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Implications of covariate induced test dependence on the diagnostic accuracy of latent class analysis in pulmonary tuberculosis

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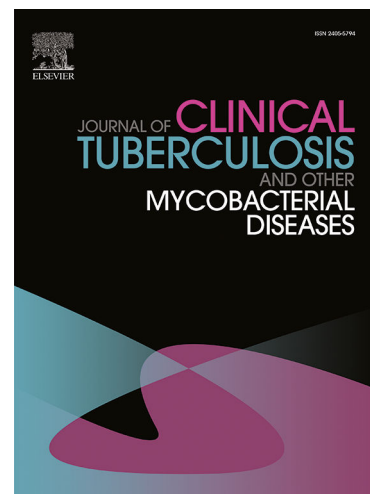
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1 **Implications of covariate induced test dependence on the diagnostic accuracy of latent class analysis in**
2 **pulmonary tuberculosis**

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55 **Abstract**

56 **Background** In application studies of latent class analysis (LCA) evaluating imperfect diagnostic tests, residual
57 dependence among the diagnostic tests still remain even after conditioning on the true disease status due to
58 measured variables known to affect prevalence and/or alter diagnostic test accuracy. Presence of severe
59 comorbidities such as HIV in pulmonary tuberculosis (PTB) diagnosis alter the prevalence of PTB and affect the
60 diagnostic performance of the available imperfect tests in use. This violates two key assumptions of LCA: (1) that
61 the diagnostic tests are independent conditional on the true disease status (2) that the sensitivity and specificity
62 remain constant across subpopulations. This leads to incorrect inferences.

63 **Methods** Through simulation we examined implications of likely model violations on estimation of prevalence,
64 sensitivity and specificity among passive case-finding presumptive PTB patients with or without HIV. Jointly
65 conditioning on PTB and HIV, we generated independent results for five diagnostic tests and analyzed using
66 Bayesian LCA with Probit regression, separately for sets of five and three diagnostic tests using four working
67 models allowing: (1) constant PTB prevalence and diagnostic accuracy (2) varying PTB prevalence but constant
68 diagnostic accuracy (3) constant PTB prevalence but varying diagnostic accuracy (4) varying PTB prevalence and
69 diagnostic accuracy across HIV subpopulations. Vague Gaussian priors with mean 1 and unknown variance were
70 assigned to the model parameters with unknown variance assigned Inverse Gamma prior.

71 **Results** Models accounting for heterogeneity in diagnostic accuracy produced consistent estimates while the model
72 ignoring it produces biased estimates. The model ignoring heterogeneity in PTB prevalence only is less problematic.
73 With five diagnostic tests, the model assuming homogenous population is robust to violation of the assumptions.

74 **Conclusion** Well-chosen covariate-specific adaptations of the model can avoid bias implied by recognized
75 heterogeneity in PTB patient populations generating otherwise dependent test results in LCA.

76 **Key words:** Sensitivity, Specificity, Prevalence, Tuberculosis, Simulation, Bayesian Latent Class Analysis

77

78 1. Introduction

79 Lack of a perfect reference standard complicates evaluation of new diagnostic tests and quantification of disease
80 prevalence. Ideally, new diagnostic tests are evaluated by comparison to a gold standard (GS) test that conclusively
81 determines the diagnosis. However, in practice, the GS test is rarely available. As a result, new diagnostic tests are
82 assessed by comparison to available imperfect reference tests. Due to the inherent limitation of imperfect reference
83 tests, discrepant resolution and composite reference standard methods were proposed to alleviate imperfect reference
84 standard bias.[1] Both methods, however, yield biased estimates.[2–4] Another promising approach is the use of
85 latent class analysis (LCA).[5–7] This approach is used for identifying unobserved subgroups in the population. [8]
86 It has enjoyed extensive application in many disciplines. [9] Over the past few decades, it has attracted attention in
87 biomedical field, including evaluation of diagnostic tests in the absence of a gold standard in the field of infectious
88 disease. [9,10]

89 Consider, for example, the diagnosis of pulmonary tuberculosis (PTB). The current conventional diagnostic methods
90 for PTB involve culture, smear microscopy, Xpert MTB/RIF, Xpert MTB/RIF Ultra, and imaging (chest X-ray,
91 Computed Tomography) in a patient with presumptive TB. Recently, Computer-Aided Detection for TB (CAD4TB)
92 and C-reactive protein (CRP) were proposed as triage tests in presumptive TB patients before ordering an expensive
93 but more accurate Xpert MTB/RIF.[11] Lateral Flow test for lipoarabinomannan (LAM) in urine is recommended
94 for diagnosis of TB in patients with advanced HIV disease.[12–14] The conventional reference standard for
95 diagnosis of PTB is culture for *Mycobacterium tuberculosis complex*. While culture is the most specific test
96 available, an imperfect sensitivity (76%–92%) is a limitation.[15] Thus, a negative culture test result does not rule
97 out the presence of TB. PTB diagnosis could use as few as two symptoms/tests e.g. ‘cough lasting more than two
98 weeks and chest X-ray’ or ‘any TB symptom and chest X-ray’, to more elaborate combinations of three e.g. ‘any TB
99 symptom, chest X-ray and Xpert MTB/RIF’ or four e.g. adding culture to the set.[16] In this context we consider
100 any TB symptom as a diagnostic test. A combination of tests that does not include TB symptoms in the set has also
101 been considered.[17] Using a combination of imperfect diagnostic tests as the reference standard will potentially
102 lead to biased estimates.[18] Composite reference standard (CRS) does not take into account the underlying
103 uncertainties attributable to each imperfect test while assessing the diagnostic accuracy of the new test. A detailed
104 discussion on the concerns of CRS has been provided elsewhere.[4] Alternatively, with such a set of test results
105 jointly available for a sample of patients, LCA allows not only for improved patient diagnosis but further allows

106 evaluation of the diagnostic tests themselves. It yields correct estimates of disease prevalence and diagnostic test
107 accuracy under nontrivial assumptions.[19] These strong assumptions are violated when a serious comorbidity
108 affects the diagnostic test accuracy and/or risk of the targeted disease. This then results in biased estimates of disease
109 prevalence and diagnostic test accuracy. [3,20–22] However, there is scanty evidence on the performance of latent
110 class models in the presence of differential diagnostic test accuracy induced by an observed external covariate that is
111 also associated with the risk of the targeted disease.

112 Previous authors in their work have adjusted for covariates known to influence diagnostic test accuracy based on
113 expert opinion, [22,23] some did not adjust for covariates [24] while others adjust for the effect of covariates on
114 disease prevalence only. [5,6,21] Thus, the differing approaches on how to conduct LCA leaves an important gap in
115 diagnostic test evaluation, especially in TB where factors such as HIV status, history of TB and malnutrition affect
116 the performance of Xpert MTB/RIF, TB symptoms and tuberculin skin test among others. [16,22,25] It is unclear
117 whether studies that fail to adjust for measured covariates as well as those that partly adjust for the effects of
118 measured covariates on diagnostic test accuracy only yield biased estimates while those that correctly adjust for the
119 effect of measured covariates have a better chance of obtaining correct inferences. Using simulation, we performed
120 Bayesian LCA separately for a set of three (any PTB symptom, CAD4TB, Xpert MTB/RIF) and a set of five
121 diagnostic tests (any PTB symptom, CRP, CAD4TB, Xpert MTB/RIF and culture) for PTB with the aim of
122 assessing the impact of covariate induced diagnostic test dependence on the performance of latent class models. We
123 evaluated the likelihood of four proposed models, representing common situations under which the standard
124 assumptions are violated for a set of three and a set of five diagnostic tests and offer recommendations for analysis.

125

126 **2. Simulation conditions: The generated data**

127 We generated data mimicking a setting of passive case-finding among presumptive PTB patients with or without
128 HIV. Our goal is to show the effect of residual dependence induced by a measured covariate on the diagnostic
129 performance of LCA after conditioning on the true PTB status and isolating the dependence between the diagnostic
130 tests attributable to other sources. Based on realistic sensitivities and specificities of five diagnostic tests for PTB
131 (any PTB symptom, CRP, CAD4TB, Xpert MTB/RIF and culture) we simulated independent test results conditional
132 on PTB and HIV (Table A.1 in Appendix A). We thus simulated 20% HIV+ patients with 5% PTB prevalence in

133 HIV- and 10% in HIV+, for an overall prevalence of 6%.[26,27] The accuracy used for culture was based on a
 134 composite reference standard of BACTEC 960/MGIT, BACTEC 460 and solid media [15] For the other diagnostic
 135 tests it was based on culture as the reference standard. The overall sensitivity (specificity) averages the test-related
 136 sensitivity (specificity) over the HIV subpopulations. Thus, the joint probability of the j^{th} diagnostic test $Y_j, j =$
 137 $1, 2, 3, \dots, J$, PTB status D and covariate (HIV status) X was generated using the following model

$$Pr(Y_j, D, X) = Pr(Y_j | D, X) Pr(D | X) Pr(X)$$

139 Hence for the set of test results under conditional independence given D and X:

$$Pr(Y_1, Y_2, \dots, Y_J, D, X) = \prod_{j=1}^J Pr(Y_j | D, X) Pr(D | X) Pr(X)$$

141 Where $Y_j = 1$ if the j^{th} test result is positive, 0 otherwise; $D = 1$ if the latent PTB status is positive, 0 otherwise;
 142 $X = 1$ if HIV status is positive (i.e HIV +), 0 otherwise.

143 We introduced the observed covariate X in the relevant models to handle dependence of diagnostic tests induced by
 144 this covariate.

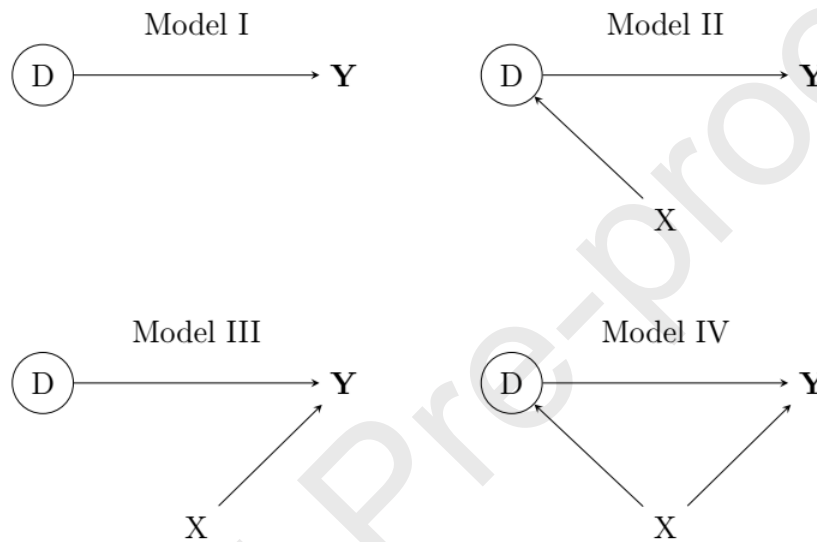
145 We thus generated three pseudo-random populations of 1000, 2000 and 5000 individuals with their true PTB and
 146 HIV status. Each of the three pseudo-random populations were replicated 100 times. The covariance and correlation
 147 structures are presented in Appendix A (Tables A.2–A.5).

148

149 3. Working Models

150 The standard two-class LCA assumes that the study population consists of at least two separate, internally
 151 homogenous latent classes. We consider a person's true PTB status consisting of two mutually exclusive and
 152 exhaustive categories: 'PTB' and 'non-PTB'. We acknowledge that this may not be true in practice because PTB
 153 status for an individual may be any of (1) active-TB (2) no TB (3) latent/subclinical TB. [28] However, we restrict
 154 ourselves to the case where we have two classes: PTB and non-PTB, for the purpose of assessing violation of model
 155 assumptions. The model further assumes that the result of one diagnostic test does not depend on the results of other
 156 tests (and persons) in the latent class, with a constant chance of error across individuals in a latent class, implying
 157 constant test sensitivity and specificity across subpopulations.[5] In practice, these standard latent class model

158 assumptions are violated, especially in the field of TB where, for example, HIV disease is known to influence the
 159 performance of some diagnostic tests including TB symptoms and Xpert MTB/RIF. To assess the effect of the
 160 measured covariate on the performance of latent class analysis, we analyzed the data using four working models:
 161 from most simple - with no HIV dependence - to the accurate (or complex) model representing the true model used
 162 to generate the data (Figure 1). These are variants of the standard two-class latent class model. Their detailed
 163 description is given in Appendix A.



→ Arrows indicate direction of effect

$\mathbf{Y} = \{Y_1, Y_2, \dots, Y_J\}$ – A vector of J diagnostic tests

D – PTB status

X – HIV Comorbidity/covariate

Model I – Model restricting PTB prevalence and the diagnostic test accuracy to remain constant across the HIV subpopulations

Model II – Model allowing PTB prevalence but not the diagnostic test accuracy to vary across the HIV subpopulations

Model III - Model restricting PTB prevalence but not the diagnostic test accuracy to remain constant across the HIV subpopulations

Model IV - Model allowing PTB prevalence and the diagnostic test accuracy to vary across the HIV subpopulations

164

165 Figure 1: Graphical presentation of the working models

166

167 The joint probability $Pr(Y_{i1} = y_{i1}, Y_{i2} = y_{i2}, Y_{i3} = y_{i3}, \dots, Y_{ij} = y_{ij} | X_i = x_i) = Pr(Y_i = \mathbf{y}_i | X_i = x_i)$ of observing a
 168 combination of J test results $y_{i1}, y_{i2}, y_{i3}, \dots, y_{ij}$ applied to the i^{th} individual, $i = 1, 2, 3, \dots, N$, was derived from the
 169 assumption of constant (or varying) PTB prevalence and diagnostic test accuracy across the HIV subpopulations as
 170 Model I: assuming independence of (Y, D) from X

$$171 \quad Pr(\mathbf{y}_i) = \sum_{d=0}^1 \prod_{j=1}^J Pr(Y_{ij} = y_{ij} | D_i = d) Pr(D_i = d)$$

172 Model II: assuming $Pr(Y_{ij} = y_{ij} | D_i = d, X_i = x_i) = Pr(Y_{ij} = y_{ij} | D_i = d)$

$$173 \quad Pr(\mathbf{y}_i | X_i = x_i) = \sum_{d=0}^1 \prod_{j=1}^J Pr(Y_{ij} = y_{ij} | D_i = d) Pr(D_i = d | X_i = x_i)$$

174 Model III: assuming $Pr(D_i = d | X_i = x_i) = Pr(D_i = d)$

$$175 \quad Pr(\mathbf{y}_i | X_i = x_i) = \sum_{d=0}^1 \prod_{j=1}^J Pr(Y_{ij} = y_{ij} | D_i = d, X_i = x_i) Pr(D_i = d)$$

176 Model IV

$$177 \quad Pr(\mathbf{y}_i | X_i = x_i) = \sum_{d=0}^1 \prod_{j=1}^J Pr(Y_{ij} = y_{ij} | D_i = d, X_i = x_i) Pr(D_i = d | X_i = x_i)$$

178

179 4. Analysis

180 We implemented Bayesian LCA to evaluate diagnostic test properties of a set of five diagnostic tests: any PTB
 181 symptom, CAD4TB, CRP, Culture and Xpert MTB/RIF. A subset of any PTB symptom, CAD4TB and Xpert
 182 MTB/RIF were also evaluated. The number of parameters to be estimated for LCA with five diagnostic tests is less
 183 than the number estimable from the degrees of freedom in the data. Hence the data could support estimation of
 184 disease prevalence and diagnostic accuracy of the five diagnostic tests. With three diagnostic tests, however, there
 185 are more parameters than degrees of freedom in the data. This introduces a statistical non-identifiability problem
 186 unless additional information enters, for instance through informative prior distributions for some parameters.[8]

187 The dependence of sensitivity and specificity on the covariate was expressed through a Probit model. Similarly, for
188 PTB prevalence. (Appendix A). Independent Gaussian priors $N(\mu, \sigma^2)$ with unknown variance $\sigma^2 \sim \text{InvGamma}$
189 ($\text{shape} = \alpha, \text{rate} = \beta$) were used to model the uncertainty in sensitivity and specificity as well as the PTB
190 prevalence.

191 Amongst the HIV- ($x=0$), sensitivity and specificity were assigned a normal prior with mean of 1 on the Probit scale
192 translating to 84% on the probability scale, for the prevalence this was mean -1 on the Probit scale translating to
193 16% on the probability scale. The difference in sensitivity and specificity and the difference in prevalence between
194 the HIV- and HIV+ subpopulations were assigned priors from normal distributions with mean 0 and unknown
195 variance. When evaluating five diagnostic tests, the variance parameters were assigned near-uninformative
196 $\text{InvGamma}(1.0^{-3}, 1.0^{-3})$ priors (Appendix A: Figures A.3–A.13). Given the identifiability issues when
197 evaluating three diagnostic tests, the variance parameters for prevalence, sensitivity and specificity were assigned
198 informative $\text{InvGamma}(2, 3)$ priors (Appendix A: Figures A.14–A.20). The variance parameters of the difference
199 in prevalence and the difference in sensitivity and specificity between the HIV- and HIV+ were assigned priors from
200 $\text{InvGamma}(3, 1)$. The values of the inverse Gamma distribution were chosen such that the variation in the estimate
201 would span the range of plausible values for the parameter (Appendix A: Table A.6, Figures A.1 and A.2). Marginal
202 sensitivity, specificity and prevalence were assigned priors similar to those of the HIV- subpopulation. Given the
203 lack of a perfect reference standard, correct informative priors for the parameters of the model may not be readily
204 known. Nonetheless, based on expert knowledge, using the most accurate imperfect reference standard a diagnostic
205 test that is promising for diagnosis of a disease often has a sensitivity and a specificity $>50\%$. Thus, we chose prior
206 distributions for sensitivity and specificity with mode around 84% on the probability scale that reflected the degree
207 of confidence in the performance of the diagnostic tests. The prior chosen for the prevalence was based on the
208 general understanding about the prevalence of the disease spanning a range of plausible values in the population
209 rather than knowledge of the actual estimate.

210 For each replicate dataset, we calculated the median of the posterior distribution of PTB prevalence, diagnostic test
211 sensitivity and specificity as our point estimate with the corresponding 95% credible intervals (95% CrI), defined as
212 2.5%–97.5% percentiles of the posterior distribution. For each combination of the four working models and three
213 sample sizes, we calculated the median of the distribution of posterior median estimates of the one hundred replicate

214 datasets. The corresponding 2·5% and 97·5% percentiles of the distribution of the one hundred posterior median
215 estimates were derived. These intervals were referred to as 95% reference intervals (95% RI). We also calculated the
216 mean and the corresponding 95% confidence intervals (95% CI) as well as the root mean squared error (RMSE)
217 from the distribution of the one hundred posterior median estimates. Using the lower and upper estimates of the 95%
218 CrI for each posterior distribution of the one hundred replicate datasets we derived the coverage rates. Posterior
219 inferences were based on 50000 Monte Carlo iterations with the first 25000 discarded as “burn-in”. Convergence in
220 model fitting was assessed by running three chains. In order to reduce autocorrelation between consecutive values in
221 the chain, every 10th iteration was saved (“thinning”).[29] Trace plots and Gelman-Rubin convergence statistic
222 $<1\cdot05$ were used to monitor mixing in the chains.[30] Trace plots for the posterior samples of the parameters
223 obtained from analysis of the first replicate dataset of size 1000, 2000 and 5000 using working model IV are
224 provided in Appendix A (Figures A.21–A.22). Analysis was implemented in R version 4.0.3 using R2jags package
225 for R version 4.0.3.[31,32]

226

227 5. Simulation Results

228 5.1 Pulmonary TB prevalence

229 In Table 1 we present the frequentist evaluation of the posterior distributions of total population pulmonary TB
230 (PTB) prevalence. *True values* as presented in Table 1 in this section and in the following sections refers to the
231 *actual values* used in the simulation. We present the frequentist median with 95% reference intervals (95% RI),
232 mean with 95% confidence intervals (95% CI) and the true value of the total population PTB prevalence for five and
233 three diagnostic tests analyzed using the four working models. We also present the root mean squared error (RMSE)
234 and coverage rates of the 95% credible intervals (95% CrI) around the median estimates of the posterior
235 distributions. All estimates are based on the analysis of one hundred replicate datasets. From this point going
236 forward we refer to the coverage rates of the 95% CrI around the median estimates of the posterior distributions as
237 coverages of the 95% CrI.

238 When evaluating five diagnostic tests, the working models accounting for heterogeneity in diagnostic test
239 performance (working models III and IV) as well as the model assuming homogeneous population produced
240 consistent estimates of the total population PTB prevalence. There was evidence of some systematic bias for smaller
241 sample size. The model assuming heterogeneity in PTB prevalence but constant diagnostic accuracy across the
242 subpopulations yielded systematically biased but consistent estimates of total population PTB prevalence.

243 In the evaluation of three diagnostic tests, working models I and II yielded systematically biased estimates of the
244 total population PTB prevalence. Model II yielded large RMSE and poor coverages of 95% CrI. Working models III
245 and IV yielded consistent estimates of total population PTB prevalence with modest systematic bias.

246

247 Table 1: Frequentist evaluation of Bayesian estimates of total population pulmonary tuberculosis (PTB) prevalence
 248 obtained using four working models in the analysis of five and three diagnostic test results

			Five diagnostic tests			
Model	N	True value	Median (95% RI)	Mean (95% CI)	RMSEx100	Coverage
I	1000	6.0	6.3 (4.4, 8.9)	6.4 (6.1, 6.6)	1.2	95.0
	2000	6.0	6.1 (5.0, 7.3)	6.1 (6.0, 6.2)	0.6	95.0
	5000	6.0	6.0 (5.2, 6.9)	6.9 (5.2, 8.6)	8.8	93.0
II	1000	6.0	7.3 (5.5, 10.6)	7.5 (7.1, 7.8)	2.3	81.0
	2000	6.0	6.5 (5.1, 7.9)	6.5 (6.4, 6.6)	0.8	90.0
	5000	6.0	6.2 (5.4, 6.9)	6.2 (6.1, 6.3)	0.4	93.0
III	1000	6.0	6.4 (4.4, 11.4)	6.6 (6.3, 6.9)	1.7	94.0
	2000	6.0	6.0 (4.7, 7.4)	6.0 (5.9, 6.1)	0.7	93.0
	5000	6.0	5.9 (5.2, 6.7)	5.9 (5.9, 6.0)	0.4	95.0
IV	1000	6.0	6.7 (4.7, 9.5)	6.8 (6.5, 7.1)	1.6	93.0
	2000	6.0	6.2 (4.8, 7.7)	6.3 (6.1, 6.4)	0.7	93.0
	5000	6.0	6.1 (5.3, 6.9)	6.1 (6.0, 6.2)	0.4	94.0
			Three diagnostic tests			
Model	N	True value	Median (95% RI)	Mean (95% CI)	RMSEx100	Coverage
I	1000	6.0	5.7 (3.0, 16.2)	6.6 (5.9, 7.3)	3.4	96.0
	2000	6.0	6.1 (4.0, 13.1)	6.6 (6.2, 7.1)	2.4	98.0
	5000	6.0	6.7 (4.4, 12.7)	7.3 (6.8, 7.7)	2.4	94.0
II	1000	6.0	23.1 (7.0, 39.8)	23.4 (21.7, 25.1)	19.4	17.0
	2000	6.0	23.0 (14.6, 37.3)	24.0 (22.8, 25.2)	19.0	0.0
	5000	6.0	25.5 (18.7, 36.3)	26.1 (24.8, 27.4)	21.2	0.0
III	1000	6.0	4.9 (2.7, 12.9)	5.3 (4.8, 5.8)	2.5	94.0
	2000	6.0	5.0 (3.2, 8.5)	5.3 (5.0, 5.5)	1.6	98.0
	5000	6.0	5.6 (3.8, 10.5)	5.9 (5.6, 6.2)	1.6	92.0
IV	1000	6.0	5.1 (2.9, 15.2)	5.7 (5.1, 6.4)	3.2	96.0
	2000	6.0	5.4 (3.5, 8.0)	5.5 (5.2, 5.7)	1.3	99.0
	5000	6.0	5.6 (4.1, 8.2)	5.7 (5.4, 5.9)	1.2	95.0

249 N – Sample size

250 RI – Reference Intervals and was calculated as the 2.5% and 97.5% percentiles of the distribution of median
 251 estimates of the posterior distributions from the one hundred replicate datasets

252 CI – Confidence Intervals

253 RMSE – Root Mean Square Error

254 Five diagnostic tests: any PTB symptom, CAD4TB, CRP, culture and Xpert MTB/RIF

255 Three diagnostic tests: any PTB symptom, CAD4TB and Xpert MTB/RIF

256 Model I – Model restricting PTB prevalence and the diagnostic test accuracy to remain constant across the HIV
 257 subpopulations

258 Model II – Model allowing PTB prevalence but not the diagnostic test accuracy to vary across the HIV
 259 subpopulations

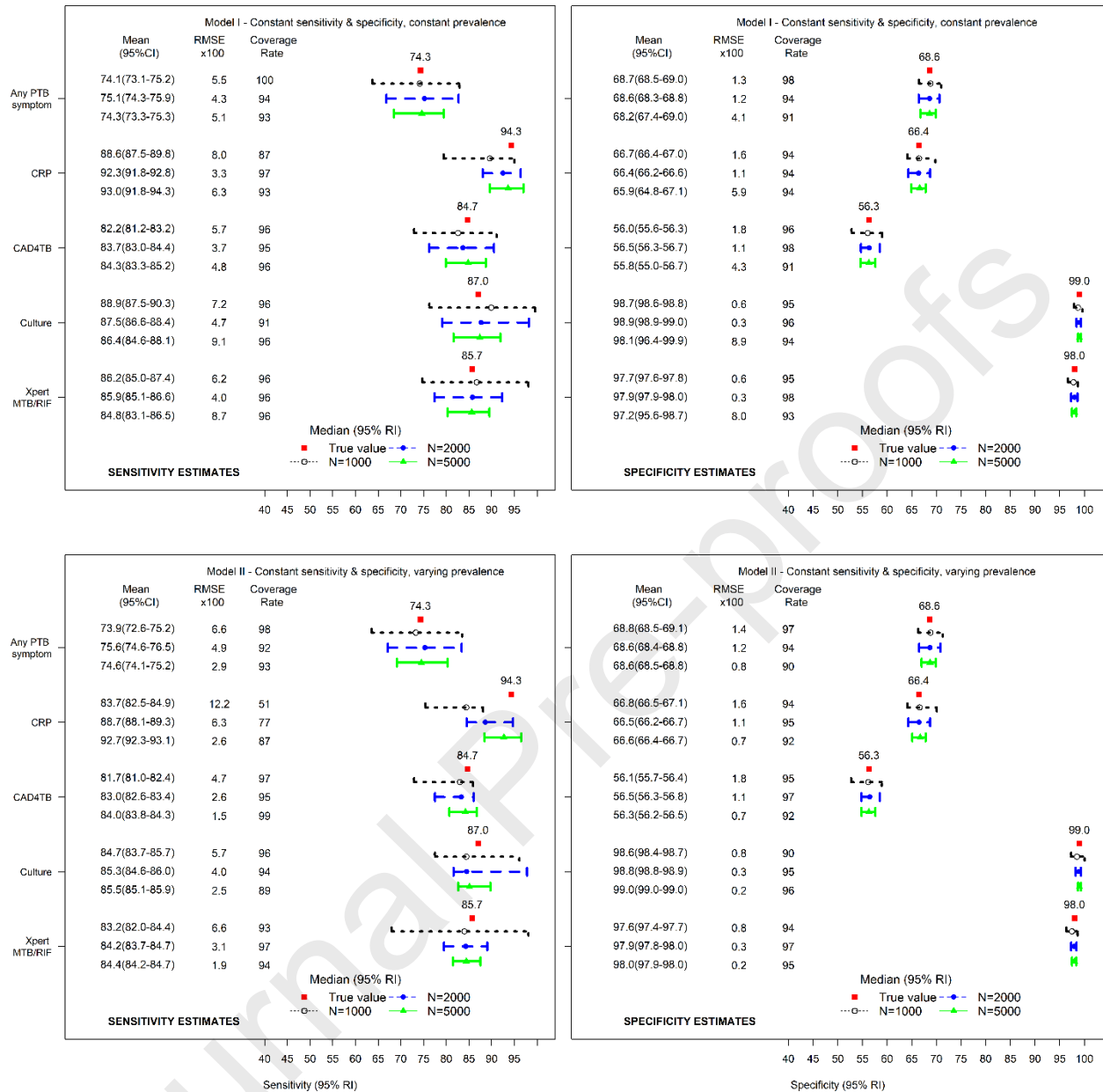
260 Model III - Model restricting PTB prevalence but not the diagnostic test accuracy to remain constant across the HIV
 261 subpopulations

262 Model IV - Model allowing PTB prevalence and the diagnostic test accuracy to vary across the HIV subpopulations

263 **5.2 Sensitivity and specificity of the diagnostic tests**

264 **5.2.1 Evaluation of five diagnostic tests**

265 Figure 2 presents the estimates of sensitivity and specificity for five diagnostic tests analyzed using working models
266 I and II. The models produced asymptotically consistent estimates of the total population sensitivity and specificity
267 with small systematic bias. The RMSE were good with acceptable coverages of the 95% credible intervals (95%
268 CrI). Working model II, however, yielded estimates of sensitivity for CRP that were different from the true value
269 with tendency towards the mean of prior distribution.



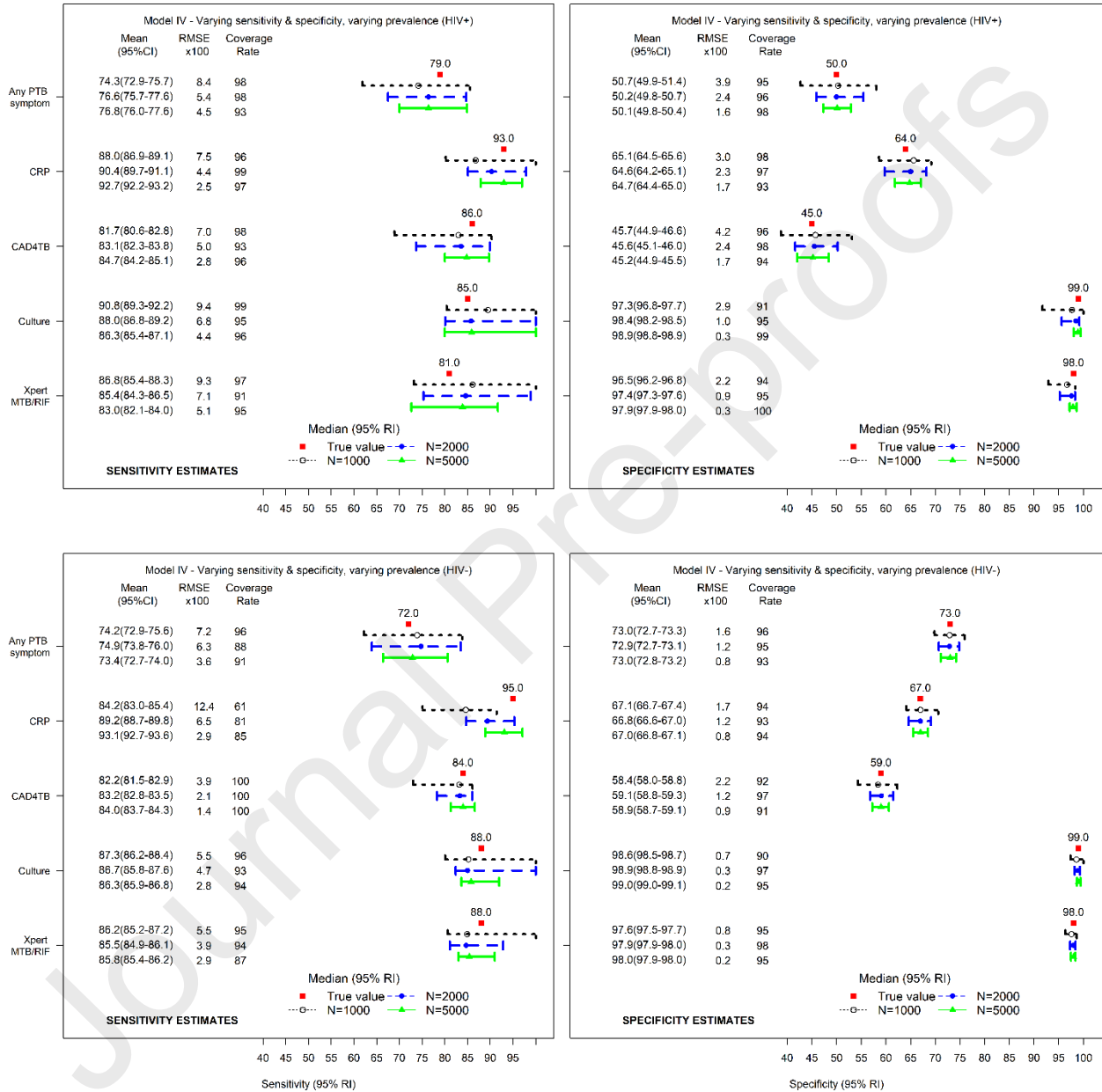
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271 Figure 2: Median (95% reference intervals (RI)) and mean (95% confidence intervals (CI)) estimates of total
 272 population sensitivity (left) and specificity (right) with corresponding root mean squared error (RMSE) and
 273 coverages of 95% credible intervals (CrI) for true total population sensitivity and specificity for five diagnostic tests
 274 evaluated using working model I (top panel) and working model II (lower panel) – Working model I restricts the
 275 diagnostic test accuracy and disease prevalence to remain constant across the HIV subpopulations, Working model
 276 II restricts the diagnostic test accuracy to remain constant but allows the disease prevalence to vary across the HIV
 277 subpopulations

278

279 Figure 3 presents the estimates of sensitivity and specificity by HIV status for five diagnostic tests evaluated using
 280 working model IV (true model). The model yielded estimates of sensitivity that matched the true values. The

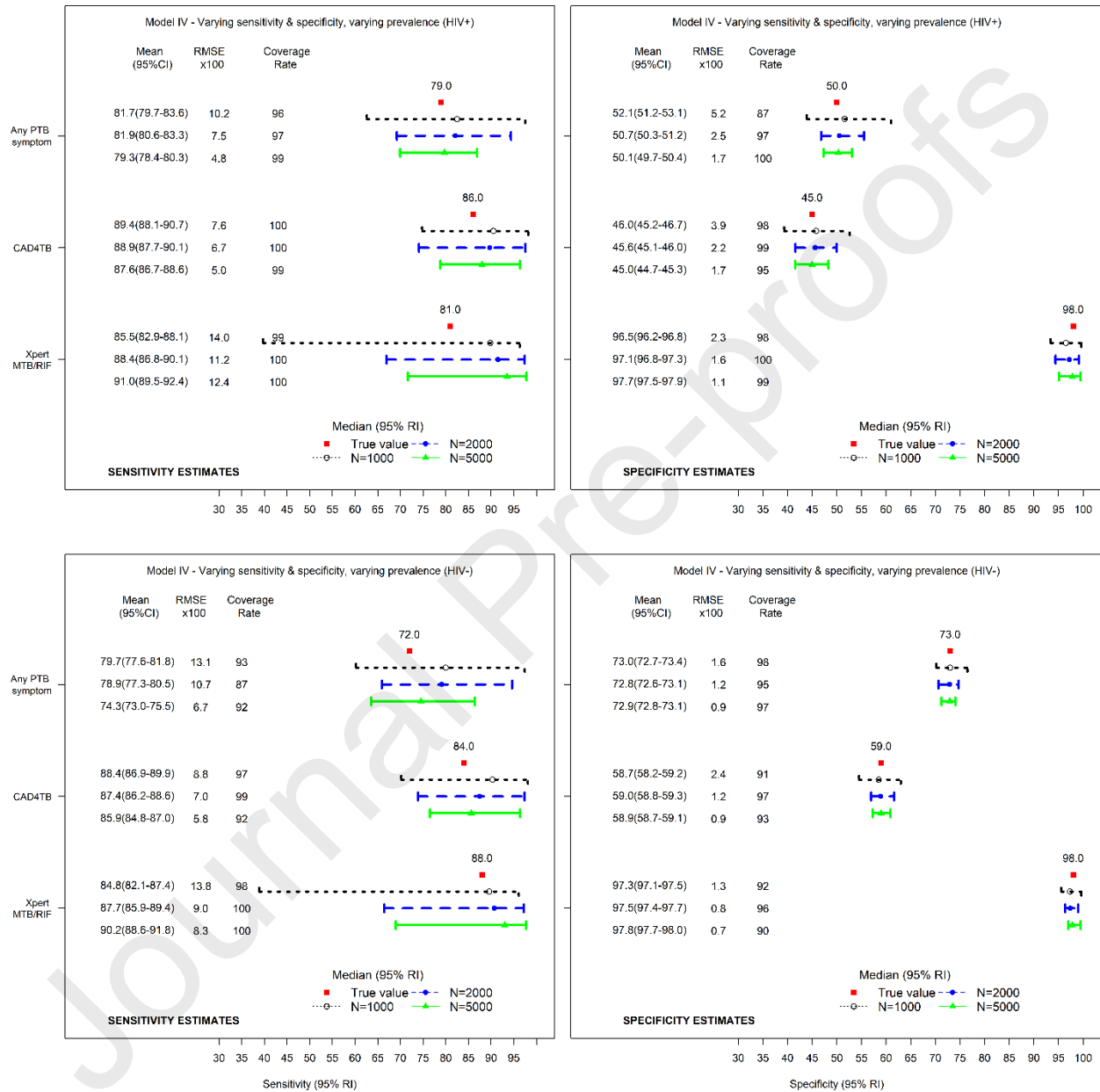
281 estimates of sensitivity among the HIV- were skewed in the direction of the prior. There was no evidence of serious
 282 systematic bias in the estimates of specificity. Similar findings were obtained using working model III (Figure B.1 in
 283 Appendix B).



284
 285 Figure 3: Median (95% reference intervals (RI)) and mean (95% confidence intervals (CI)) estimates of sensitivity
 286 (left) and specificity (right) for HIV+ (top panel) and HIV- (lower panel) with corresponding root mean squared
 287 error (RMSE) and coverages of 95% credible intervals (CrI) for true sensitivity and specificity for five diagnostic
 288 tests evaluated using the model allowing the diagnostic test accuracy and disease prevalence to vary across the HIV
 289 subpopulations (working model IV)
 290

291 **5.2.2 Evaluation of three diagnostic tests**

292 Figure 4 shows the estimates of sensitivity and specificity by HIV status for three diagnostic tests evaluated using
 293 working model IV (true model).



294

295 Figure 4: Median (95% reference intervals (RI)) and mean (95% confidence intervals (CI)) estimates of sensitivity
 296 (left) and specificity (right) for HIV+ (top panel) and HIV- (lower panel) with corresponding root mean squared
 297 error (RMSE) and coverages of 95% credible intervals (CrI) for true sensitivity and specificity for three diagnostic
 298 tests evaluated using the model allowing the diagnostic test accuracy and disease prevalence to vary across the HIV
 299 subpopulations (working model IV)

300

301 The estimates of sensitivity and specificity indicate some systematic bias. Similar findings were obtained using
302 working model III (Figure B.3 in Appendix B). Figure B.2 in Appendix B shows the estimates of sensitivity and
303 specificity for three diagnostic tests analyzed using working models I & II.

304 **6. Discussion**

305 Our aim was to investigate implications of violation of model assumptions induced by an observed external
306 covariate that is associated with diagnostic test accuracy and risk of the targeted disease. We assessed some likely
307 model violations on estimation of total population prevalence of the disease, sensitivity and specificity. We
308 supported our results with finite sample simulations mimicking a setting of passive case-finding among presumptive
309 pulmonary tuberculosis (PTB) patients with or without HIV. Based on realistic sensitivities and specificities of five
310 diagnostic tests used for PTB, we simulated independent test results in samples of various sizes with different PTB
311 prevalence within the HIV subpopulations. Due to instability of the estimates with small sample size, we endeavored
312 to be as realistic as possible by choosing different sample sizes (1000, 2000, 5000) to help us evaluate the
313 performance of LCA when the number of true PTB cases is as low as 60 (20 in the HIV+ and 40 in the HIV-
314 subpopulations) when N=1000 and when it is as high as 300 (100 in the HIV+ and 200 in the HIV- subpopulations)
315 when N=5000 with 6% overall TB prevalence (5% in HIV- and 10% HIV+). For five and three diagnostic tests, we
316 performed Bayesian LCA using four working models assuming constant (or varying) PTB prevalence and diagnostic
317 test accuracy across the HIV subpopulations. We have shown that in the analysis of five and three diagnostic tests
318 the model ignoring heterogeneity in diagnostic test accuracy but allowing the prevalence of PTB to vary across the
319 subpopulations (working model II) produced systematically biased estimates of total population PTB prevalence and
320 diagnostic test accuracy. However, the models accounting for heterogeneity in diagnostic test accuracy across the
321 subpopulations (working models III and IV) yielded consistent estimates with modest systematic bias.

322 Working models I and II violated the assumption of conditional independence when the diagnostic test accuracy was
323 restricted to remain constant. When used to evaluate five diagnostic tests, working model I appeared robust to
324 violation of the assumption of conditional independence. Working model II yielded systematically biased but
325 consistent estimates. Working models III and IV produced consistent estimates of total population PTB prevalence
326 and modestly biased estimates of sensitivity with greater uncertainty. The specificity estimates matched the true
327 values while the sensitivity estimates were skewed in the direction of the prior in the HIV- subpopulation. With

328 small sample size (few cases with PTB) Bayesian estimation is driven more by the prior rather than the likelihood.
329 This finding emphasizes the need to carefully choose the prior distribution as alluded to by others.[33–35] An
330 additional analysis evaluating three diagnostic tests using the same working models but different priors revealed the
331 unavoidable dependency of the results on the (informative) prior (Table B.1 and Figures B.4 – B.6 in Appendix B).
332 In our analyses we chose prior distributions that reflected the degree of confidence in the performance of the
333 diagnostic tests and the general understanding about the prevalence of the disease rather than knowledge of the
334 actual estimate. This was intentional to avoid presuming knowledge of the performance of the diagnostic tests given
335 the lack of a gold standard.

336 In the analysis of three diagnostic tests, working models I and II yielded systematically biased estimates of total
337 population PTB prevalence. The models also produced systematically biased and highly unstable estimates of total
338 population sensitivity. Thus, Bayesian LCA with fewer diagnostic tests that violate the assumption of constant
339 diagnostic test accuracy across the underlying subpopulations may suffer from limited information that contribute to
340 bias as established by others.[20,33,34,36] Using working models III and IV demonstrated modest bias in the
341 sensitivity but reliable estimates of specificity. Failure to account for varying disease prevalence in working model
342 III did not noticeably impact the estimates of diagnostic test accuracy.

343 Residual dependence induced by a measured covariate remains even after conditioning analysis on the latent disease
344 status. This leads to incorrect inferences. Potential remedies to such problems in real studies was evaluated through
345 simulations. Though not applied to real dataset, this may not be viewed as a weakness of the study but should serve
346 as a guide to experts intending to apply LCA to carefully consider plausibility of the model, especially in TB where
347 severe comorbidities are known to affect diagnostic test performance. LCA uses all the available imperfect
348 diagnostic tests, including symptoms, to determine the likelihood of the presence of PTB for an individual.
349 Therefore, incorrectly specified model not only yields biased inferences for diagnostic test accuracy and disease
350 prevalence but also contributes to incorrect diagnosis and treatment of cases. This has serious implications in terms
351 of allocation of resources, unnecessary harm to individuals without the disease, and onward transmission of
352 infectious disease by those missed due to incorrect diagnosis. Our approach reveals the need for a rigorous process
353 that involves experts in the field of study. Besides their knowledge on the diagnostic tests known to be dependent
354 conditional on the (unknown) disease status, their input regarding potential covariates that affect the disease

355 prevalence as well as the diagnostic test accuracy can be harnessed and incorporated into the model. In addition,
356 correct statistical methods can be used to evaluate the importance of the proposed covariates in influencing disease
357 prevalence and the diagnostic test accuracy. All these ideas put together should yield a plausible model that best
358 explains the diagnostic accuracy of the tests and the prevalence of the disease.

359

360 7. Conclusion

361 In the presence of measured covariates known to affect the diagnostic accuracy and disease prevalence, experts
362 should avoid the model that allows the disease prevalence to vary but restricts the diagnostic test sensitivity and
363 specificity to remain constant across the different subpopulations. This model yield severely biased estimates of
364 PTB prevalence and diagnostic test accuracy. The model that allows the disease prevalence to remain constant but
365 allows the diagnostic test sensitivity and specificity to vary across the different subpopulations yields correct
366 estimates of overall disease prevalence (averaged across the different subpopulations) and the subpopulation specific
367 estimates of sensitivity and specificity. The model that allows disease prevalence and diagnostic test sensitivity and
368 specificity to vary across the different subpopulations defined by the covariates known to induce test dependence
369 should be applied. When the interest is also to understand the drivers of disease prevalence then this model should
370 be applied. In the absence of measured covariates or when the conditions do not allow adjusting for covariates due
371 to small sample size (or few PTB cases), the model that allows the disease prevalence and the diagnostic test
372 sensitivity and specificity to remain constant across the different subpopulations can be applied since it would yield
373 less biased estimates.

374 In light of these findings, we recommend diagnostic studies to be as inclusive as possible in collecting important
375 covariates known to influence diagnostic test performance e.g HIV status, history of TB treatment, miners etc.
376 Because of the obvious concerns regarding imperfect reference standard, correctly specified latent class model
377 should be used to evaluate new diagnostic tests as well as determine disease prevalence. Interpretation of results
378 based on small sample sizes should be done carefully since they may lack precision. We saw a potential influence of
379 the prior distribution on the posterior estimates of sensitivity attributed to small sample size. Therefore, correct
380 choice of the prior for modelling uncertainty in diagnostic test sensitivity and prevalence is imperative, particularly
381 for few diagnostic tests or small sample sizes. Different experts have applied different latent class models, some

382 adjusting for measured covariates and others failing to do so. Therefore, following robust model evaluation, our
383 work provides an invaluable guidance on the correct approach for analysis of imperfect diagnostic tests in the
384 presence of a measured covariate that affects the prevalence of the disease and/or diagnostic accuracy of the tests.
385 Thus our findings complement the findings of the already published work. [37] Future research should look into
386 predictive models that can promptly give correct diagnosis for an individual based on clinical history, diagnostic test
387 results and measured covariates.

388 **Availability of data and materials**

389 The parameters used in the simulation have been provided in Table A.1 in Appendix A. The simulation as well the
390 analysis scripts written in R language can be obtained from the corresponding author upon request.

391

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397

398 **CRedit authorship contribution statement**

399 **Alfred Kipyegon Keter:** Conceptualized the idea, developed the methodology, performed simulation, analysis and
400 interpretation of results. Wrote the original manuscript draft. **Lutgarde Lynen:** Participated in grant acquisition,
401 conceptualization of the idea, development of the methodology, and review and editing of the manuscript. **Alastair**
402 **Van Heerden:** Participated in grant acquisition, conceptualization of the idea, development of the methodology, and
403 review and editing of the manuscript. **Els Goetghebeur:** Conceptualization of the idea, development of the
404 methodology, and review and editing of the manuscript. **Bart K. M. Jacobs:** Conceptualization of the idea,
405 development of the methodology, review of simulation and analysis, and review and editing of the manuscript.
406 All authors read and approved the final manuscript.

407

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409

Journal Pre-proofs

410 **References**

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413 [sim205>3.0.co;2-b](https://doi.org/10.1002/(sici)1097-0258(19991130)18:22<2987::aid-sim205>3.0.co;2-b).
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504

505 **Highlights**

- 506 • Residual dependence induced by a measured covariate remains even after conditioning analysis on the latent
507 disease status. This violates the key assumptions of latent class analysis (LCA) hence incorrect inferences.
- 508 • Models accounting for heterogeneity in diagnostic test accuracy induced by the covariate yield realistic
509 estimates
- 510 • Experts intending to apply LCA should carefully consider plausibility of the model, especially in TB where
511 severe comorbidities are known to affect diagnostic test performance
- 512 • Covariate-adjusted LCA alleviate bias implied by heterogeneity in diagnostic test accuracy. Therefore, we
513 recommended it.

514

515

516

517 **CRedit authorship contribution statement**

518 **Alfred Kipyegon Keter:** Conceptualized the idea, developed the methodology, performed simulation, analysis and
519 interpretation of results. Wrote the original manuscript draft. **Lutgarde Lynen:** Participated in grant acquisition,
520 conceptualization of the idea, development of the methodology, and review and editing of the manuscript. **Alastair**
521 **Van Heerden:** Participated in grant acquisition, conceptualization of the idea, development of the methodology, and
522 review and editing of the manuscript. **Els Goetghebeur:** Conceptualization of the idea, development of the
523 methodology, and review and editing of the manuscript. **Bart K. M. Jacobs:** Conceptualization of the idea,

524 development of the methodology, review of simulation and analysis, and review and editing of the manuscript. All
525 authors read and approved the final manuscript.

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