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A Class of Odd Order Numerical Integrators and its Applications to the Study of Nature and Location of Singularity of Non-Autonomous Initial Value Problems and their Applications to Climate Change

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ABSTRACT

Even though, there a number of environmental models that seek to evaluate arbitrary even (odd) number of nodes in climate change problems, in this research, we seek to present a method of obtaining a trigonometric interpolation polynomial through polynomial interpolation. A class of formulae for the numerical solution of Initial Value Problems (IVP), in Ordinary Differential Equations (ODEs) is considered. This class of integrators is imbedded with the capacity of determining the nature and location of catastrophe in a system of IVP that is non-autonomous in nature. The study firstly, presents the method to compute a trig-polynomial interpolation, with a specific focus on the coefficients of the trigonometric polynomial, also the derivation of odd numerical integrators. Subsequently, discussions on the numerical solutions of the problem using the study approach is presented by implementing the numerical integrators to non-autonomous initial value problems. The numerical schemes afford us the opportunity to control the performance of schemes because of the two complex function control parameters. Lastly, performance of the proposed methods is assessed using analytical numerical simulations. Results from the proposed methods are compared with observed data and found compare favorably well. Owing to these results, the proposed methods can be used in both theoretical and practical applications of solving climate change problems. Moreover, these integrators can be developed into user friendly software that can be used to study the frequency of global emissions, excitation energies of the molecular composition of components that contribute to climate changes.

Keywords: Interpolant, Trig-polynomial, Numerical integrators, Non-autonomous IVP's, Singularity, Undetermined coefficients

Introduction

Climate change is threat to humans and animals and the occurrence and reoccurrence of climate change has become a global challenge, Sebo (2021). In previous research work by Kong et.al., (2021), the researchers point toward appropriate comprehension of mechanisms and model solutions as an approach that could enhance the development of solutions. Models and solutions addressing climate change have also been studied, in existing literature Enoch *et.al.,* (2012), Ibijola *et.al.,* (2011) and Lambert *et.al.,* (1966), the authors have considered a class of formulae for the numerical solution of:

$$y' = f(x,y); \qquad y(x) = a \qquad (1)$$

The results obtained from mentioned authors have improved the numerical solution closed to the point of singularity of solution.

Theoretically, Lambert *et.al.,* (1966) and Schreiber (2011, 2016) considered an interpolation of a trigonometric polynomial with analyzing the environmental models and arbitrary even (odd) number of nodes. Many one step numerical methods are without inherent control parameters, but these integrators have two complex control functions. These parameters enable us to control the integrators to deliver the desired solutions.

This research will focus on extension of the mentioned authors research work to further investigate the nature and location of singularity of non-autonomous IVPs and their applications to the

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study of climate change and global warming. In addition, the numerical schemes will afford us the opportunity to control the performance of schemes because of the two complex function control parameters.

Aim of study

The aims/goals of this study are as follows:

i. To present an explicit method for the computation of numerical Integrators which are based on trig-polynomial interpolants,

ii. Develop numerical schemes that will afford us the opportunity to control the performance of schemes because of the two complex function control parameters,

iii. More precisely, we aim to derive and generalize the coefficients of the trigonometric polynomial.

If the assumed solution to the IVP models of the equations that represents some climate change/ global warming scenarios is the trigonometric polynomial;

$$F(x) = P_m(x) + bcos(Nx + A), \qquad m > 0 \qquad (2)$$

With:

$$P_m(x) = \sum_{j=0}^{m} a_j x^j, \theta_x = Nx + A$$

We initialize equation (1) as:

$$F(x) = \sum_{j=0}^{m} a_j x^j, + bcos(Nx + A), \qquad m > 0 \qquad (3)$$

Where a_j, b, are real such that A and N are complex functions, and m is a positive integer.

Assumptions

1. We assume that F(*x*) is continuously differentiable in the interval x(*j*) and x(*j+1*)

2. F'(x) = f (*x, y*) such that the n[th] +1 derivative of F(x) coincides with the nth derivative of the initial value problems, represented as f (*x, y*) [4].

Authors [5,4] adopted that: F(x(*j*)) approximately equals to y(x(*j*)) and F(x(*j+1*)) coincides with y(x(*j+1*)) as contained in equation *(i)* to *(ii)* below. These assumptions shall be subsequently used to determine the undetermined coefficients that are needed for the development of the numerical schemes.

$$F(x) = P_m(x) + bcos(Nx + A), \qquad m > 0 \qquad (4)$$

Where the polynomial P_m (x) is given as;

$$P_m(x) = \sum_{j=0}^{m} a_j\, x^j \qquad (5)$$

$$y(x_n) - P_m(x_n) - bcos(Nx + A) = 0 \qquad (i)$$

and;

$$y(x_{n+1}) - P_m(x_{n+1}) - bcos(Nx_{n+1} + A) = 0$$

$$y^{(s)}(x_n) - P_m^{(s)}(x_n) - \frac{bd^s}{dx^s}cos(Nx + A)x = x_n \qquad (ii)$$

for s = 1, 2, ..., m+1.

We now define the linear function;

$\theta(x)$ as $\theta_x = Nx + A$

This implies that;

$$q_n = q(x_n) = N(x_n) + A \qquad (6)$$

and;

$$\theta_{n+1} = Nx_n + A + nh = \theta_n + Nh$$

Materials and Methods

In this section we proceed to formulate and provide solutions to the proposed problem by deriving the class of one step integrators from the assumed solution as contained in [5,4,3].

Derivation of order one numerical integrator

Where m = 1 (i.e., the polynomial $P_m(x)$ in linear)

$F(x_n) = a_0 + a_1x_n + b\cos\theta_n$
$F(x_n) = a_1 + b[N(-\sin\theta_n)] = a_1 - bN\sin\theta_n = f(x,y)$ (7)
$F^{(2)}(x_n) = -bN^2\cos\theta_n = f(x,y)$ (8)
$a_1 = f(x,y) + b\sin\theta_n$

and;

$$b = \frac{-f'(x,y)}{N^2\cos\theta_n} = -f'(x,y)N^{-2}\sec\theta_n$$

$$a_1 = f(x,y) - \frac{-f'(x,y)}{N\cos\theta_n}\sin\theta_n$$

$$= f(x,y) - f'(x,y)N^{-1}\tan\theta_n$$

$$y_{n+1} - y_n = a_1(x_{n+1} - x_n) + b(\cos\theta_{n+1} - \cos\theta_n)$$

$$y_{n+1} = y_n - hf(x,y) - N^{-1}hf'(x,y)\tan\theta_n - N^{-2}f'(x,y)(\cos\theta_{n+1} - \cos\theta_n)\sec\theta_n \qquad (9)$$

Derivation of order three numerical integrator

When m = 3

$$P_m(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$

$$F(x_n) = a_0 + a_1x_n + a_2x_n^2 + a_3x_n^3 + b\cos(Nx_n + A) \qquad (10)$$

$$= a_0 + a_1x_n + a_2x_n^2 + a_3x_n^3 + b\cos\theta_n \qquad (11)$$

$$F'(x_n) = a_1 + 2a_2x_n + 3a_3x_n^2 - bN\sin\theta_n \qquad (12)$$

$$F^{(2)}(x_n) = 2a_2 + 6a_3x_n + bN^2\cos\theta_n = f'(x,y) \qquad (13)$$

$$F^{(3)}(x_n) = 6a_3 + bN^3\sin\theta_n = f^{(2)}(x,y) \qquad (14)$$

$$F^{(4)}(x_n) = bN^4\cos\theta_n = f^{(3)}(x,y) \qquad (15)$$

$$b = \frac{f^{(3)}(x,y)}{N^4\cos\theta_n} = f^{(3)}(x,y)N^4\cos\theta_n$$

From Eq. (14) we then compute:

$$6a_3 + bN^3\sin\theta_n = f^{(2)}(x,y)$$

$$6a_3 = f^{(2)}(x,y) - bN^3\sin\theta_n$$

We can show that,

$$a_3 = \frac{1}{6}\left[f^{(2)}(x,y) - f^{(3)}(x,y)N^{-1}\tan\theta_n\right] \qquad (15a)$$

From Eq. (13);

$$2a_2 + 6a_3x_n + bN^2\cos\theta_n = f'(x,y)$$

$$2a_2 = f'(x,y) - +6a_3x_n + bN^2\cos\theta_n$$

it can be deduced that,

$$a_2 = \frac{1}{2}\left[f'(x,y) - x_n f^{(2)}(x,y) + x_n f^{(3)}(x,y)N^{-1}\tan\theta_n + f^{(3)}(x,y)N^{-2}\right] \qquad (15b)$$

From Eq. (12);

$$a_1 + 2a_2x_n + 3a_3x_n^2 - bN\sin\theta_n = f(x,y)$$

$$a_1 = f(x,y) - 2a_2x_n - 3a_3x_n^2 + bN\sin\theta_n$$

It can be seen that,

$$a_1 = f(x,y) - x_n\left[f^{(1)}(x,y) + f^{(3)}(x,y)N^{-2}\right] + \frac{x_n^2}{2}\left[f^{(2)}(x,y) + f^{(3)}(x,y)N^{-1}\tan\theta_n\right] - f^{(3)}(x,y)N^{-3}\tan\theta_n \qquad (15c)$$

Subsequently,

$$y_{n+1} = y_n + hf(x,y) - x_nh\left[f^{(1)}(x,y) + f^{(3)}(x,y)N^{-2}\right] + \frac{x_n^2}{2}h\left[f^{(2)}(x,y) + f^{(3)}(x,y)N^{-1}\tan\theta_n\right] - hf^{(3)}(x,y)\tan\theta_n + hf^{(3)}(x,y)N^{-4}\sec\theta_n\,(\cos\theta_{n+1} - \cos\theta_n) \qquad (16)$$

Derivation of order five numerical integrator

Where m = 5,

$$P_m(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$$

$$F(x_n) = a_0 + a_1x_n + a_2x_n^2 + a_3x_n^3 + a_4x^4 + a_5x^5 + b\cos(Nx_n + A) \qquad (17)$$

$$= a_0 + a_1x_n + a_2x_n^2 + a_3x_n^3 + a_4x_n^4 + a_5x_n^5 + b\cos\theta_n \qquad (18)$$

$$F'^{(x_n)} = a_1 + 2a_2x_n + 3a_3x_n^2 + 4a_4x_n^3 + 5a_5x_n^4 - bN\sin\theta_n = \mathrm{f(x,y)} \qquad (19)$$

$$F^{(2)}(x_n) = 2a_2 + 6a_3x_n + 12a_4x_n^3 + 20a_5x_n^4 + bN^2\cos\theta_n = f'(x,y) \qquad (20)$$

$$F^{(3)}(x_n) = 6a_3 + 24a_4x_n + 60a_5x_n^3 + bN^3\sin\theta_n = f^{(2)}(x,y) \qquad (21)$$

$$F^{(4)}(x_n) = 24a_4 + 120a_5x_n + bN^4\cos\theta_n = f^{(3)}(x,y) \qquad (22)$$

$$F^{(5)}(x_n) = 120a_5 - bN^5\sin\theta_n = f^{(4)}(x,y) \qquad (23)$$

$$F^{(6)}(x_n) = -bN^6\cos\theta_n = f^{(5)}(x,y) \qquad (24)$$

$$b = -\frac{f^{(5)}(x,y)}{N^6\cos\theta_n} = -f^{(5)}(x,y)N^6\cos\theta_n$$

From Eq. (23):

$$120a_5 - bN^5\sin\theta_n = f^{(4)}(x,y) \qquad (24a)$$

it can be seen that,

From Eq. (22):

$$120a_5 = f^{(4)}(x,y) + bN^5\sin\theta_n$$

$$a_5 = \frac{1}{120}\left[f^{(4)}(x,y) - f^{(5)}(x,y)N^{-1}\tan\theta_n\right] \quad (24a)$$

$$24a_4 + 120a_5x_n + bN^4\cos\theta_n = f^{(3)}(x,y)$$

$$24a_4 = f^{(3)}(x,y) - 120a_5x_n - bN^4\cos\theta_n$$

it can be shown that,

$$a_2 = \frac{1}{2}\left[f^{(3)}(x,y) - x_nf^{(4)}(x,y) + x_nf^{(5)}(x,y)N^{-1}\tan\theta_n + f^{(5)}(x,y)N^{-2}\right] \quad (24b)$$

From Eq. (21):

$$6a_3 + 24a_4x_n + 60a_5x_n^4 + bN\sin\theta_n = f^{(2)}(x,y)$$

$$6a_3 = f^{(2)}(x,y) - 24a_4x_n - 60a_5x_n^2 - bN^3\sin\theta_n$$

It can be deduced [6,8] that,

$$a_3 = \frac{1}{6}\left[f^{(2)}(x,y) - x_n\left(f^{(3)}(x,y) + f^{(5)}(x,y)N^{-2}\right) + \frac{x_n^2}{2}\left(f^{(4)}(x,y) - f^{(5)}(x,y)N^{-1}\tan\theta_n\right) + f^{(5)}(x,y)N^{-3}\tan\theta_n\right] \quad (24c)$$

$$a_2 + 6a_3x_n + 12a_4x_n^3 + 20a_5x_n^4 + bN^2\cos\theta_n = f'(x,y)$$

$$a_2 = f'^{(x,y)} - 6a_3x_n - 12a_4x_n^3 - 20a_5x_n^4 - bN^2\cos\theta_n$$

$$a_2 = \left[f'(x,y) - \left(f^{(2)}(x,y) + f^{(5)}(x,y)N^3\sin\theta_n\right) + \frac{x_n^2}{2}\left(f^{(3)}(x,y) + f^{(5)}(x,y)N^{-2}\right) - \frac{x_n^3}{6}\left(f^{(4)}(x,y) - f^{(5)}(x,y)N^{-1}\tan\theta_n\right) - f^{(5)}(x,y)N^{-4}\right] \quad (24d)$$

$$a_1 + 2a_2x_n + 3a_3x_n^2 + 4a_4x_n^3 + 5a_5x_n^4 - bN\sin\theta_n = \mathrm{f(x,y)}$$

$$a_1 = \mathrm{f(x,y)} - 2a_2x_n - 3a_3x_n^2 - 4a_4x_n^3 - 5a_5 + bN\sin\theta_n$$

$$a_1 = \mathrm{f(x,y)} - x_n\left(f'(x,y) - f^{(5)}(x,y)N^{-4}\right) + \frac{x_n^2}{2}\left(f^{(3)}(x,y) + f^{(5)}(x,y)N^{-3}\tan\theta_n\right) - \frac{x_n^3}{6}\left(f^{(3)}(x,y) - f^{(5)}(x,y)N^{-2}\right) + \frac{x_n^4}{24}\left(f^{(4)}(x,y) - f^{(5)}(x,y)N^{-1}\tan\theta_n\right) - f^{(5)}(x,y)N^{-5}\tan\theta_n \quad (24e)$$

Thus,

$$y_{n+1} = y_n + hf(x,y) - x_nh\left(f'(x,y) + f^{(5)}(x,y)N^{-4}\right) + \frac{x_n^2}{2}h\left(f^{(2)}(x,y) + f^{(5)}(x,y)N^{-3}\tan\theta_n\right) - \frac{x_n^3}{6}h\left(f^{(3)}(x,y) + f^{(5)}(x,y)N^{-2}\right) + \frac{x_n^4}{24}h\left(f^{(4)}(x,y) + f^{(5)}(x,y)N^{-1}\tan\theta_n\right) - hf^{(5)}(x,y)N^{-3}\tan\theta_n + hf^{(5)}(x,y)N^{-6}\sec\theta_n\left(\cos\theta_{n+1} - \cos\theta_{n+1}\right) \quad (25)$$

Implementation of the numerical integrators to non-autonomous initial value problems

Tables 1-4 provide the numerical solution of the problem.

Table 1: y' = $4xy^{0.5}$, y (0.51) = 1.58785200119; h = 0.50.
The exact solution is y(x) = $(1.0 + x^2)^2$

x_n	Y_n	y_n	A_n	N_n	**Error**
0.51	1.58785200119	1.58785212040	0.0000000	4.0000000	0.00000012

Table 1: y' = $4xy^{0.5}$, y (0.51) = 1.58785200119; h = 0.50.
The exact solution is y(x) = $(1.0 + x^2)^2$

x_n	Y_n	y_n	A_n	N_n	Error
1.01	4.08080434799	4.08080434799	0.0000000	4.0000000	0.000000009
1.51	10.75905990601	10.75905704498	0.0000000	4.0000000	0.00000286
2.01	25.40261840820	25.40261268616	0.0000000	4.0000000	0.00000572
2.51	53.29144287109	53.29146575928	0.0000000	4.0000000	0.00002289
3.01	101.20552825928	101.20560455322	0.0000000	4.0000000	0.00007629
3.51	177.42483520508	177.42500305176	0.0000000	4.0000000	0.00016785
4.01	291.72933959961	291.72970581055	0.0000000	4.0000000	0.00036621
4.51	455.39901733398	455.39962768555	0.0000000	4.0000000	0.00061035
5.01	681.21380615234	681.21478271484	0.0000000	4.0000000	0.00097656
5.51	983.45373535156	983.45520019531	0.0000000	4.0000000	0.00146484
6.01	1377.89855957031	1377.90100097656	0.0000000	4.0000000	0.00244141
6.51	1881.82849121094	1881.83178710938	0.0000000	4.0000000	0.00329590
7.01	2514.02343750000	2514.02807617188	0.0000000	4.0000000	0.00463867
7.51	3294.76269531250	3294.76953125000	0.0000000	4.0000000	0.00683594
8.01	4245.82714843750	4245.83544921875	0.0000000	4.0000000	0.00830078
8.51	5390.50195312500	5390.50683593750	0.0000000	4.0000000	0.00488281
9.01	6753.56396484375	6753.56445312500	0.0000000	4.0000000	0.00048828
9.51	8361.29199218750	8361.28515625000	0.0000000	4.0000000	0.00683594
10.01	10241.4687500000	10241.45214843750	0.0000000	4.0000000	0.01660156
10.41	11512.46679687500	11512.44531250000	0.0000000	4.0000000	0.02148438

Table 2: y' = -(2y+ 5x), y (0) = 1.22139859; h = 0.5.
The exact solution is y(x) =$2e^{-x(\cos 2x+\sin 2x)}$

x_n	Y_n	y_n	A_n	N_n	Error
1.000	1.221398591995239	1.221300005912781	1.1957120	8.2365437	0.00000000000000
1.500	7.145311832427979	7.145311355590820	1.6957121	8.2365437	0.00000047683715
2.000	33.583660125732422	33.583660125732422	2.1957123	8.2365437	0.00000000000000
2.500	10.818999290466309	10.818999290466309	2.6957119	8.2365437	0.00000000000000
3.000	0.259470701217651	0.364375352859497	3.1957114	8.2365437	0.10490465164184
3.500	0.014336288906634	0.014336283318698	3.6957109	8.2365437	0.00000000558793
4.000	0.068405680358410	0.068405628204346	4.1957102	8.2365437	0.00000005215406
4.500	18.89082908630371	18.890829086303711	4.6957102	8.2365437	0.00000000000000
5.000	2015.454956054688	2015.4548339843750	5.1957097	8.2365437	0.00012207031250
5.500	477.6034240722656	477.60342407226562	5.6957092	8.2365437	0.00000000000000
6.000	0.316483914852142	0.316483795642853	6.1957088	8.2365437	0.00000011920929
6.500	0.000357522134436	0.000357417826308	6.6957083	8.2365437	0.00000010430812
7.000	0.000747847603634	0.000747424026486	7.1957078	8.2365437	0.00000042357714
7.500	4.542812347412109	4.542811393737793	7.6957073	8.2365437	0.00000095367431
8.000	42513.92578125000	42513.9257812500000	8.1957064	8.2365437	0.00000000000000
8.500	73420.17187500000	73420.1718750000000	8.6957083	8.2365437	0.00000000000000
9.000	4.523249149322510	4.523248672485352	9.1957102	8.2365437	0.00000047683715
9.500	0.000040130249545	0.000039512811782	9.6957121	8.2365437	0.00000061743776
10.000	0.000003663434882	0.000003205784424	10.1957140	8.2365437	0.00000045765045
10.500	0.096282377839088	0.096281319856644	10.6957159	8.2365437	0.00000105798244
11.000	131954.32812500000	131954.328125000000	11.1957178	8.2365437	0.00000000000000

Table 3: y' = -(2 y+ x), y (0) = 10; h = 0.5.
The exact solution is y(x) = 10(1-x)e^{-x}

x_n	Y_n	y_n	A_n	N_n	Error
0.0000	10.000000000000000	10.000000000000000	42.0000000	55.0000000	0.00000000000000
0.5000	3.032653331756592	3.032653331756592	42.8339920	55.3596306	0.00000000000000
1.0000	-0.000000438546408	0.000000000000000	43.6869812	55.7396126	0.00000043854640
1.5000	-1.115651011466980	-1.115650177001953	44.5606270	56.1417084	0.00000083446502
2.0000	-1.353352785110474	-1.353353500366211	45.4567909	56.5679016	0.00000071525573
2.5000	-1.231275081634522	-1.231273651123047	46.3775520	57.0204086	0.00000143051147
3.0000	-0.995741724967957	-0.995742797851563	47.3252563	57.5017281	0.00000107288360
3.5000	-0.754935145378113	-0.754936218261719	48.3025703	58.0146904	0.00000107288360
4.0000	-0.549469769001007	-0.549468994140625	49.3124962	58.5625000	0.00000077486038
4.5000	-0.388815402984619	-0.388816833496094	50.3584785	59.1487999	0.00000143051147
5.0000	-0.269518345594406	-0.269515991210938	51.4444389	59.7777748	0.00000235438346
5.5000	-0.183905124664307	-0.183906555175781	52.5749054	60.4542160	0.00000143051147
6.0000	-0.123937942087650	-0.123939514160156	53.7550926	61.1836700	0.00000157207250
6.5000	-0.082689419388771	-0.082687377929688	54.9910736	61.9725571	0.00000204145908
7.0000	-0.054713111370802	-0.054710388183594	56.2899284	62.8283958	0.00000272318720
7.5000	-0.035950627177954	-0.035949707031250	57.6599884	63.7599907	0.00000092014670
8.0000	-0.023482490330935	-0.023483276367188	59.1110954	64.7777634	0.00000078603625
8.5000	-0.015260177664459	-0.015258789062500	60.6549988	65.8941345	0.00000138860195
9.0000	-0.009872800670564	-0.009872436523438	62.3057785	67.1239624	0.00000036414712
9.5000	-0.006362405605614	-0.006362915039063	64.0804977	68.4852600	0.00000050943344
10.0000	-0.004085986874998	-0.004089355468750	66.0000076	70.0000076	0.00000336859375
10.4000	-0.002853393553456	-0.002853393554688	67.6569595	71.3402863	0.00000725104473

Table 4: y' = 10(y- x^3) + 3x^2 ; y(0)= 1.0; h = 0.1.
The exact solution is y(x) = x^3e^{10x}

x_n	$y(x)$	y_n	A_n	N_n	Error
0.00	1.000000000	1.000000000	16.76666641235	163.66667175293	0.0000
0.10	2.719281435	2.719280958	45.50468444824	450.04681396484	0.0000
0.20	7.397057533	7.397054672	123.45091247559	1228.50915527344	0.0000
0.30	20.112533569	20.112537384	335.15896606445	3344.58959960938	0.0000
0.40	54.662086487	54.662136078	910.46893310547	9096.68945312500	0.0000
0.50	148.537857056	148.538116455	2474.15185546875	24732.51953125000	0.0002
0.60	403.643676758	403.644653321	6724.51074218750	67235.10937500000	0.0009
0.70	1096.972167969	1096.975830078	18278.01367187500	182769.14062500000	0.0036
0.80	2981.456054688	2981.469238282	49683.51953125000	496823.21875000000	0.0131
0.90	8103.767578125	8103.810058594	135052.343750000	1350510.50000000000	0.0424
1.00	22027.322265625	22027.457031250	367108.718750000	3671073.2500000000	0.1347

Discussions

Proposed methods with the assumptions have subsequently used to determine the undetermined coefficients that were needed for the development of the numerical schemes. Further numerical analysis and solution to the problem has been evaluated and observing the error terms the results mimic very well the exact values.

Conclusion

A class of formulae as models to integrate initial value problem has been developed and implemented in this study. These models directly determine the oscillatory parameters in a system of IVP, considering non-autonomous initial value problems only. The numerical solutions compare favorably well with the exact values.

Owing to these results, it can be concluded that the proposed methods can be used in both theoretical and practical applications of solving climate change problems. In addition, integrators can be developed into user friendly software that can be used to study the frequency of global emissions, excitation energies of the molecular composition of components that contribute to climate changes;

Data availability

This study considers numerically simulated data and has no ethical implication.

Conflicts of interest

All authors declare not to have any interests that could be perceived as conflicting to this research work.

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