, et cetera. Since these con= cepts are applicable to practically all sections of Mathematics and also indicate its essentials, they add particular perspective to Mathematics as a whole and make possible a particular economy in the explication of Mathematics contents. On the strength of this the didactical hypothesis that inclusion of these abstract concepts in school syllabuses would make a particular contribu= tion towards more effective instruction, developed (3). Thom (1972) puts it in this way: "In particular, it is argued that the introduction into teaching of the great mathematical 'struc= tures' will, in a natural way, simplify this teaching, for, by so doing, one offers the universal schemata which govern mathe= matical thought" (21, p. 195). The development psychological conclusions of Piaget served as further motivation for the inclusion of "modern Algebra" in school syllabuses. These conclusions comprised, <u>inter alia,</u> the fact that the abstract concepts of modern Mathematics are analogous to "thought struc= tures" which appear spontaneously in the child's development. Piaget summarises his point of view as follows in a paper read at the ICME congress at Exeter: "We believe ..... that there exists, as a function of the development of intelligence as a whole, a spontaneous and gradual construction of elementary logico-mathematical structures and that these 'natural' ...... structures are much closer to those being used in 'modern' mathematics than to those being used in traditional mathematics" (18, p. 79).

Innovation of Mathematics instruction initially (in the sixties) comprised mainly the introduction at school level of the above-mentioned abstract concepts, as appears from the syllabuses in= cluded in the appendix. In some countries two further altera= tions were gradually made in traditional syllabuses, namely the introduction of Linear Algebra and Transformation Geometry, the latter as a replacement for Euclidean Geometry, which, in the light of contemporary mathematical insight, is not such a good example of axiomatic-deductive arrangement. The forementioned alterations were introduced not only at secondary school level, but also at primary school level (as in South Africa since 1966).

6.2 "CONTRA-INNOVATION"

The above-mentioned "modernization" of syllabus content was not supported to the same degree by all Mathematics didacticians, and during the late sixties different trends with regard to syllabus composition gradually gained in popularity. Mathema= tics didacticians began to realise that Mathematics instruction could not be regarded as a detached activity of which the cri= teria of success are situated only in the acquisition of subject knowledge. Persons such as Kline (9) (USA), and in recent years Freudenthal (5), Lighthill (11), Christiansen (2), Steiner (20) and Bishop (1), to mention a few, emphasised the fact that it is pedagogically meaningless to instruct Mathematics at school level as an isolated subject, and that Mathematics instruction can only contribute effectively towards the education of children if the interdependence between Mathematics and the total reality of life (including other subjects) at school, is raised. This "relationship" of Mathematics with the reality of life, according to the forementioned authors, is not only a condition for the realization of the pedagogical sense of Mathematics instruction, but also, as motivation for learning, a condition for the effecti= vity of the instruction. Ormell writes as follows in this con= nection (1972): "As in all teaching, motivation is the key factor. If we can get this right there is some hope of getting the system to work in a fruitful way for student, teacher, tertiary lecturer/ teacher and employer alike. The most natural way to create motivation is to let the subject speak for itself: to show what it <u>does</u>, or can do" (16, p. 127). As already indicated in Chapter 4, the above-mentioned assessment is at present being implemented in the Sixth Form Mathematics Project by introducing a new theme every time with practical applications and treating the abstract mathematical theory afterwards (and not the other way round as in the majority of South African Mathematics text=

books). Lighthill, present president of ICME, in his presi= dential address at the 1972 ICME congress, inter alia, said the following: "My personal emphasis in this message is on those aspects of mathematical education that are concerned with communicating a working knowledge of how mathematics interacts with other subjects and with the external world; in one word, a knowledge of how mathematics is applied. .... more abstract curricula contain many attractive features; in particular, they may often succeed in imparting an enthusiastic appreciation of the beauty of mathematical structures and mathematical deductions. Many young minds show a keen response to that beauty, and some of you may regard an educator as myself as doomed to failure because in place of beauty all I could offer to those young minds would be utility; prosaic utility!

In reality, however, no such stark contrast is exhibited by the alternative of a curriculum based on integrated pure and applied mathematics. The values in such a curriculum involve inte= grated beauty and utility: they lie in a space of two dimensions, and this has certain educational advantages. The most obvious of these derives from the observation that a class may contain some pupils who can be induced to respond mainly to the beauty of mathematical ideas and arguments and some pupils whose interest can be aroused mainly from realisation of their utility ....

Whether for this or other reasons, the trend in modern mathema= tics teaching projects in Britain and some other countries has been to give continual illustrations of how the Mathematics taught can be applied. They bring in constantly the concrete example, and are particularly concerned to emphasise the variety of uses and applications of mathematics.

This personal message of mine is concentrated, however, on a slightly different educational goal. It says: let us go beyond mere use of the concrete example as an aid to understand= ing or of reference to utility as an aid to widening the circle of those in whom interest is aroused. There is a still more important prize to be won: a prize concerned with a deeper integration of mathematics into the total education of the individual.

I want to suggest that educators may have most benefited their pupils when they have succeeded in giving a feel for what is involved in the process of applying mathematics. This is the process of building a bridge between the abstract ideas and inferences of mathematics and the concrete problems arising in some field of application. It seems to be increasingly recog= nised that there may be more skill, more art, in that bridge-building process than in the associated mathematical problem-solving" (11, p. 94).

In his address "What groups mean in mathematics and what they should mean in mathematical education", Freudenthal draws atten= tion to a number of examples of authentic applications of the group concept in complex numbers, analytical geometry, proba= bility, celestial mechanics and electromagnetic fields. He (Freudenthal is a world-famous group theorist) comes to the conclusion that "Groups are important because they arise from structures as systems of automorphisms of those structures" (4, p. 109), and on the strength of the examples which he gives, it must be deduced that with "structures" he means the same as Ormell (see Paragraph 4.5.2) means with "situations". Referring to the earlier trends in innovation (see Paragraph 6.1), Freu= denthal says: "Groups as taught, or proposed by curriculum designers to be taught, at school, are a different thing. They usually begin with the 2-cyclic or the Klein group ...." (4, p. 111). He then describes the well-known instruction procedure of subjecting a geometrical figure to certain trans= formations and then to establish empirically that the transfor= mations form a group. "In itself this procedure is sound; what is wrong with it, is that by this prelude the stage is set for generalisations which are wrong - mathematically and pedagogic= ally. And so it continues. New groups are introduced, of 6, 8, 12, 24 elements - all of them try summing up its elements, one by one, usually as mappings of different kinds, with the stress on constructing group tables .... Group tables and group diagrams are devices to make groups explicit or to visua= lize them, but they are utterly inefficient tools to introduce groups or to prove that some system is a group.

It is true that in such school group theory things are finally put straight if one lands in the safe harbour of algorithmics. It is algorithmics in a finite set of at most twenty-six letters each of which means a constant group element. It is a particu= larly dangerous kind of algorithmics because if it comes early it may frustrate the interpretation of letters as symbols to denote variables - a dangerous tendency which is today very strongly felt in set theory at school.

Are there valid arguments to teach a school group theory, different from the genuine one? I would doubt it ......

We have to be careful and honest if we want to adapt some piece of high mathematics to a lower level. Simplifying is a good thing but wrong elementarisations are a danger, and so is imitating superficial features while destroying the great ideas of some mathematical theory. If children are taught groups they are entitled to learn genuine group theory rather than a childish version. In the past, mathematics has seriously suf= fered under the falsifying tendencies in adaptations of mathe= matical subject matter to school level. Let us be more cautious in the future. Honesty is a cardinal virtue in education. Nothing is lost if some subject matter cannot be taught prema= turely and much is gained if it can in an honest way" (4, p. 111).

It should be emphasised that Freudenthal's insistence on "genuine" group theory does not imply that he advocates emphasising the abstract-structural characteristics of Mathematics at school. The specimens of groups which he mentions (see previous para= graphs) point to emphasising the relations between Mathematics and the reality outside Mathematics. Elsewhere (5, p. 74) Freudenthal emphasises his didactical supposition as follows: "In principle it is a healthy idea not to teach isolated pieces but coherent material. Connected matter is faster learned and longer retained. But there is more than one kind of connection. There is one kind which is understood by the teacher, and another which is only understood by the textbook author. Both of them are of little use but unfortunately most connections constructed in a logically coherent school programme are of this kind. They are connections <u>within</u> mathematics, constructed in order to teach a unified mathematics, which is constructed at the expense of the outside connections of mathematics which are possibly more natural and more important . ... While I do not urge that the pupil learns applied mathematics, I do wish that he learns how to apply mathematics. This does not mean utilita= rianism. Therefore instead of applied mathematics, I would prefer to speak of <u>multirelated</u> mathematics. .... To teach connected mathematics it is not wise to start looking for direct connections (here Freudenthal differs from Ormell and the approach of the Sixth Form Mathematics Project), they should rather be found between the contact points where mathematics is attached to the lived-through reality of the learner. Reality is the framework to which mathematics attaches itself, and though these are initially seemingly unrelated elements of mathematics, in due process of maturation connections will develop. Let the mathe= maticians enjoy the free-wheeling system of mathematics - for the non-mathematician the relations with the lived-through reality are incomparably more momentous." As third example of a dis= tinguished contemporary mathematician who advocates "contra-innovation", a number of quotations are given from the address of Thom to the ICME Congress (21). In the first instance he draws attention to the fact that the modernization of syllabuses which has taken place in most European countries, is inconsistent with the aim of inducing didactical innovation. He calls atten=

tion to the fact that some didacticians use the following argument: "For pedagogical reform to succeed, one must overcome inertia, the routine of teachers; with this object, one must change syllabuses. In changing the content, one will more easily be able to change the methods." Thom continues: "This tactical argument only has validity if it becomes evident that the new materials introduced into teaching definitely encourage a constructive, heuristic approach. Now it happens that the reformers (at least those of Continental Europe) have been induced, by their philosophical bias, on the one hand to abandon that terrain which is an ideal apprenticeship for investigation, that inexhaustible mine of exercises, Euclidean geometry, and on the other hand, to substitute for it the generalities of sets and logic, that is to say, material which is as poor, empty and discouraging to intuition as can be" (21, p. 197).

Thom continues to reject the didactical hypothesis (see also previous paragraph) "......... - structures of sets and logic, algebraic structures, topological structures - teaching the child, at an early enough age, the definition and the use of these structures would suffice to give him easy access to con= temporary mathematical theories." He calls attention to the fact that "...... the standard structures of mathematics ..... only represent its most superficial aspects", and also "one knows that any hope of giving mathematics a rigorously formal basis was irreparably shattered by Gödel's theorem. However, it does not seem as if mathematicians suffer greatly in their professional activities from this. Why? Because in practice, a mathematician's thought is never a formalised one. The mathe= matician gives a meaning to every proposition, one which allows him to forget the formal statement of this proposition within any existing formalised theory (the meaning confers on the pro= position an ontological status independent of all formalism). ...... Thus the emphasis placed by modernists on axiomatics is not only a pedagogical aberration (which is obvious enough) but also a truly mathematical one .... One has ...... very pro= bably overestimated the importance of rigour in mathematics. Of all the scientific disciplines, mathematics is the one where rigour is a priori least necessary" (21, p. 203).

The foregoing quotations are probably adequate to indicate that West European mathematicians and Mathematics didacticians at present very much doubt the didactical-pedagogical accountability and effectivity of the initial attempts at innovation (which gave direct cause to the present (new) South African Mathematics syllabuses for the primary and secondary school). It is there= fore also noticeable (see Chapters 2 and 4) that more recent attempts at innovation such as the Sixth Form Mathematics Project, the Mathematics for the Majority project and the projects under= taken by the IOWO Institute in the Netherlands, break away radi= cally from mathematical modernization which has become tradi= tional. In the following paragraph a few wider indications are given of the syllabus tendencies which characterise "contra-innovation".

6.3 PRESENT TRENDS IN RESPECT OF SYLLABUS COMPOSITION

Although few Mathematics didacticians at present risk formu= lating explicit criteria of subject-matter selection, it is obvious that the suitability of learning contents as examples of the methods of work and significance of Mathematics in the wider sense of the word ("multirelated mathematics"); and the learning readiness of children at various school levels, as empirically evaluated (see Paragraph 7.2), serve as important implicit criteria.

Most West European syllabuses are still at the stage of "first phase" innovation (see Paragraph 6.1), but a few contextual con= sequences of the "contra-innovation" are already crystallising. In the first place the abstract concepts of the "first phase" innovation are now placed in perspective, and are seen rather as possible terminal points of Mathematics instruction at school level, as the outcome of the pupil's attempts to organise the mathematical knowledge which he has (5). (In the "first phase" the concepts were often regarded as starting-points for the instruction.) Secondly, "applied" part-disciplines of Mathe= matics such as Statistics, Calculation of Probability and Computer Science are regarded as important sources of suitable learning contents for the school (5, p. 280).

Thirdly, as in some "first phase" innovation attempts, Trans= formation Geometry is regarded as a suitable alternative for Euclidean Geometry.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 ORGANIZATION OF INNOVATION: TEACHER PARTICIPATION

It was immediately noticeable that serving teachers in Western European countries are involved in innovation projects to a particular extent. In a country such as West Germany, where it is not the case yet, it is definitely planned for the future (see Chapter 3). Persons involved with innovation and with whom interviews were conducted, without exception agree expli= citly that the active, productive participation of serving teachers in the designing of new syllabuses, is an essential condition for effective syllabus innovation. It is also notice= able that innovation projects in which serving teachers had been involved from the beginning, until now have apparently been more successful than others.

In West Germany and Sweden, for example, a large percentage of Mathematics teachers at secondary school level are still in favour of "traditional" subject-matter and methods of work, whereas in Belgium, the Netherlands and Britain considerably more enthusiasm for and implementation of "innovation programmes" can be detected. (The foregoing was conspicuous during visits to several schools in each of the countries mentioned.)

In a report on an international congress on curriculum develop= ment held in 1967, Maclure writes as follows: "In the United Kingdom much innovation has been teacher-inspired. The School Mathematics project began with a small group of public schools who were mainly interested in bringing the syllabuses up to date; the Midlands Mathematical Experiment began with the co-operation of the head of a technical school and a lecturer in a college of education; the Shropshire Mathematical Experiment began with a group of teachers in the county, and St Dunstan's and Abbey Wood were two individual schools who compiled their own mathematics syllabuses", and further:

"In the United Kingdom teachers met to organize projects and write texts and they then met other teachers at conferences and on courses in order to discuss the work involved; the Associa= tion of Teachers of Mathematics and the Mathematical Association organized meetings at national and local levels for less specific exchange of ideas and information; the Schools Council/Nuffield project for children between the ages of 5 and 13 was engaged in preparing guides to its material for certain pilot areas, and a condition for joining the scheme was that a teachers' centre should be established; many centres which originated as mathe= matics centres have become more general than this, but the setting up of some more specifically mathematical centres was the result of a report on in-service training issued in 1965 by the Joint Mathematics Council" (12, p. 64).

On the occasion of the above-mentioned conference, Mr Joslyn Owen, secretary of the Schools Council, had the following to say on the manner of syllabus innovation in Britain:

"Only by being able to assess his own performance in the daily aspects of new work can we hope that any teacher will be able to move towards the connection between the particular and the general - between his discrete effort at innovation in his own room and his renewal in the total role of educator. This <u>has</u> to be the aim of our strategy, to improve the child's educa= tional experience but at the same time and in terms of the future more important, to connect particular innovation with the process of professional self-renewal. If we claim that our strategy in Britain works towards meeting these needs, we do it, I repeat, in a framework of partnership: the universi= ties, the researchers, the mammoth projects - all have their part to play, but if there <u>is</u> dominance in a system which, we claim, has no hierarchy, it <u>is</u> that of the serving teacher. It is the teachers - men and women on the leading edge, respected by other teachers ..... who are the majority of the School Council's committees, who are the sifters and improvers; it is the teachers who pass realistic judgment on our programmes and projects. It is serving teachers who form, on secondment, our project teams. Away from the Council, it is they, too who experiment with, and appraise, what issues from our projects.

Our projects are, we think, manned by brilliant and diligent teachers ..... People who have done difficult and responsible jobs in their own schools and colleges and who want to get back to them ...... One example: in the difficult area of curriculum which is concerned with those in secondary schools who don't want to stay, who don't want to learn, and who seem, in every particu= lar, to be allergic to education, the most impossible subjects seem to be the humanities. To find out what, if anything, could be done in Britain, the Schools Council asked three of its staff, with a working party sponsored by the Nuffield Foun= dation, to find out what was happening in schools, what was good, ... less good, what seemed promising and what seemed showy. In one year a very large number of teachers showed what they were doing and expressed their hopes and desires and dissatisfactions: ideas were sifted and compared, practices were noted, objectives were analysed and a very practical judgment was made of what was and what might be.

All this at the end of the year was well written up, published and welcomed. Teachers recognized the description as being one about the real world and they could therefore accept the assumptions, cock-shies and proposals which were included, too.

Then, the Council and the Foundation jointly set up a project - a director with a team of six to work for three years in devising materials which would catch such motivations as do exist. ..... the team has to provide materials which will achieve what the teachers who had been probed thought should be achieved.

This project is now at work, like some dozen others on other subjects, producing trial materials which a very small number of schools will test immediately: the material will be worked through, attacked and returned: it will be revised and tested again, with a larger number of schools: these again will .... feed their comments and suggestions back .... Further revision, and a still wider trial (and) in three years' time we hope, with some confidence, that we shall have some very good materials.

But a devoted team and an inspired director achieve very little by themselves: it is the appraisal and trial which count for most."

With reference to the "teachers' centres" Mr Owen said:

"These are the strength, so we think, of our system of curri= culum development: very local, very accessible centres where teachers ...... can meet, regularly and informally, to test, to display, to devise and to discuss their own work and the work of others. If we are having a curriculum ...... this is how we hope to achieve it" (17, pp. 15-17).

The report of the work group on Mathematics in developing countries of the 1972 ICME conference is illuminating in this connection, as appears from the following quotation from it: "In many countries the educational system is so constructed that teacher-training, curriculum development and classroom practice are three separate activities. We are coming to realize that change can only be effective when the three are seen as aspects of a single process. It is one thing to pre= scribe a new series of mathematical textbooks for classroom use, but quite another to ensure that the mathematical education of the pupils is thereby improved. It is naïve to define curri= culum as the content of a textbook, or its development as the introduction of a new one. It is both more realistic and more constructive to define curriculum as what actually takes place in the classroom. This immediately gives the teacher a key role in curriculum innovation. Many developing countries are already acting on this principle, by giving teachers a major share of responsibility for developing new teaching materials, and by recognising their crucial role in the evaluation of them in the classroom" (6, p. 68).

There seems to be no reason to doubt the validity of the fore= going verdicts. However, it is necessary to remark that in all cases (within the scope of this investigation) where actively productive teacher participation in syllabus development has been perceived, this participation was based on the active allocation of responsibility to teachers, as well as establishing the neces= sary facilities (freedom to follow "experimental" syllabuses, teachers' centres). In a publication of the Schools Council the following is written in this connection: "Only teachers can change this curriculum. But there are limits to what they can achieve without support. There are some things which they cannot do for themselves - things which are not part of curri= culum development as such, but which determine the pace, the effectiveness and even the possibility of development" (19). In this publication, implicitly as recommendations to local education authorities, attention is drawn to the following con= ditions for effective syllabus development:

(a) Adequate physical accommodation at schools.
(b) Co-operation concerning syllabus development between heads of schools and teachers.
(c) Adequate training of teachers.
(d) Availability of adequate funds, and the freedom of teachers to apply these funds where and when necessary.
(e) Provision of travelling expenses and time to enable teachers to have regular discussions with other teachers engaged in syllabus development.

7.2 ORGANIZATION OF INNOVATION: PRACTICAL EVALUATION OF ALTERATIONS IN SYLLABUS

One of the most striking characteristics of present Western European innovation strategies is the consistent preservation of the principle that proposed alterations in syllabuses should first be evaluated on a restricted experimental scale in schools before they are finally implemented (see quotations in previous paragraph as well as the expositions in Chapters 2 to 5). Such evaluation, as appears from the previous paragraph, is primarily regarded as the task of serving teachers. In a country such as England with decentralised control over examining (see Chapter 4), from the nature of the case it is easier to evaluate experimental syllabuses practically, since in any case there is a variety of different final examinations for every subject, and the presen= tation of a special final examination for pupils who follow experimental syllabuses, creates no problem. In countries with centralised control over examinations, such as the Netherlands, it was necessary to exempt schools in which experimental sylla= buses were followed from the normal final examinations.

It is important to note that the implementation of experimental syllabuses in schools is regarded at present in the Netherlands, Belgium and Britain (and according to present planning, in the future also in West Germany) as an integral part of syllabus development, and not merely as evaluation of "completed" sylla= buses. The normal procedure in the forementioned countries is that experimental syllabuses are consistently being changed on the strength of the comment of the teachers who implement them, until the teachers are of the opinion that further amendments are unnecessary.

7.3 ORGANIZATION OF INNOVATION: FULL-TIME STAFF FOR THE PLANNING OF INNOVATION

The above-mentioned aspect of the organization of innovation was explicitly treated in the previous chapters and it appeared in all the countries which were visited, that provision is made (or such provision is planned) for teachers to be seconded full-time (or appointed permanently) to organizations where new syllabuses are planned. In the light of the degree of detail with which innovation is tackled in Western European countries (production of practically evaluated textbooks in contrast with the mere alteration of prescribed syllabuses), it is obvious that this task cannot be undertaken by persons who are involved on a part-time basis only. It is noticeable, however, that these full-time "innovation planners" in most cases are seconded teachers who return to the classroom after a number of years.

Provision for full-time innovation planners has not been made in all cases since the start of innovation attempts in Western Europe. In the Netherlands, for example, such full-time officers have only been appointed since 1971. From this it should be deduced that where initially no provision was made for such staff, it was gradually realised that it is an essential prerequisite for effective innovation.

In Belgium, the Netherlands and Britain, where institutional provision is made for full-time innovation planners (see previous chapters), it is noticeable that the persons appointed as planners, for all practical purposes, have the final say in the composi= tion of new syllabuses.

7.4 ORGANIZATION OF INNOVATION: SYNOPSIS

Broadly speaking, the experience gained by education administra= tors (in the countries with which this report deals) in innova= tion of Mathematics instruction, indicates that a mere announce= ment of new syllabuses, however well-considered the changes may be, is not an effective manner of innovation. During personal interviews persons such as members of the Schools Council, the directors of the IOWO Institute (the Netherlands) and the BCMW (Belgium), a member of the Deutsche Bildungsrat and school inspectors, heads of schools and teachers in the various coun= tries, throughout stated, without mincing matters, that the method of work mentioned above has especially two defects, namely -

(a) that adequate re-orientation of teachers is simply not possible unless teachers contribute productively to the planning of innovation, and are therefore unable to imple= ment new syllabuses and didactical directives effectively, and

(b) that it is not possible to introduce effective textbooks for new syllabuses without thorough practical evaluation (and alteration on the strength thereof) by a variety of teachers in different schools.

The present Western European approach to innovation of Mathe= matics instruction is therefore that teachers' participation and practical evaluation, supported by the services of full-time planners, are the only really effective manner of innova= tion. There appear to be no reasons to doubt the correctness of this approach.

7.5 RECOMMENDATIONS

In the light of the foregoing expositions, and on the strength of the fact that Mathematics instruction in the Republic of South Africa is at present still virtually predominantly at the stage described in Chapter 6 as first-phase innovation, it is recom= mended -

(a) that in their planning, authorities make provision that the principle of consistent revision of Mathematics sylla= buses on the strength of research be maintained,

(b) that the Joint Matriculation Board make provision for certain schools to be exempted from the final examinations based on core syllabuses in order to make effective evalua= tion of possible syllabus alterations possible,

(c) that, where at all possible, substantial syllabus altera= tions in future be implemented after they have been evalua= ted on an experimental basis in practice,

(d) that for the evaluation of such syllabus alterations, full-time researchers be appointed, and serving Mathematics teachers be involved.

7.6 CONCLUDING REMARK

It is hoped that this report provides a picture of Mathematics instruction and its innovation in a number of overseas countries, and that a contribution has been made with regard to the advance= ment of Mathematics instruction in the Republic of South Africa.

APPENDIX

A NUMBER OF WESTERN EUROPEAN MATHEMATICS SYLLABUSES

1 THE NETHERLANDS

Examinations have been conducted since 1972 with reference to the following syllabuses.

1.1 GENERAL CONTINUED EDUCATION

(a) MAVO-3 (Three years' secondary education)

Transformation geometry: linear reflection, point reflection, translation, rotation and multiplication.

Congruence and similarity of figures. (According to CMLW advice these themes should be treated with reference to transformations.)

Lengths, area and content and the Theorem of Pythagoras.

Sets of points in the plane (i.e. geometrical figures described as sets of points).

Linear equalities and inequalities with one variable.

Relations: the graph of linear relations; two linear equalities with two variables.

Functions; the graph of a function; linear and simple quadratic functions; simple quadratic equalities.

The goniometric ratios sine, cosine and tangent.

Simple calculations of angles and distances in the plane and in the space.

Simple descriptive statistics.

(b) MAVO-4 (Four years secondary education)

As for MAVO-3, with the following added to it:

Linear inequalities with two variables.

Quadratic equalities and inequalities.

Tangents on these curves.

Determination of the intersections of straight lines and curves.

Coaxial lines and circles.

Sets of points.

1.3 VWO (SINCE 1974)

Paper 1

As in 1.2 above without the option of omitting certain themes and with addition of an introduction to the theory of probabi= lity and mathematical statistics.

Paper 2

In the plane: equations of the line and the circle; inter= sections and tangents; vectors; vector representation of a line; inner product; angles and distances.

In the space: equations of the plane and ball; vectors; vector representations of the line and plane; inner product; angles and distances.

Sets of points and simple sets of lines.

Dependence and independence of a system of vectors; determi= nants of the second and third order; situation of lines and planes in respect of one another.

Orthonormal basis; normal equation.

Circle and ball; tangent and tangent plane.

The group of translations.

Linear mappings; matrices; the regular linear mapping as a group; orthogonal mappings; isometrics.

One of the following themes: complex numbers, topology, number theory, numerical mathematics, projective geometry, non-Euclidean geometry, logic, history of mathematics, applications of analysis, or another theme approved by the inspectorate.

2 WEST GERMANY. SKK CORE SYLLABUS - 1968

Forms 1 to 4 (6 to 9 year-old pupils)

Natural numbers as cardinal numbers (whole numbers).

The number 0.

Number symbols (digits).

Illustration of numbers.

Place value.

The fundamental operations, addition and multiplication with their converses.

The equal sign.

Arrangement of natural numbers. The signs $<$ and $>$.

The natural numbers to 1 000 000.

Computations.

Use of brackets.

Simple equalities and inequalities (no transformations).

True and false sentences.

Solution sets.

Weight, mass, distance and time calculations and simple applications.

Fundamental geometrical concepts: cube and square, cuboid and rectangle, ball and circle (handling, construction and naming of these).

Forms 5 and 6 (10-11 year-old pupils)

Compound magnitudes: area of square and rectangle, volume of cube and cuboid.

Point, line, radius, straight line, parallel lines, angle, triangle, quadrilateral, cylinder, prism, cone.

Translation, rotation and reflection and the connection of these transformations (mappings) with triangle, quadrilateral and circle, cube, cuboid, cylinder, pyramid, cone and ball. Treatment on the basis of concrete examples (models).

Place value systems. The Roman number system may serve as example of a number system without local value.

Powers.

Base.

Various bases, especially 2 and 10.

Examples of computations in the binary system.

Divisibility and divisibility rules.

Divisibility as characteristic of numbers.

Particular divisibility rules as characteristics of number systems with particular bases.

Prime numbers and resolution into prime factors.

Greatest common divisor and least common multiple, in connection with intersection and union of sets.

Common fractions and system fractions (with comma and local value for example decimal fractions).

As different notations for fractions.

Equivalent fractions.

Simplification.

Basic operations applied to fractions.

Exercise in fraction arithmetic (common fractions).

System fractions, with particular reference to decimal and binary fractions.

Computation with decimal fractions.

Rounding and approximation.

Conversion of common and decimal fractions.

Arrangement of fraction numbers.

Natural numbers as subset of fraction numbers.

Fraction numbers as points on half line.

Simple equalities and inequalities.

<u>Forms 7 and 8 (12-13 year-old pupils): all groups</u>

The function as mapping and set of number pairs.

The functions $f: x \rightarrow ax$ $(\ (x;y)/y = ax)$
and $f: x \rightarrow \frac{b}{x}$ $(\ (x;y)/y = \frac{b}{x})$
and their graphs.

Use of the slide rule.

Percentage and interest.

The concurrent relation.

The arrangement relation.

Reflection, translation, rotation and simple symmetrical figures.

Properties of triangles and quadrangles (characteristics).

Simple constructions of triangles.

Specimen treatment of the connection between a theorem and the suppositions (axioms) on which it is based.

Angle and angular measure, with applications in the determi= nation of direction and place.

Theorems in connection with angles.

Mapping by "shearing".

Area of triangles and parallelograms.

Volume of the right prism.

Linear equations with one unknown.

Domain and solution set.

Forms 7 and 8: only gymnasium pupils

Mathematical (algebraic) statements and connection between mathematical statements. The connections "and" ($\wedge$) and "or" ($\vee$).

Connection between $\wedge$ and $\vee$ with diameter and union.

Transformation of mathematical statements to equivalent state= ments.

Linear inequalities in one unknown.

The linear function $x \rightarrow ax+b$ and its graph.

Linear equalities and inequalities in two unknowns.

Graphical representation.

Linear programming.

Connection between domain and solution set.

The concept "group", on the basis of known examples from geometry and arithmetic.

The integers as ring .

Introduction of negative numbers.

The rational numbers as field.

The arranged rational numbers as field.

Practising the ability to compute with rational numbers.

Forms 9 and 10 (14 and 15 year-old pupils): gymnasiums and Realschulen

The function $x \rightarrow ax^2+bx+c$ and its graph. The graph may be drawn from the graph of $x \rightarrow x^2$ by translation, reflection and "stretching".

The inverse function $x \rightarrow \sqrt{x}$ of $x \rightarrow x^2$ for $x \geq 0$.

The quadratic equation (only real solutions). Numerical and analytical methods of solution.

Conversion to linear factors.

The real numbers.

The theorem of Vieta.

Theorems in connection with the rectangular triangle and circle.

Multiplication of vectors with numbers (only for gymnasium).

Powers with rational exponents.

The relation $x \rightarrow ax^2$.

Principles of exponents.

Power function.

Exponent function.

Logarithmic function.

Calculation of function values (slide rule).

Practising skill in using slide rule and tables.

Area and circumference of a circle.

Preparation on the limit concept.

Circular segments.

Area and volume of the pyramid, cylinder, cone and ball.

Perpendicular and orthogonal projections.

The sine, cosine and tangent functions.

Calculations with regard to triangles.

Scalar product (only gymnasium).

Forms 11–13 (16 to 19 year-old pupils): only gymnasiums

All gymnasium pupils

Characteristics of the real numbers, arrangement and closeness.

Limits, continuity.

Functions, Progressions, Inverse functions, Intervalue theorem.

Continuous functions.

Maximum and minimum values in closed intervals.

Differential calculus, differentiability.

Integral calculus.

Integral, primitive function, principal theorem of the differential and integral calculi.

Applications of the differential and integral calculus.

Vector space, Point space.

Linear independence, basis, co-ordinates, models of vector spaces.

Linear forms in the plane and the space.

Scalar product.

Analytical geometry of the plane and the space with the aid of vectors.

Congruence relation.

Relation of similitude.

Affine relation.

(All three the above-mentioned in connection with the concept "group".)

Cone sections, with particular reference to the connection between the cone sections.

Only Mathematics-General Science gymnasium

Projective mapping (projection relation).

Example of a non-linear mapping.

Matrices (in connection with the above).

Domains and fields: example of the extension of number systems.

Integers, rationals, real and complex numbers.

Cosets.

The Gaussian plane.

Boolean algebra (models, theory of sets, logic, circuits).
or
Example of a finite or non-Euclidean geometry.

Theory of probability.

Statistics.

Modern mathematical techniques.

3 BRITAIN

The syllabuses of the University of London, as applicable for examination purposes from June 1974, are given as examples.

3.1 ORDINARY LEVEL

In Mathematics at Ordinary level there are two syllabuses, Syllabus C and Syllabus D.

Mathematical formulae will be provided for use of candidates in all papers except multiple-choice tests.

The examination will consist of two papers each carrying one half of the maximum mark.

Paper 1

This will be a multiple-choice test of $1\frac{1}{2}$ hours.

Paper 2

This will be of $2\frac{1}{2}$ hours and will consist of twelve questions of which candidates will be required to answer seven.

Four-figure mathematical tables will be provided by the University. The use of these tables, and of slide-rules, is permitted in all questions which do not specifically exclude them. In all such questions, three-figure accuracy will be sufficient. No questions will be set specifically on the use or manipulation of logarithms. Centres may, if they wish, provide acetate sheets or tracing paper for use by the candidates in this paper.

Introduction

When Syllabus C was introduced in 1968, it was decided to make the syllabus very full and comprehensive so as to give teachers a relatively free hand in choosing what they would teach. The papers gave a wide choice of question, and candidates were not expected to have covered the whole syllabus. It is now believed that teachers in general prefer a more precise syllabus, and it is possible that the introduction of the multiple-choice paper, in which there is no choice of questions, could place the candidates who do not cover the whole syllabus at a disadvantage. The syllabus has therefore been revised to permit a fuller teaching of a smaller number of topics, those topics which have proved unpopular being deleted together with others which were not altogether in keeping with the spirit of 'modern' mathematics. A few small additions have been made so that certain topics can be taught and examined more fully.

The format of Paper 2 has also been changed, and this, again, is in response to the introduction of the multiple-choice paper as Paper 1. Since Paper 1 now consists entirely of very short questions, it is thought that the examination as a whole will be better balanced if Paper 2 is made up of questions of the problem type, requiring deeper and more continuous thought. The previous two-section paper has therefore been replaced by a one-section paper consisting of longer questions of the problem type and containing a wide choice of questions.

Syllabus

The idea of a set, set language and notation. (Recommended symbols are given at the end of the syllabus.) Union, inter=section, complement, subset, empty set, universal set, number of elements in a set. Venn diagrams and their use in simple logical problems. Use of symbols to represent numbers and sets. Operation tables, closure; identity and inverse elements.

Natural numbers, integers, rationals, irrationals. Sets of numbers, sequences, prime numbers, factors, multiples, indices. Whole numbers and simple fractions in any base.

The ordinary processes of arithmetic. Approximations, estimates, significant figures, limits of accuracy. Numbers in 'standard form', i.e. $a \times 10^n$ where n is an integer and $1 \leqslant a < 10$.

Money, Weights, measures in British and metric units. Ratio, proportion and percentage.

Length, area, volume. Use of Pythagoras' theorem. Mensuration of the rectangle, parallelogram, triangle, circle, cylinder, cone, sphere. Length of an arc of a circle, area of a sector of a circle.

Manipulation of simple algebraic expressions; simple factor=isation. Use of the remainder theorem to find factors of alge=braic expressions. Use of the symbols $\Rightarrow, \Leftrightarrow$. Solution of simple equations, quadratic equations, linear simultaneous equations in two unknowns. Distinction between conditional equations and identities. Inequalities, linear and quadratic: their solution and the representation of the solution on the real number line. Linear programming in two variables. Formulae: construction, interpretation, simple manipulation, numerical applications.

Representation of data by a matrix. Addition, multiplication of two matrices. Multiplication of a matrix by a scalar. Unit matrix, null matrix, transpose of a matrix. Determinant, inverse of a non-singular 2 x 2 matrix. Use of 2 x 2 matrices as trans=formations. Combination of transformations. Invariant properties of simple rectilinear figures under these transformations. Application of matrices to the solution of simultaneous equations in two variables. Route matrices, incidence matrices.

Scalar and vector quantities. Representation of a vector by a column matrix$\binom{x}{y}$. Modulus of a vector $\binom{x}{y}$. Multiplication of a vector by a matrix. Representation of a vector by a directed line-segment. Sum and difference of two vectors. Expression of a vector in terms of two coplanar vectors. Use of the results (i) $a=b \Rightarrow a=b$ and a is parallel to b; (ii) $ka=hb \Rightarrow$ a is parallel to b or h=0 and k=0 in proving properties of equivalence, parallelism and incidence in rectilinear figures.

Geometry of Euclidean space based on the transformations of translation, rotation, reflection, enlargement. Similarity: areas and volumes of similar figures, scale and simple map problems. Symmetry about planes, lines and points. Symmetrical properties of circles, triangles, quadrilaterals. Angle and tangent properties of circles. Intersecting chords property of a circle for an internal or an external point. Simple loci.

Sines, cosines and tangents of angles from 0 to 360 degrees (0 to 2π radians) and their graphs. Solution of triangles. Solution of simple problems in two and three dimensions, by calculation and by drawing. Bearings.

Function as a one-one or as a many-one mapping or as a corres= pondence between elements of two sets. Domain and range. Com= posite functions, inverse functions. Use of symbols to repre= sent operators, transformations, functions, mappings. Repre= sentation of a point in a plane by an ordered pair (x, y). Line, ray, line segment. Rectangular Cartesian coordinates. Methods of expressing functions: mappings, tables, ordered pairs.

Relationships, especially linear, square, reciprocal, exponen= tial, (i.e. $y=ka^{bx}$) and their graphs. Gradients, significance of zero gradient. Area under a curve. Applications to kine= matics including time-distance and time-speed graphs and to other rates of change. (Candidates who can differentiate and integrate sums of integral powers of x, excluding the integra= tion of x^{-1}, may find opportunities to use their knowledge in certain questions: other candidates will be able to obtain full marks for these questions by the use of approximate methods based on drawing and measurement).

Statistics: graphical representation of numerical data including bar diagrams, circular diagrams ('pie' charts), histograms, cumulative frequency curves. Determination of the mean and median of a sample.

Simple ideas of probability with application of the product and sum rules to easy problems.

Note: The notation used will include the following:

$\{\ \}$	The set of
$n\{\ \}$	the number of elements in the set
$\{x: \quad \}$	the set of values of x such that
$\in$	is an element of
$\notin$	is not an element of
ϕ	the empty (null) set
$\mathcal{E}$	the universal set
$\cup$	union of
$\cap$	intersection of
$\subset$	is a subset of
A'	the complement of the set A
PQ	operation Q followed by operation P
$f: x \rightarrow y$	the function of mapping the set X into the set Y
$f(x)$	the image of x under the function f
f^{-1}	the inverse of the function f
fg	the function f of the function g
—○—○—	open interval on the number line
—●—●—	closed interval on the number line

SYLLABUS D

The examination will consist of three papers each carrying one-third of the maximum mark:

(a) Paper 1 - multiple-choice test of $1\frac{1}{4}$ hours.

(b) Papers 2 and 3 - two papers, each of two hours, each consisting of two sections, Sections A and B.

Section A will contain separate questions of an elementary type all of which may be attempted, and will carry about one-third of the maximum marks for the paper.

Section B will contain a choice of questions.

All papers may contain questions from any part of the syllabus and the solution of any question may require knowledge of more than one section of the syllabus.

Each paper will carry the same number of marks.

In papers 2 and 3 unless limitations are imposed in the questions a candidate may use

(a) any appropriate method,

(b) any drawing instruments,

(c) any of the four-figure mathematical tables provided by the University, or a slide rule if this gives the required degree of accuracy, but its use must be stated.

Syllabus

1. The ordinary processes of number manipulation.

The commoner systems of weights, measures and money.

The nautical mile and the knot.

Fractions, decimals, averages, ratio, proportion, percentages.

Use of four-figure tables.

Expressing answers to a given degree of accuracy, i.e. a given number of decimal places or significant figures.

Questions may involve the application of these processes to problems of everyday, personal, domestic or community life.

(Questions will not be set involving the 'long rules' for the determination of square roots, H.C.F. etc.)

2. The basic processes of algebra.

The symbolic expression of general results in arithmetic.

The interpretation, evaluation and easy manipulation of formulae.

The use of indices and the expression of numbers in the form $A10^n$ or $B10^{-n}$ where n is an integer.

Factorisation of simple expressions including use of factor theorem.

Manipulation of simple fractions.

Solution of equations of the first and second degree containing one unknown quantity.

Solution of simultaneous equations either both linear or one linear and one quadratic.

Use of simple arithmetic and geometric progressions.

Applications of algebra to the solution of problems.

3. Graphs form statistical data.

The idea of a function of a variable.

Translation into symbols of relations such as 'y is inversely proportional to x' or 'v varies as x^3' and their illustration by sketch graphs.

Graphs and graphical treatment of simple cases of the function

$$y = Ax^3 + Bx^2 + Cx + D + \frac{E}{x} + \frac{F}{x^2}$$

in which the constants are numerical and at least three of them are zero.

The gradients of these graphs by drawing.

Differentiation of powers of x.

Determination of gradients, rates of change, maxima and minima.

Applications to easy linear kinematics including distance-time and speed-time curves.

Integration of powers of x excluding x^{-1}.

Applications to areas, volumes of revolution, linear kinematics and determination of a function from its gradient.

4. In proving a theorem any preceding concepts may be assumed without proof.

In solving riders candidates may use any knowledge they possess and may apply trigonometry.

Basic concepts - proofs of theorems in italics may be required	Associated concepts
Angle at a point. Angle properties of parallel lines	Acute and obtuse angles. Complementary and supple= mentary angles. Angles of elevation and depression. Bearings from the North measured clockwise.
<u>Exterior angle property and angle sum property of a triangle</u>	Angle sum properties of polygons.
Congruency and similarity of triangles. Definition of a parallelogram. Straight line joining the mid-point of two sides of a triangle is parallel to the third side and equal to half the third side.	Special properties of isosceles and equilateral triangles. Definitions of sine, cosine and tangent of acute and obtuse angles Sine rule for any triangle. Properties of angles, sides and diagonals of the paral= lelogram, rhombus, rectangle, square, trapezium and kite. Symmetry about a point or a line.
<u>Parallelograms on the same base and between same parallels are equal in area</u>	Area of a triangle including $\frac{1}{2}bc \sin A$. Area of a trapezium.
Pythagoras' Theorem	Converse of Pythagoras' Theorem $\sin^2 A + \cos^2 A = 1$ $a^2 = b^2 + c^2 - 2bc \cos A$
Symmetrical properties of chords of a circle	

Basic concepts – proofs of theorems in italics may be required	Associated concepts
The angle at the centre of a circle is twice any angle at the circumference standing on the same arc Perpendicularity of tangent and radius. The alternate segment theorem	Other angle properties of the circle. Angle properties of cyclic quadrilateral. Equality of tangents from an external point.
The intersecting chord theorem (a) for an internal point AP.AQ = AR.AS (b) for an external point $BP.BQ = BR.BS = BT^2$	
Intercept theorem of a line drawn parallel to one side. The relationship between areas of similar triangles	Corresponding results for similar figures and ex= tension to volumes of similar solids.
The bisector of any angle of a triangle divides the opposite side in the ratio of the sides containing the angle.	Analogous property for an exterior angle.
Definition of a locus	Simple two and three dimensional loci
Properties of two and three dimensional figures	Circle, cube, rectangular block, pyramid, tetrahedron, prism, right circular cylinder, right circular cone and sphere.

5. Practical Applications.

(a) Construction of:

Parallel lines; an angle equal to a given angle; division of a straight line into a given number of equal parts or in a given ratio; angles of 90°, 45°, 60°, 30°; bisectors of angles and straight lines; triangles and polygons; a perpendicular to a given line; a triangle equal in area to a quadrilateral; circles from simple data including inscribed and circumscribed circles of a triangle; a segment of a circle containing a given angle; tangents to a circle from an external point; a tangent to a circle from a point on its circumference; a square equal in area to a given rectangle.

(b) Other practical applications:

Determination of positions by two bearings; use of coordinates; heights and distances in two or three dimensions; length of an arc in terms of the radius and the angle at the centre; latitude and longitude - great and small circles of a sphere; calculation of the angles between two planes and the angle between a straight line and a plane; solution of three dimensional problems by analysis into plane figures.

3.2 ADDITIONAL MATHEMATICS

Ordinary Level Only

Mathematical formulae will be provided. Statistical tables will be included with the formulae for use in Paper 6 where necessary

There will be six papers, each of two hours:

Paper 1 Pure Mathematics 1

Paper 2 Pure Mathematics 2

Paper 3 Mechanics 1

Paper 4 Mechanics 2

Paper 5 Statistics 1

Paper 6 Statistics 2.

Candidates may offer one of the following subjects:

370 Papers 1 and 2

371 Papers 1 and 3

372 Papers 1 and 5

373 Papers 3 and 4

374 Papers 3 and 5

375 Papers 5 and 6

Each paper will contain eight questions of which candidates will be required to answer six.

Questions will be set in the S.I. system of units.

Four-figure mathematical tables will be provided by the University for use at the examination. The use of slide rules will also be permitted.

Syllabuses

Paper 1

The use of properties of logarithms. The theory of quadratic equations. Arithmetic and geometric progressions. Use of the remainder theorem. The use of the binomial series for a positive integral index. Graphs of polynomials and of simple rational functions.

Solution of triangles (proofs of sine and cosine formulae only will be required). Graphs (and their applications) of sine, cosine and tangent and of functions of the form $a \cos x^{\circ} + b \sin x^{\circ}$, for $0 \leqslant x \leqslant 360$. Solution of simple trigonometric equations.

Cartesian rectangular coordinates of a point in a plane, distance between two points, mid-point of a line joining two points. The straight lines $y = mx + c$, $x = a$, $y - y_1 = m(x - x_1)$, conditions for two lines to be parallel or perpendicular, equation of a line joining two points.

Gradients, the gradient at any point of $y = x^n$ and $y = f(x)$ where $f(x)$ is a polynomial. Differentiation of a sum, a product and a quotient. Integration of x^n, excluding $n = -1$. Applications of the calculus to stationary values, maxima and minima, tangents, normals, areas and simple kinematics.

Paper 2

(The syllabus for Paper 1 will be assumed to be known, and questions will be more searching.)

The theory of the quadratic function $ax^2 + bx + c$. The use of the binomial series for a rational index.

The use of the formulae for $\sin (A \pm B)$, $\tan (A \pm B)$, $\sin A \pm \sin B$, $\cos A \pm \cos B$.

Trigonometric functions of angles of any magnitude. Circular measure. Three-dimensional problems including applications to simple solids.

The idea of a locus, the equation of a circle, determination of the centre and the radius of a circle.

Differentiation of a function of a function. Differentiation and integration of sin x, cos x (no proofs required). Applications of the calculus to rates of change and volumes of revolution.

Paper 3

The addition and subtraction of co-planar vectors and the multiplication of a vector by a scalar. Components and resolved parts of a vector. Applications of these properties of vectors to displacements, velocities and forces. Relative velocity.

The moment of a force.

Parallel forces (excluding couples).

The centres of gravity of uniform bodies which have centres of symmetry and of bodies formed by combining two such bodies or by removing one from another. Weight, normal reaction, friction and the coefficient of friction, tension and thrust.

The equilibrium of a rigid body acted on by coplanar forces. Two forces in equilibrium. Three forces in equilibrium. The triangle and polygon of forces. Lami's theorem.

The principle of work applied to simple machines, e.g. the lever, pulley systems, the wheel and axle, the screw. Mechanical advantage, velocity ratio, efficiency.

Uniformly accelerated rectilinear motion.

Newton's laws of motion. Simple applications, including the motion of connected particles.

Work, kinetic energy, power.

Momentum. Impulse. The principle of conservation of momentum applied to two particles moving along a straight line.

Questions may be set requiring a knowledge of graphical methods and of trigonometrical functions of angles between 0° and 360°.

Paper 4

(The syllabus for Paper 3 will be assumed to be known, and questions will be more searching.)

A knowledge of radian measure, the differentiation of x^n, cos nx, sin nx and the integration of x^n (excluding x^{-1}) will be assumed. Position vectors. The position vector of a point dividing the straight line joining two points in a given ratio.

Couples. Equivalent systems of coplanar forces. The angle of friction. The centre of gravity of a uniform triangular lamina. The use of integration in finding a centre of gravity.

Hooke's law.

Projectiles. The greatest height attained and the range on a horizontal plane.

The equation of the path.

Applications of the calculus to rectilinear motion with a distance as a function of the time.

Angular velocity.

Uniform circular motion. The acceleration in such motion.

Simple harmonic motion given by s = a cos nt or s = a sin nt. The relation of simple harmonic motion to uniform circular motion.

Potential energy. The principle of conservation of energy.

Paper 5

Methods of collection, classification and tabulation of statistical data.

Design of simple questionnaires bias in sampling.

Discrete and continuous distribution of data.

Diagrammatic representation of data ; bar charts, block diagrams, histograms, frequency polygons, pictograms, cumulative frequency diagrams, pie-charts.

Grouped data; class intervals, relative frequency.

Measures of location and central tendency; median and percentiles (from cumulative frequency table or diagram), arithmetic mean and geometric mean (by calculation).

Measures of dispersion: range, interpercentile range, mean deviation, variance and standard deviation from a set of numbers.

Time series, moving averages, index numbers, crude and standardised rates (e.g. birth, marriage and death rates).

Elementary ideas of probability as applied to discrete events, combination of probabilities, expected value, probabilities deduced from possibility spaces.

Mutually exclusive, exhaustive, independent events. Tree diagrams for independent events.

Scatter diagrams. Means of arrays. Calculation of rank correlation coefficients (Kendall's or Spearman's).

Paper 6

(The syllabus for Paper 5 will be assumed to be known, and questions will be more searching.)

Mean and variance of $aX + b$ in terms of the mean and variance of X. Linear transformations of data to a given mean and variance.

Simple permutations and combinations.

Discrete probability distribution functions (and their necessary restrictions) particularly the rectangular, triangular and binomial distributions. Calculation of expected values (means) and variances (including the binomial distribution).

Elementary ideas of probability density functions for continuous data, and the graphs of such functions. The rectangular distribution. Applications of calculus to finding probabilities, expected values and variances. The shape and use of the normal probability curve. The use of the tables of the normal probability integral. The normal distribution as an approximation to the binomial distribution.

3.3 MATHEMATICS

Advanced Level

Mathematical formulae will be provided.

The subjects that may be offered are:

Pure Mathematics

Applied Mathematics

Mathematics (Pure and Applied)

Higher Mathematics. Higher Mathematics may be taken in the June

Examination only.

A candidate mav not take anv of these subjects with either of the subjects under the heading Mathematics (Alternative Syllabus) at the Advanced level.

PURE MATHEMATICS

Advanced Level

This subject may not be taken by candidates taking either Mathematics at the Ordinary level or Mathematics (Pure and Applied) at the Advanced level.

Candidates who offer Pure Mathematics and Applied Mathematics, and who are present for all the papers concerned, may be awarded an Advanced level pass in Mathematics (Pure and Applied) if they satisfy the examiners in Paper 1 of each subject. The result in Mathematics (Pure and Applied) will be alternative to both of the results obtained in Pure Mathematics and Applied Mathematics.

Questions will be set in the S.I. system of units.

Four-figure mathematical tables will be provided by the University.

The use of slide rules will also be permitted.

There will be three papers, each of 3 hours. Candidates must take Paper 1 and one other paper. It is desirable that candidates for Paper 3 should have had access to a computer. Candidates taking this paper must have followed a course of practical work, and may be required to produce a practical note=book for inspection by the examiners.

Paper 1

The theory of the quadratic function and of quadratic equations.

Simple algebraic functions. The theory of indices and logarithms.

Complex numbers: sum, product and quotient of complex numbers in algebraic form, Argand diagram.

Permutations and combinations. Elementary examples in the use of induction. The binomial theorem for a positive integral index.

The use of the expansion $(1 + x)^n$, where n is fractional or negative; simple approximations. Determination of a linear law from experimental data.

Rectangular cartesian coordinates, including parameters, applied to the straight line, circle and parabola. Easy problems on loci.

Circular measure; small angles, including the use of limits such as $\sin\theta/\theta$ as θ tends to zero. The formulae for $\sin(A \pm B)$ and $\cos(A \pm B)$. Periodic properties of trigonometric functions. Sine and cosine formulae for a triangle with elementary applications. General solution of equations such as $\cos\theta = \cos\alpha$. Expression of $a\cos\theta + b\sin\theta$ in the form $r\cos(\theta - \alpha)$. De Moivre's theorem for a positive integral index.

The differentiation of simple algebraic, trigonometric (excluding inverse functions), exponential and logarithmic functions. Differentiation of a sum, product, quotient and simple cases of a function of a function and of implicit functions. Applications to gradients, maxima and minima. Simple curve tracing.

Definite and indefinite integration of simple functions. Applications to area and volume.

Paper 2

The convergence of geometric series. Summation of simple finite algebraic series.

The relations between the roots of an algebraic equation and its coefficients.

The exponential and logarithmic series.

Identities, including partial fractions; the remainder theorem.

Handling of simple inequalities.

Further applications of de Moivre's theorem: the cube roots of ± 1.

Rectangular cartesian coordinates, including parameters, applied to the ellipse and hyperbola.

The determination of the angles made by planes and straight lines with one another. The mensuration and simpler properties of common solids including the tetrahedron, cone and sphere; any appropriate methods may be used.

Cartesian coordinates in three dimensions; direction cosines, equations of a plane and of a straight line.

Differentiation of arcsin x and arctan x. Simple examples of integration by substitution and integration by parts. Applications of the calculus to rates of change, tangents and normals and centroids.

Further sketching of graphs including those of algebraic functions such as $(ax^2 + bx + c)/(px^2 + qx + r)$ and of simple transcendental functions.

Paper 3

The mathematical content of the work is indicated in the paragraphs below. Candidates will be expected to be able to write, where relevant, computer programs in a suitable language (e.g. Algol, Fortran, etc.) for the solution of problems involving a knowledge of these topics. To assist the examiners, centres and candidates may be required to submit to the University the language and data to be used by candidates in the examination.

Flow diagrams. The convergence of geometric series. Summation of simple finite algebraic series. The use of Taylor's series and Maclaurin's series for the expansion of a function; the exponential, logarithmic and trigonometric series. Simple properties of the hyperbolic functions.

Differentiation of inverse functions. Simple examples of integration by substitution, by partial fractions and by parts. Integration of first-order differential equations in which the variables are separable. Numerical solution of first-order differential equations by step-by-step methods, including the modified Euler method. Numerical integration by the trapezium rule, the mid-ordinate rule and by Simpson's rule.

Addition, subtraction and multiplication of matrices; the determinant, inverse and characteristic equation of a 3 x 3 matrix. The application of matrices to linear transformations and to the solution of a system of linear equations. The solution of a system of simultaneous linear equations by Gaussian elimination and by the Gauss-Seidel method.

Residuals.

The relations between the roots of an algebraic equation and its coefficients. Solution of non-linear equations by graphical and iterative methods, including the Newton-Raphson method. Linear and quadratic convergence.

Elementary Boolean algebra, including 'and', 'or', 'not', 'nand' and 'nor' connectives.

A special paper will be set in this subject; see Regulation III. This paper will be suitable for candidates taking Papers 1 and 2 or Papers 1 and 3.

APPLIED MATHEMATICS

Advanced Level

This subject may not be taken by candidates taking Mathematics (Pure and Applied) at the Advanced level.

Candidates who offer Pure Mathematics and Applied Mathematics, and who are present for all the papers concerned, may be awarded an Advanced level pass in Mathematics (Pure and Applied) if they satisfy the examiners in Paper 1 of each subject. The results in Mathematics (Pure and Applied) will be alternative to both of the results obtained in Pure Mathematics and Applied Mathematics.

Questions will be set in the S.I. system of units.

Four-figure mathematical tables will be provided by the University. The use of slide rules will also be permitted.

There will be two papers, each of 3 hours.

Paper 1

The addition and subtraction of vectors. Moments and couples. Equilibrium of a particle and of a rigid body under the action of a system of coplanar forces.

Centre of mass. Friction. Hooke's law.

Kinematics of a particle (a) moving in a straight line with acceleration a function of the time or a function of the distance (but excluding the determination of the position of the particle from the given data) (b) moving with simple harmonic motion (c) moving in a circle with uniform speed.

Concept of relative velocity.

Newton's laws of motion, mass, momentum, force, impulse, work, energy and power. The principles of conservation of energy and momentum.

Direct impact of perfectly elastic spheres and extension to the case of imperfectly elastic spheres.

Elementary treatment of projectiles.

Paper 2

(Candidates are expected to be familiar with the notion that such diverse physical quantities as displacement, velocity, acceleration and force can be represented by directed line segments (vectors) obeying the same law of addition.)

Vectors; addition, subtraction and multiplication by a scalar. Direction of a vector, direction cosines, components of a vector, magnitude of a vector, unit vector. Position vector, position vector of a point dividing two points in a given ratio. Vector equation of a straight line, equivalent cartesian forms in two and three dimensions. Position vector in terms of a parameter, parametric representation of a simple curve, such as the circle $r = a \cos p\, i + a \sin p\, j$ and the parabola $r = ap^2 i + 2ap\, j$. Scalar product of two vectors.

Vector functions of time; displacement, velocity, acceleration. Relative velocity. Components of velocity and acceleration referred to cartesian axes.

Further treatment of projectiles.

Rectilinear motion of a pair of connected particles; direct and oblique impact of smooth spheres.

Position vector of centre of mass. Second moments in simple cases, parallel axis theorem, perpendicular axis theorem for a lamina.

Motion in a vertical circle under gravity.

Motion of a rigid body about a fixed axis. Angular velocity. Moment of momentum about a fixed axis. Motion of the centre of mass of a system.

Kinetic energy of a rigid body. Compound pendulum.

A special paper will be set in this subject; see Regulation III.

MATHEMATICS (PURE AND APPLIED)

Advanced Level

This subject may not be taken by candidates taking either Mathematics at the Ordinary level or Pure Mathematics or Applied Mathematics at the Advanced level.

Questions will be set in the S.I. system of units.

Four-figure mathematical tables will be provided by the University. The use of slide rules will also be permitted.

There will be two papers, each of 3 hours, which will be identical with Paper 1 of Pure Mathematics and Paper 1 of Applied Mathematics respectively.

Paper 1

The theory of the quadratic function and of quadratic equations. Simple algebraic functions. The theory of indices and logarithms.

Complex numbers: sum, product and quotient of complex numbers in algebraic form, Argand diagram.

Permutations and combinations. Elementary examples in the use of induction. The binomial theorem for a positive integral index. The use of the expansion $(1 + x)^n$, where n is fractional or negative; simple approximations. Determination of a linear law from experimental data.

Rectangular cartesian coordinates, including parameters, applied to the straight line, circle and parabola. Easy problems on loci. Circular measure; small angles, including the use of limits such as $\sin\theta/\theta$ as θ tends to zero. The formulae for $\sin(A \pm B)$ and $\cos(A \pm B)$. Periodic properties of trigonometric functions. Sine and cosine formulae for a triangle with elementary applications. General solution of equations such as $\cos\theta = \cos\alpha$. Expression of $a\cos\theta + b\sin\theta$ in the form $r\cos(\theta - \alpha)$. De Moivre's theorem for a positive integral index.

The differentiation of simple algebraic, trigonometric (excluding inverse functions), exponential and logarithmic functions. Differentiation of a sum, product, quotient and simple cases of a function of a function and of implicit functions.

Applications to gradients, maxima and minima. Simple curve tracing. Definite and indefinite integration of simple func= tions. Applications to area and volume.

Paper 2

The addition and subtraction of vectors. Moments and couples. Equilibrium of a particle and of a rigid body under the action of a system of coplanar forces. Centre of mass. Friction. Hooke's law.

Kinematics of a particle (a) moving in a straight line with acceleration a function of the time or a function of the distance (but excluding the determination of the position of the particle from the given data) (b) moving with simple harmonic motion (c) moving in a circle with uniform speed.

Concept of relative velocity.

Newton's laws of motion, mass, momentum, force, impulse, work, energy and power. The principles of conservation of energy and momentum. Direct impact of perfectly elastic spheres and extension to the case of imperfectly elastic spheres.

Elementary treatment of projectiles.

HIGHER MATHEMATICS

Advanced Level

This subject may be taken at the June examination only.

Questions will be set in the S.I. system of units.

Four-figure mathematical tables will be provided by the University. The use of slide rules will also be permitted.

There will be two papers, each of 3 hours. Each paper may contain questions on any part of the syllabus. The questions will be designed to test the understanding of principles rather than the ability to carry out intricate calculations.

Papers 1 and 2

Matrices (not greater than 3 x 3), multiplication of matrices; determinants, inversion of square matrices and application to linear equations.

Sequences and series, limits, convergence of simple series. Hyperbolic functions and their inverses.

Elementary properties of the triangle and its associated points and circles.

Concurrence and collinearity. Radical axis, coaxal circles. Inversion.

Curvature of a plane curve using cartesian or parametric coordinates.

Polar coordinates, polar equations of simple plane curves. Area = $\frac{1}{2}\int r^2 d\theta$. Use of $\tan\varphi = r d\theta/dr$.

Length of arc of a curve, area of surface of revolution, theorems of Pappus.

Curve tracing. Simple properties of particular curve, including the catenary, cycloid, cardioid and equiangular spiral.

Numerical methods: Newton's method of approximate solution of $f(x) = 0$ and other simple iterative methods. Approximate integration: discussion of methods such as the trapezoidal rule and Simpson's rule. Step by step integration of $dy/dx = f(x,y)$.

Leibnitz theorem for the nth derivative of a product.

Easy cases of reduction formulae for integrals.

Elementary differential equations: variables separable;

$\frac{dy}{dx} + Py = Q$, where P and Q are functions of x; $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$, where a, b, c are constants.

Scalar product of two vectors, perpendicular and parallel vectors. Scalar product in terms of components of vectors. Angle between vectors. Normal to a plane. Vector equation of a plane, cartesian equivalent. Angle between two planes.

The parametric respresentation of curves such as the helix $r = a \cos p\, i + a \sin p\, j + bp\, k$. Vector product of two vectors. Rate of change of unit vector in a plane. Velocity and acceleration of a particle in non-uniform motion in a circle. Radial and transverse components of velocity and acceleration. Tangential and normal components of velocity and acceleration of a particle moving in a plane.

Two-dimensional motion of a rigid body; instantaneous centre of rotation.

Impulse, momentum and energy applied to rods, cylinders and spheres.

Linear motion in a resisting medium.

Uniform chain hanging freely under gravity.

3.4 MATHEMATICS (ALTERNATIVE SYLLABUS)

Advanced Level

Mathematical formulae will be provided. Statistical formulae and tables will also be provided.

The subjects that may be offered are:

Mathematics

Further Mathematics

These syllabuses offer a 'modern' mathematics alternative to the subjects under the general heading Mathematics, Advanced level.

A candidate may not take either of these subjects with any of the subjects under the heading Mathematics at the Advanced level or with Mathematics at the Ordinary level.

Introduction

In framing a single-subject syllabus, Mathematics, as far as is practicable those parts of the pure mathematics have been emphasised which are important in applied fields. For example, linear algebra figures more fully than in the past, along with simple three-dimensional coordinate geometry and vector work (at the expense of manipulative work on conics), and complex number.

In differential calculus the emphasis is, as has now become generally accepted, on the understanding and application of the basic techniques for dealing with functions and their graphs. In integral calculus however, the understanding of principles is paramount and e.g. ability to make a change of variable is not taken to include knowing, except in very simple cases, what change to choose.

Numerical methods have been emphasised in connection (i) with integration, (ii) with evaluation of functions by expansion and (iii) with solution of equations. The break-down of a process into steps expressed in a flow-chart is a necessary part of this topic.

Changes in emphasis in manipulative algebra and trigonometry are made which are consistent with the attitudes expressed above - any easing of manipulative load being compensated by higher demand for cogent treatment in given situations (e.g. in partial fractions, in establishing sums of finite series, or in simple inequalities).

Elements of probability theory figure in the syllabus. Here as elsewhere, ability to understand and use the language of elementary set theory is demanded.

In applied mathematics the emphasis is on the ability to handle a simple model of a real situation whether deterministic or stochastic. In an examination the chance to exploit the provisional character of such a model is extremely limited, but one would wish where possible to give expression to it.

The range of physical topics amenable to model treatment in examination questions is inevitably restricted. As material for models Newtonian mechanics must of necessity figure largely. In particular it is not practicable to show applications of vectors in applied fields other than this. Nevertheless, subject to limitations of space and time, simple application to other fields can be tested, e.g. in the setting up of differential equations from assumptions expressed in verbal form.

In Further Mathematics the same general considerations apply, though to an extended range and to greater depth. The main difference of emphasis in pure mathematics at this stage is directed to the specific needs of those who may be studying mathematics as such at the university. Accordingly (a) in pure mathematics the axiomatic character of the subject must be made apparent by suitable case-studies in abstract treatment, and (b) in applied, there should be not only a quantitative broadening in applied technique but, above all, a growth in critical analysis of its application, e.g. an introduction to the idea of signi= ficance in statistics.

MATHEMATICS

Advanced Level

The examination will consist of two papers, each of 3 hours. Each paper will contain twelve questions and will be divided into three sections, containing questions on pure mathematics, applied mathematics and statistics respectively. Questions involving numerical methods, when set, may occur in any section. Candidates will be required to answer eight questions, at least one from each section, and will therefore have a wide choice.

Where appropriate, questions will be set in S.I. units. Four-figure mathematical tables will be provided by the University. The use of slide rules will also be permitted.

It is assumed that candidates will have a knowledge of the following topics which will not be made specifically the subject of questions: All topics in Ordinary level Mathematics Syllabus C which are relevant to this syllabus. Indices and logarithms Arithmetic progressions. Relations between the roots and the coefficients of a quadratic equation. Permutations and combinations. Radian measure. Sine and cosine rules for triangles.

List of symbols

$\mathbb{N}$ The natural numbers

$\mathbb{Z}$ The integers

$\mathbb{Q}$ The rational numbers

$\mathbb{R}$ The real numbers

$\mathbb{C}$ The complex numbers

Syllabus

Idea of mapping (function). Laws of composition (binary ope= rations). Inverse functions. Proof by induction. The concept of a group with special emphasis on integers and rational numbers and on groups of symmetries of simple plane figures, permutation groups and the isometries of Euclidean geometry.

The integers modulo n. Isomorphism.

The cartesian plane referred to rectangular axes, rectangular coordinates and vectors. Simple extensions to three dimensions. Polar coordinates in two dimensions. The relation of the cosine formula to orthogonality and the scalar product of vectors. Translation and linear transformations. Representation of linear transformations by matrices. Inverse linear transformations; determinants of order not greater than 3.

Treatment of curves with simple parametric equations; sketching of such curves. Trigonometric functions of angles of any magnitude. Addition formulae and their relation to rotations in the plane. General solution of simple equations of the form $a\cos\theta + b\sin\theta = c$.

Real and complex numbers. The real number line and the Argand plane. Inequalities in $\mathbb{R}$. Modulus and argument of complex numbers; simple applications of de Moivre's theorem for a positive integral index.

Polynomials, factor and remainder theorems, partial fractions and simple applications. Binomial theorem for positive inte= gral exponents. The method of induction applied to the summa= tion of simple finite series. Infinite geometric series.

Functions from $\mathbb{R}$ to $\mathbb{R}$; simple algebraic and trigonometric functions, logarithmic and exponential functions. Differen= tiation. Differentiation of sums, products, quotients and composite functions (function of a function). Implicit func= tions and inverse functions.

Applications of the differential calculus to rates of change, tangents, normals, maxima and minima.

The definite integral and the relation between definite and indefenite integrals. Simple integration including integration by parts and by substitution. (Substitutions will be given except in easiest cases.) Applications to areas, volumes and centroids.

Differentiation and integration applied to velocity and accele= ration. Construction of differential equations from verbal data in various contexts. Solution of simple first order equations with variables separable, the equation $\frac{dy}{dx} + ay = f(x)$ and the equation $D^2y \pm n^2y = 0$.

Applications to simple harmonic motion, rate of growth, etc.

Elementary knowledge of momentum, force, moment of a force, Newton's laws of motion, impulse, work, energy, power conservation of momentum and of energy, projectiles, motion in a circle with uniform speed.

Composition and resolution of vectors in two dimensions.

Equilibrium of a particle and rigid body under the action of coplanar forces.

Moments and couples. Applications of vectors to velocity, relative velocity and force, including simple instances in three dimensions.

Units and dimensions.

Elementary numerical methods; approximate evaluation of a function from an expansion either given or obtained by Maclaurin's method. Linear interpolation. Simple examples of iterative methods including Newton's method for finding a root of a simple equation. Numerical integration by trapezium rule and by Simpson's rule. Simple flow diagrams.

Probability of events in finite sample spaces. Application of the relation $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Sum and product laws of probability. Mean, variance and standard deviation. Binomial distribution. Simple applications of the normal distribution, regarded as an empirical distribution.

A special paper will be set in this subject; see Regulation III.

FURTHER MATHEMATICS

Advanced Level

The examination will consist of two papers, each of 3 hours. Each paper will contain twelve questions and will be divided into three sections, containing questions on pure mathematics, applied mathematics and statistics respectively. Candidates will be required to answer eight questions, at least one from each section, and will therefore have a wide choice.

Where appropriate, questions will be set in S.I. units. Four-figure mathematical tables will be provided by the University. The use of slide rules will also be permitted.

Knowledge of the syllabus for Advanced level Mathematics (Subject number 480) will be assumed.

Candidates are expected to understand the role of logic in mathematics (for example, the distinction between if and only if).

Syllabus

Relations, equivalence relations.

The general idea of algebraic structure with reference to groups, elementary Boolean algebra and the examples mentioned in this paragraph. The integers as an (ordered) integral domain. Divisibility properties. Euclid's algorithm. H.c.f. and l.c.m. defined by divisibility properties. The existence of integers x, y such that ax + by = h where h is the h.c.f. of a and b. The solution of linear congruences to a prime modulus. Simple examples of fields. Vector spaces of not more than four dimensions; bases, linear mappings and associated matrices.

Rectangular axes, coordinates and vectors in three dimensions. Linear equations in three unknowns and their relation to geometry. Cartesian and vector forms of equations of lines and planes including the use of parameters. Scalar and vector products and their geometrical applications. Simple problems on line, plane, sphere, cone and cylinder.

Sequences and series, limits, elementary ideas of convergence based on the comparison test. The exponential and logarithmic series.

The binomial series for $(1 + x)^{\alpha}$ for $|x| < 1$, where $\alpha \in R$

The exponential form of a complex number. De Moivre's theorem for any rational index. Elementary transformations in the com= plex plane of the form w = az + b where a, b $\in$ C.

Hyperbolic and inverse hyperbolic functions and their derivatives.

Polar coordinates, simple polar equations of plane curves, area= $\int \frac{1}{2}r^2 d\theta$.

Length of arc of a curve in cartesian coordinates.

Differential equations of the form

(i) $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x,

(ii) $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$, where a, b, c are constants, in

cases where a particular integral can be found by inspection.

Simple applications of scalar and vector products to work and moments.

Differentiation of vectors with respect to a scalar variable and applications to velocity and acceleration. Simple cases of integration of vector equations.

Motion of a particle in two dimensions, components of velocity and acceleration along and perpendicular to the radius vector; simple orbit problems, problems involving variable acceleration expressed as a function of time, distance or speed. Simple applications to the motion of two or more particles connected by, for example, light rods or strings.

Moments of inertia including the parallel and perpendicular axes theorems. Motion of a rigid body about a fixed axis, moment of momentum. General motion of a rigid body in two dimensions, instantaneous centre of rotation.

Change of motion of a rigid body following a blow.

Harder problems on basic probability theory as in the syllabus for Advanced level Mathematics. Random variables, discrete and continuous. Probability density.

Expectation. Derivation of the mean and variance of (a) a simple given probability density function and (b) the rectangular, binomial, Poisson and exponential distributions. The normal distribution given that, for the normal,

$\frac{1}{\sqrt{(2\pi)}}\int_{-\infty}^{+\infty} \exp(-\frac{1}{2}t^2)dt = 1$. Simple applications of these distributions.

Scatter diagrams. Calculation of the equations of regression lines by the method of least squares. Product-moment correlation coefficient and Kendall's and Spearman's rank correlation coefficients; the use and limitation of these. (Questions will not be set which will involve a long series of calculations.)

General idea of sampling. Estimation, from a random sample, of the mean and variance of a population. Standard error of means of samples and applications to problems involving large samples.

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